Question 1:

On the two-dimension coordinate, Sam and Daniel throw the dice twice respectively. The first throw represents the position on x-axis and second represents y-axis, what is the probability that Sam and Daniel form a straight line with the origin? (Supposed that Sam and Daniel are at different position)

Answer 1:

To form the straight line, Sam and Daniel should have the same slope.

a. m=1 → (1,1), (2,2), (3,3), (4,4), (5,5), (6,6)
b. m=2 → (1,2), (2,4), (3,6)
c. m=1/2 →(2,1), (4,2), (6,3)
d. m=3 →(1,3), (2,6)
e. m=1/3 →(3,1), (6,2)
f. m=2/3 →(3,2),(6,4)
g. m=3/2 →(2,3),(4,6)

Above are the positions that may form a straight line.

$$P = \frac{\binom{6}{2}*2!+\binom{3}{2}*2!*2+\binom{2}{2}*2!*2+\binom{2}{2}*2!*2}{6*6*6*6} = \frac{25}{648}$$

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Question 2 :

You have one 4x4 Rubik's Cube, each point (x,y,z) was marked from (0,0,0) , (1/4,0,0) to (1,1,1)
You also have a smooth cube, there are infinite point in each side.

What is the probability about x + y + z <= 1 and (x,y,z) = (1/4,1/2,3/4)
For Rubik's Cube and smooth cube

Answer 2 :

Rubik's Cube

(1) 26/75
(2) 1/75
(3)
Smooth cube

(1) 1/6
(2) We can’t assign probability to specific outcome.

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Question 3 :

A child's DNA is (A, a). There are two candidates of her father, F_1(a, a) and F_2(A, a), F_1's probability of being the child's father is p, and the other one is 1-p. Now knowing that the child's mom's DNA is (A, A), find the probability that F1 is the child's father.

Answer 3 :

Mi denotes the event Fi is the child's dad.
N denotes the event of the child's DNA is (A, a)

$$P(M_1|N) = \frac{P(N|M_1)P(M_1)}{P(N|M_1)P(M_1) + P(N|M_2)P(M_2)}$$
$$= \frac{1*p}{1*p+(\frac{1}{2})(1-p)}$$
$$= \frac{2p}{1+p}$$

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Question 4 :

Suppose there are six dice in a box, one of which is called the "all-6-die", having 6 on all sides, another is called the "all-1-die", having 1 on all sides, the other four dice are all regular, with each face having distinct numbers from 1 to 6. A person reaches into the box and picks two dice and tosses it, the sum of the 2 numbers facing up is 7, what is the probability for the 2 dice being the "all-6-die" and "all-1-die" ?

Answer 4 :

Our problem involves 3 cases,
Case 1: numbers 1 and 6 facing up
Case 2: numbers 2 and 5 facing up
Case 3: numbers 3 and 4 facing up

For Case 1, the person has a $\frac{1}{15}$ chance of choosing the "all-6-die" and "all-1-die". In this case, the chances of seeing numbers 1 and 6 facing up is 1. The person also has a $\frac{2}{5}$ chance of choosing two regular dice. In this case, the chances of seeing numbers 1 and 6 facing up is $\frac{1}{18}$. The person also has a $\frac{4}{15}$ chance of choosing a regular die and the "all-6-die". In this case, the chances of seeing numbers 1 and 6 facing up is $\frac{1}{6}$. The person also has a $\frac{4}{15}$ chance of choosing a regular die and the "all-1-die". In this case, the chances of seeing numbers 1 and 6 facing up is $\frac{1}{6}$. Therefore, the probability for Case 1 is $\frac{8}{45}$.

For Case 2, the person has a $\frac{2}{5}$ chance of choosing two regular dice. In this case, the chances of seeing numbers 2 and 5 facing up is $\frac{1}{18}$. Therefore, the probability for Case 2 is $\frac{1}{45}$.

Case 3 has the same probability as Case 2.

To sum up, the probability for the sum of the two numbers being 7 is $\frac{8}{45}+\frac{1}{45}+\frac{1}{45}=\frac{2}{9}$. The probability for the sum of the two numbers being 7 and the two dice being the "all-6-die" and "all-1-die" is $\frac{1}{15}$. Therefore, our answer is $\frac{3}{10}$.

Case 1:

$$\left(\frac{C_2^4}{C_2^6}\times\frac{1}{6}\times\frac{1}{6}\times 2\right)+\left(\frac{C_2^2}{C_2^6}\times 1\right)+\left(\frac{C_1^1\times C_1^4}{C_2^6}\times 1\times\frac{1}{6}\right)+\left(\frac{C_1^1\times C_1^4}{C_2^6}\times 1\times\frac{1}{6}\right)=\frac{8}{45}$$

- 2 regular dice
- "all-6-die" and "all-1-die"
- "all-6-die" and regular die
- "all-1-die" and regular die

Case 2:

$$\frac{C_2^4}{C_2^6}\times\frac{1}{6}\times\frac{1}{6}\times 2=\frac{1}{45}$$

Case 3:

$$\frac{C_2^4}{C_2^6}\times\frac{1}{6}\times\frac{1}{6}\times 2=\frac{1}{45}$$

answer:

$$\frac{\frac{1}{15}}{\frac{1}{45}+\frac{1}{45}+\frac{8}{45}}=\frac{3}{10}$$

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Question 5 :

The probability that A tells truth is 2/3, and the probability that B tells truth is 4/5. There are 5 white balls and 3 blue balls in a bag. The probability of each ball being selected is equal. Now, one ball is selected from the bag. If A and B both say it is blue, what is the probability of the ball to be white?

Answer 5 :

There are two circumstances: 1. The ball is white and both A and B lie. 2. The ball is blue A tells truth, and B lies.

So, the probability of A and B saying the ball is blue is 5/8 * 1/3 * 1/5 + 3/8 * 2/3 * 4/5 = 29/120

And the answer is (5/8 * 1/3 * 1/5) / (29/120).