Problem_HW2_Group5

Question 1:
Consider four independent rolls of a 6-sided die. Let X be the number of 1's and let Y be the number of 2's obtained. What is the joint PMF of X and Y ?

Answer 1:

$$p_Y(y) = \binom{4}{y}\left(\frac{1}{6}\right)^y\left(\frac{5}{6}\right)^{4-y}, \qquad y = 0, 1, \ldots, 4.$$

To compute the conditional PMF $p_{X|Y}$, note that given that $Y = y$, X is the number of 1's in the remaining $4 - y$ rolls, each of which can take the 5 values $1, 3, 4, 5, 6$ with equal probability $1/5$. Thus, the conditional PMF $p_{X|Y}$ is binomial with parameters $4 - y$ and $p = 1/5$:

$$p_{X|Y}(x \mid y) = \binom{4-y}{x}\left(\frac{1}{5}\right)^x\left(\frac{4}{5}\right)^{4-y-x},$$

for all nonnegative integers x and y such that $0 \le x + y \le 4$. The joint PMF is now given by

$$\begin{aligned} p_{X,Y}(x, y) &= p_Y(y)p_{X|Y}(x \mid y) \\ &= \binom{4}{y}\left(\frac{1}{6}\right)^y\left(\frac{5}{6}\right)^{4-y}\binom{4-y}{x}\left(\frac{1}{5}\right)^x\left(\frac{4}{5}\right)^{4-y-x}, \end{aligned}$$

Question 2:
Chen is a hard-working sophomore. Now he is facing 5 exams next week but his time only allow him to prepare three of them. "Probability", "partial differential equation", "electromagnetism", "discrete math" and "signal and system" if he didn't prepare for the subject is always half chance to pass. After studying can he increase the chance to 0.6, 0.6, 0.7, 0.8, 0.9 respectively. By passing course can gain 3 credits each. And Chen chooses which subjects to read equally.
Given that he has given up studying "discrete math". And he has worked hard on "probability". what is the expectation he get credits ?(assume result only depend on this exam)

Answer 2:

Let X be the credit he get
E[X] =
= 3 * ((0.6) + (1/3*0.5+2/3*0.6) + (1/3*0.5+2/3*0.7) + (0.5) + (1/3*0.5+2/3*0.9))
= 9.2

He can get 9.2 credits as expectation.

Question 3:

In a game where we draw one card out of 32 cards. We consider the following to describe the game: 1) If we pick a heart up, we win 2 NTD. 2) If we draw a king, we win 5 NTD. 3) If we draw another card, we lose 1 NTD. Let X be the random variable that a drawn card associates a gain or a loss.

(a) Determine the PMF of X.

(b) calculate the mean, variance and standard deviation of the probability distribution of X.

Answer 3:

(a)

The random variable can take value 2, 5, -1 and 7.

if the card drawn is a heart (except the king of hearts), X = 2.

P(X=2) =7/32

if the drawn card is a king (except the king of hearts), X=5.

P(X=2) 3/32

if the drawn car is the king of hearts, X = 7.

P(X=7) = 1/32

if the drawn card is neither a heart nor a king X = -1.

P(X=-1) = 21/32

b) Find Expected value, variance and standard deviation

$$E(X) = \frac{21}{32} \times (-1) + \frac{7}{32} \times (2) + \frac{3}{32} \times (5) + \frac{1}{32} \times (7) = \frac{15}{32}$$

$$Var(X) = \frac{21}{32} \times \left(-1 - \frac{15}{32}\right)^2 + \frac{7}{32} \times \left(2 - \frac{15}{32}\right)^2 + \frac{3}{32} \times \left(5 - \frac{15}{32}\right)^2 + \frac{1}{32} \times \left(7 - \frac{15}{32}\right)^2 \approx 5.1865$$

$$\sigma(X) = \sqrt{5.1865} = 2.28$$

Question 4:

In a certain area, there are m foxes and n rabbits. During a period of time, the probability of each rabbit gets sick is 0.5, and the probability of each fox successfully catches a rabbit is 0.4. The foxes

don’t catch the sick rabbits, and it is sure that each of them tries to catch rabbit one time during the whole period. If a rabbit gets sick or being caught by a fox, it absolutely dies. After a period of time, there are only k rabbits left.
Please derive the PMF of the rabbit dies that’s because of the fox.

Answer 4:
Let X be the number of rabbits die of fox
Let A be the number of rabbits die

$$p_{X|A}(x) = \frac{C_x^m(0.5*0.4)^x(1-0.5*0.4)^{m-x}C_{n-k-x}^{n-x}(0.5)^{n-k-x}(0.5)^k}{C_{n-k}^n(0.5+0.5*0.4)^{n-k}[1-(0.5+0.5*0.4)]^k}$$
$$= \frac{C_x^m(0.2)^x(0.8)^{m-x}C_{n-k-x}^{n-x}(0.5)^{n-x}}{C_{n-k}^n(0.7)^{n-k}(0.3)^k}$$

Question 5:
Let’s say we want to send 4 bits of data. We make the transmission more reliable by adding 3 error correction parity bits, sending 7 bits in total. Error can be corrected and the data is still sent successfully, if there’s at most 1 bit corrupted (flipped) among the 7 bits. Each bit is independently corrupted with probability 0.05.

(a) What is the probability that the data is sent successfully with error correction?
(b) What about the probability without error correction (sending just 4 bits)?

Answer 5:

(a)
Let X be the number of bits corrupted.
$X \sim Bin(7,\ 0.05)$

$$P(X=0) = \binom{7}{0}(0.05)^0(0.95)^7 \approx 0.698$$

$$P(X=1) = \binom{7}{1}(0.05)^1(0.95)^6 \approx 0.257$$

$$P(\text{success}) = P(X=0) + P(X=1) \approx 0.955$$

(b)

$$P(\text{success}) = P(X=0) = \binom{4}{0}(0.05)^0(0.95)^3 \approx 0.815$$