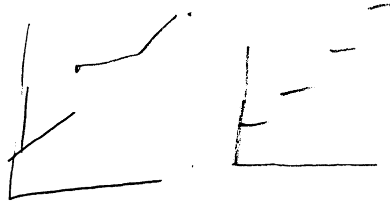

$F_Y(t) = P(Y \le t)$

$= P(\min(X,5) \le t)$

EE 3060 **Midterm Exam #2**

1. (a) (3%) What is a probability space?
(b) (3%) What is a measurable space and what is a measurable function?
(c) (2%) What is a Borel σ-algebra?
(d) (2%) What is a random variable and what is a random vector?

2. There are n_1 red balls, n_2 blue balls, and n_3 white balls in a box. These balls successively and randomly drawn from the box.
(a) (7%) What is the probability that a red ball is drawn before a blue ball is drawn if the balls are drawn *with* replacement?
(b) (8%) What is the probability that a red ball is drawn before a blue ball is drawn if the balls are drawn *without* replacement and without knowing the colors of the balls already drawn?

3. (10%) There are n_1 red balls and n_2 blue balls in box 1 and there are m_1 red balls and m_2 blue balls in box 2. A box is randomly selected and a ball is randomly drawn from the selected box and is observed to be a red ball and then returned to the selected box. If a second ball is randomly drawn from the selected box, what is the probability that it is red?

4. (10%) Let X be a random variable of a probability space $(\Omega, \mathcal{A}, P)$ and let $Y = \min\{X, 5\}$. Express the distribution function $F_Y(t)$ of Y in terms of the distribution function $F_X(t)$ of X.

5. (15%) An ordinary deck of 52 cards is well-shuffled, and then the cards are turned face up one by one until an ace appears. Let X be the number of cards that are face up. Find $E[X]$ and $\mathrm{Var}(X)$.

6. Suppose that there are N families in the world and the maximum number of children a family has is c. Let α_j be the fraction of families with j children for $j = 1, 2, \ldots, c$ (so that $\sum_{j=1}^{c} \alpha_j = 1$). A *child* is randomly selected from the set of all of the children in the world. Let the selected child be the X^{th} oldest child in his/her family.
(a) (10%) What is the probability mass function of the random variable X?
(b) (10%) What is $E[X]$?

7. (10%) Accidents occur at an intersection at a Poisson rate of three per day. What is the probability that during January there are exactly three days (not necessarily consecutive) without any accidents?

8. (10%) A fair coin is flipped repeatedly. What is the probability that the fifth tail occurs before the tenth head?

Good luck!