1.
According to a recent mortality table, the probability that a 35-year-old U.S. citizen will live to age 65 is 0.725.
(a) What is the probability that John and Jim, two 35-year-old Americans who are not relatives, both live to age 65?
(b) What is the probability that neither John nor Jim lives to that age?

(Ans)
(a) $0.725^2 = 0.525625$
(b) $(1-0.725)^2 = 0.075625$

2.
A company producing USB has three plants producing 50%, 30% & 20% of its product. Suppose that the probability that a USB by these plants is defective are 2%, 5% & 1%.
(a) If a USB is selected at random from the output of company, what's the probability that it's defective?
(b) If a USB is selected at random is found to be defective, what's the probability that it is plant B?

(Ans)
(a) 0.5*0.02 + 0.3*0.05 + 0.2*0.01 = 0.027
(b) P(B | defective) = 0.3*0.05 / 0.027 = 0.556

3.
A system consists of n identical components, each of which is operational with probability p. independent of other components. The system is operational if at least k out of the n components are operational. What is the probability that a system is operational?

(Ans)
The system is operational if k, k+1, k+2, …. n components are operational.
The probability of k components are operational :

$$\binom{n}{k}$$

Therefore the probability of a system is operational will equals to :

$$\sum_{i=k}^{n} \binom{n}{k} p^i (1-p)^{n-i}$$

4.
Suppose that 20% of the population of a city watch Korean drama, 30% watch Taiwanese drama, 15% watch American drama, 8% watch both Korean and Taiwanese drama, 5% watch both Korean and American drama, 4% watch both Taiwanese and American drama, 2% watch all kinds of drama. If a person from this city is selected at random, what is the probability that he or she does NOT watch any of these three kinds of drama?

(Ans)
First write down some information:
A: The event that a randomly selected person watches Korean drama.
B: The event that a randomly selected person watches Taiwanese drama.
C: The event that a randomly selected person watches American drama.
From probability law:

$$1 = P(A \cup B \cup C) + P((A \cup B \cup C)^c).$$

Next,

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C).$$

Next,

$$P((A \cup B \cup C)^c) = P(A^c \cap B^c \cap C^c).$$

Back,

$$1 = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$$
$$+P(A^c \cap B^c \cap C^c)$$
$$.1 = 20\% + 30\% + 15\% - 8\% - 5\% - 4\% + 2\% + P(A^c \cap B^c \cap C^c).$$

Finally,

$$P(A^c \cap B^c \cap C^c) = 50\%.$$

5.

Student A, B and C get score F at the end of probability course. However, kind professor Lee decides to let one of them pass the course and the TA knows who is the one but he can't expose the secret. A still ask TA for the result, TA only tell him/her that B will fail. A is so happy to know that because he/she thinks the probability that he/she pass the class increase from 1/3 to 1/2 . Is A is right? If A is wrong, what's the probability for A and C to pass respectively.

(Ans)

A: A pass B: B pass C: C pass D: TA says B will fail

$$P(A) = P(B) = P(C) = \frac{1}{3}$$

$$P(D \vee A) = 0.5, P(D \vee B) = 0, P(D \vee C) = 1$$

$$P(A \vee D) = \frac{P(D \vee A)P(A)}{P(D)} = \frac{P(D \vee A)P(A)}{P(D \vee A)P(A) + P(D \vee B)P(B) + P(D \vee C)P(C)} = \frac{1}{3}$$

$$P(C \vee D) = \frac{P(D \vee C)P(C)}{P(D)} = \frac{2}{3}$$

6. Bonus (10 points)

Let A1 , A2 ,...... be an infinite sequence of events. Suppose now that the events are "monotonically decreasing," i.e. , An+1⊂An for every n. Let A=∩1→∞ An . Show that P(A)=lim n→∞ P(An).

(Ans)

Suppose we got another infinite sequence of the events, which is monotonically increasing called k_1, k_2, …… meaning the $k_n \subset k^{n+1}$ for every *n*. i.e. $K = \cup_{i=1}^{n} k_i$. Suppose $L_1 = L_2$ and for n >= 2, $L_n = k_{n-1}^c$, the event L_n is disjoint, and we have $\cup_{i=1}^{n} L_i = K_n$ and $\cup_{i=1}^{\infty} L_i = K$.

$$\mathrm{P(K)} = \sum_{i=1}^{\infty} P(L_i) = \lim_{n\to\infty} \sum_{i=1}^{n} P(L_i) = \lim_{n\to\infty} P(\cup_{i=1}^{n} L_i) = \lim_{n\to\infty} P(k_n)$$

Now we set that event K is the complement of A, from the result we got above, we can write that:

$$1 - \mathrm{P(A)} = P(A^c) = P(K) = \lim_{n\to\infty} P(K_n) = \lim_{n\to\infty} (1 - P(A_n))$$

From the above result we can say that $\mathrm{P(A)} = \lim_{n\to\infty} P(A_n)$