Problems and Solutions to Smith/Hashemi
***Foundations of Materials Science and Engineering* 5/e**

Chapter 6, Problem 1

Why are cast metal sheet ingots hot-rolled first instead of being cold-rolled?

Chapter 6, Solution 1

Hot rolling is applied first because it is more efficient in reducing ingot sheet thickness than cold working.

Chapter 6, Problem 2

What type of heat treatment is given to the rolled metal sheet after hot and "warm" rolling? What is its purpose?

Chapter 6, Solution 2

In most cases, the rolled metal slabs are reheated to a relatively high temperature which allows for further hot rolling without excessive oxidation of the metal. Once all hot and "warm" rolling is complete, the metal is reheated or *annealed* to remove cold-work induced through the hot-working.

Chapter 6, Problem 3

(a) How are metal alloys made by the casting process? (b) Distinguish between wrought alloy products and cast alloy products.

Chapter 6, Solution 3

Alloying elements are added to the molten (basic) metal and allowed to melt. The molten metal is subsequently mixed to achieve uniformity and then cast into solid ingots or castings. (b) Wrought alloy products are produced using a working process such as rolling, extruding or forging.

Chapter 6, Problem 4

Describe the wire-drawing process. Why is it necessary to make sure the surface of the incoming wire is clean and lubricated?

Chapter 6, Solution 4

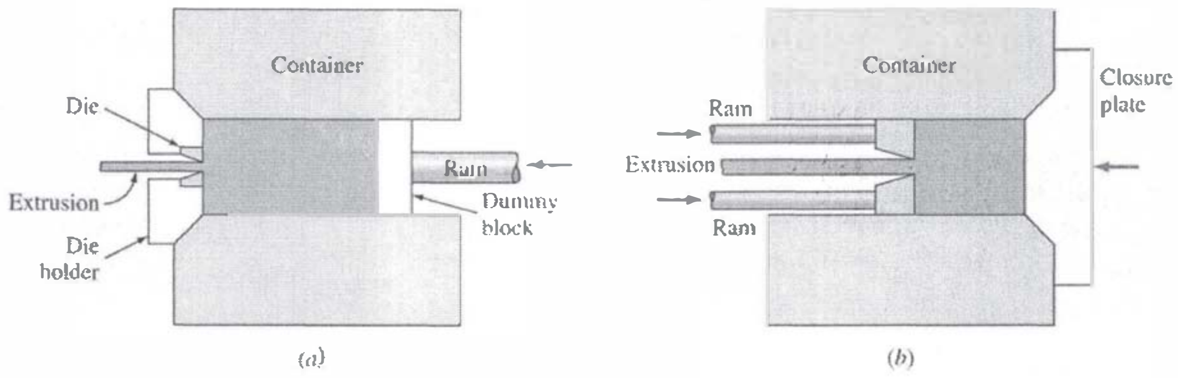
In the wire drawing process a starting rod or wire is drawn through one or more tapered dies to reduce the cross section of the wire. It is important to have the wire surface clean so that defects are not introduced into the wire. Lubrication is also necessary to prevent tearing of the metal being drawn through the die and to reduce friction.

Chapter 6, Problem 5

Describe and illustrate the following types of extrusion processes: (a) direct extrusion and (b) indirect extrusion. What is an advantage of each process?

Chapter 6, Solution 5

The direct and indirect extrusion processes are shown below in Fig. (*a*) and (*b*) respectively.



(a) In direct extrusion, the billet is forced directly through the die of the extrusion press by a solid ram.
(b) Whereas, in an indirect extrusion process, a hollow ram holds the die and forces it against the billet. Higher loads can be applied with the direct process while advantages associated with the indirect process include lower frictional forces and lower power requirements.

Chapter 6, Problem 6

Describe the forging process. What is the difference between hammer and press forging?

Chapter 6, Solution 6

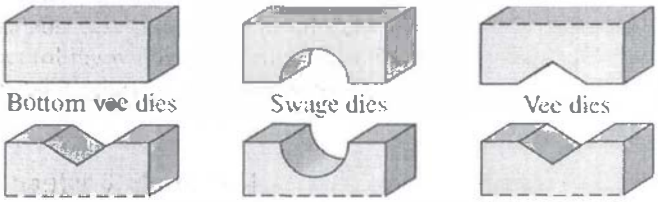
In the forging process metal is hammered or pressed into the desired shape. In hammer forging a hammer repeatedly strikes the work piece to shape it. In press forging, a slow compressive force is used to shape the metal.

Chapter 6, Problem 7

What is the difference between open-die and closed-die forging? Illustrate. Give an example of a metal product produced by each process.

Chapter 6, Solution 7

Open die forging is carried out with two flat dies or with two dies of simple shapes. Closed die forging is accomplished with a single or multiple impression set of dies with the shaped metal being entirely surrounded by the die. Examples of these die types are shown below schematically.



Examples of dies for open-die forging.

Closed forging dies used to produce an automobile connecting rod.

An example of a forging made by the open-die process is a long shaft for turbomachinery applications such as a turbine. An automobile connecting rod is an example of a closed die forging.

Chapter 6, Problem 8

Distinguish between elastic and plastic deformation (use schematics).

Chapter 6, Solution 8

Elastic deformation of metal takes place when the metal is able to return to its original dimensions once the deforming force is removed. Plastic deformation takes place if the metal does not fully recover its original dimensions after the deforming force is removed.

Chapter 6, Problem 9

Define (a) engineering stress and strain, and (b) true stress and strain. (c) What are the U.S. customary and SI units for stress and strain? (d) Distinguish between tensile/compressive stress (also called *normal stress*) and shear stress. (e) Distinguish between tensile/compressive strain (also called *normal strain*) and shear strain.

Chapter 6, Solution 9

(a) Engineering stress = s = applied normal (uniaxial) force / original cross-sectional area

Engineering strain = e = change in gage length/original gage length (lf – lo)/lo

(b) True stress = s_t = applied normal (uniaxial) force / instantaneous cross-sectional area

True strain = e_t = ln (instantaneous length / original length)

(c) where the units are expressed as:

Stress (both normal and shear): lb/in^2 or psi U. S. Customary
N/m^2 (newtons per square meter) SI
Normal strain: dimensionless (in/in, m/m, etc…)
Shear strain: radians

(d) s is designated as "normal" stress produced as a result of the application of a force that is perpendicular to a known surface. If the applied force is in tension, the normal stresses produced will also be in tension. If the applied force is in compression, the normal stresses produced will be in compression. The shearing stresses, t, on the other hand are produced as a result of the application of a force that is parallel to a known surface (shear force). Thus s and t must be treated differently (are not additive).

(e) Normal stresses produce normal strains. Tensile normal stresses produce tensile normal strains and compressive stresses produce compressive normal strains. Normal strains represent a change in length. On the other hand, shearing stresses produce shearing strains. Shearing strains represent the distortional changes or the changes in the angles with respect to the original. Thus normal and shearing strains must also be treated differently.

Chapter 6, Problem 10

(a) Define the hardness of a metal. (b) How is the hardness of a material determined by a hardness testing machine?

Chapter 6, Solution 10

(a) Hardness is a measure of the resistance of a metal to permanent deformation. (b) Hardness is measured by forcing an indenter into the metal surface. The hardness measurement is made either from the depth of penetration of the indenter or by the size of the indentation.

Chapter 6, Problem 11

What are the load used in (a) Rockwell B hardness test, and (b) Rockwell C hardness test?

Chapter 6, Solution 11

(a) The load of Rockwell B hardness test is 100 kg.
(a) The load of Rockwell C hardness test is 150 kg.

Chapter 6, Problem 12

What is the difference between Vickers and Knoop hardness tests?

Chapter 6, Solution 12

The difference between the Vickers and the Knoop hardness tests is simply the shape of the diamond pryamid indenter.The Vickers test uses a square pyramidal indenter. The Knoop test using a rhombic-based pyramidal indenter was developed which produces longer but shallower indentations.

Chapter 6, Problem 13

Describe the slip mechanism that enables a metal to be plastically deformed without fracture.

Chapter 6, Solution 13

In the metal slip mechanism, dislocations move through the metal crystals like wave fronts, allowing metallic atoms to slide over each other under low shear stress. The metal can thus deform without fracture.

Chapter 6, Problem 14

(a) Why does slip in metals usually take place on the densest-packed planes?
(b) Why does slip in metals usually take place in the closest-packed directions?

Chapter 6, Solution 14

(a) Slip usually takes place on the most densely packed planes because the atoms on these planes are in close proximity and hence require less shear energy for displacement.
(b) Slip typically occurs along the closest-packed directions because minimal energy is required to force the atoms to change positions.

Chapter 6, Problem 15

(a) What are the principal slip planes and slip directions for FCC metals? (b) What are the principal slip planes and slip directions for BCC metals? (c) What are the principal slip planes and slip directions for HCP metals?

Chapter 6, Solution 15

(a) The principal slip planes and slip directions for FCC metals are $\{1\,1\,1\}$ and $\langle 1\,\bar{1}\,0\rangle$, respectively. (b) The principal slip planes and slip directions for BCC metals are $\{1\,1\,0\}$ and $\langle \bar{1}\,1\,1\rangle$, respectively. (c) The principal slip planes and directions for HCP metals are $(0\,0\,0\,1)$ and $\langle 1\,1\,\bar{2}\,0\rangle$.

Chapter 6, Problem 16

Describe the deformation twinning process that occurs in some metals when they are plastically deformed.

Chapter 6, Solution 16

In the deformation twinning process, a part of the atomic lattice is deformed such that it forms a mirror image of the adjacent undeformed lattice.

Chapter 6, Problem 17

What is the difference between the slip and twinning mechanisms of plastic deformation of metals?

Chapter 6, Solution 17

The slip mechanism causes all atoms on one side of the slip plane to move equal distances, such that a series of slip steps are formed. Whereas in twinning, atoms only move distances that are proportional to their respective distances from the twinning plane, and thus produce a well defined region of deformation.

Chapter 6, Problem 18

What other types of slip planes are important other than the basal planes for HCP metals with low *c/a* ratios?

Chapter 6, Solution 18

The prism planes, $\{1\,0\,\bar{1}\,0\}\,\langle 1\,1\,\bar{2}\,0\rangle$, and the pyramidal planes, $\{1\,0\,\bar{1}\,1\}\,\langle 1\,1\,\bar{2}\,0\rangle$, are also important planes having low *c/a* ratios.

Chapter 6, Problem 19

What important role does twinning play in the plastic deformation of metals with regard to deformation of metals by slip?

Chapter 6, Solution 19

During twining deformation, the lattice orientations change in such a manner that new slip systems may become favorable for further slip.

Chapter 6, Problem 20

Define the critical resolved shear stress for a pure metal single crystal? What happens to the metal from the macroscale point of view and behavior point of view once critical resolved shear stress is exceeded?

Chapter 6, Solution 20

The critical resolved shear stress, τ_c, for a single crystal is the minimum shear stress required to initiate the slip process. This minima is essentially the yield stress of a single crystal. Once the critical stress level is reached on a specific plane, the top half plane will slip on the bottom half plane in the direction of slip direction vector. Excessive slip will eventually result in fracture along the slip plane.

Chapter 6, Problem 21

By what mechanism do grain boundaries strengthen metals?

Chapter 6, Solution 21

Grain boundaries strengthen metals by acting as barriers to dislocation movement.

Chapter 6, Problem 22

(a) What is solid-solution strengthening? Describe the two main types. (b) What are two important factors that affect solid-solution hardening?

Chapter 6, Solution 22

(a) Solid-solution strengthening is a method of increasing a metal's strength. By adding one or more elements, dislocation movement is impeded due to lattice distortions and the introduction of different bonding structures.

The two primary types of solid-solution strengthening are *substitutional* and *interstitial*.

(b) Two important factors that affect solid-solution hardening are: the relative size of the atoms of the elements in the solid solution and; short-range ordering of the atoms of different atoms into clusters.

Chapter 6, Problem 23

What experimental evidence shows that grain boundaries arrest slip in polycrystalline metals?

Chapter 6, Solution 23

Slip bands in polycrystalline metals are observed to be parallel within a grain but discontinuous at grain boundaries.

Chapter 6, Problem 24

(a) Describe the grain shape changes that occur when a sheet of alloyed copper with an original equiaxed grain structure is cold-rolled with 30 and 50 percent cold reductions. (b) What happens to the dislocation substructure?

Chapter 6, Solution 24

(a) When an alloyed sheet of copper with an equiaxed grain structure is cold rolled to 30-50 percent reduction, the grains are elongated in the direction of rolling.

For unalloyed copper, as plastic deformation is increased to 30%, dislocation density increases and dislocations begin to tangle and form network of dislocations resulting in a cellular structure in which high density dislocation tangles form the walls of the cell. As plastic deformation is increased to 50%, the cell size decreases and the cell structure becomes denser. (see Figure 6.44)

Chapter 6, Problem 25

How is the ductility of a metal normally affected by cold working? Why?

Chapter 6, Solution 25

Cold rolling normally decreases the ductility of metals because the dislocation density of the metal is increased and thus further slip by dislocation movement is inhibited.

Chapter 6, Problem 26

What are the three main metallurgical stages that a sheet of cold-worked metal such as aluminum or copper goes through as it is heated from room temperature to an elevated temperature just below its melting point?

Chapter 6, Solution 26

The three main stages are recovery, recrystallization and grain growth.

Chapter 6, Problem 27

When a cold-worked metal is heated into the temperature range where recrystallization takes place, how are the following affected: (*a*) internal residual stresses, (*b*) strength, (*c*) ductility, and (*d*) hardness?

Chapter 6, Solution 27

(a) Any internal stresses are relieved.
(b) The metal tensile strength is significantly reduced.
(c) The ductility of the metal is greatly increased.
(d) The hardness of the metal is substantially reduced.

Chapter 6, Problem 28

When a cold-worked metal is heated into the temperature range where recovery takes place, how are the following affected: (*a*) internal residual stresses, (*b*) strength, (*c*) ductility, and (*d*) hardness?

Chapter 6, Solution 28

(a) Internal stresses are greatly reduced.
(b) The metal strength is only slightly reduced.
(c) The metal ductility is usually significantly increased.
(d) The hardness of the metal is slightly reduced.

Chapter 6, Problem 29

Describe the microstructure of a heavily cold-worked metal of an Al-0.8% Mg alloy as observerd with an optical microscope at 100× (see Fig. 6.46a). Describe the microstructure of the same material at 20,000× (see Fig. 6.47a).

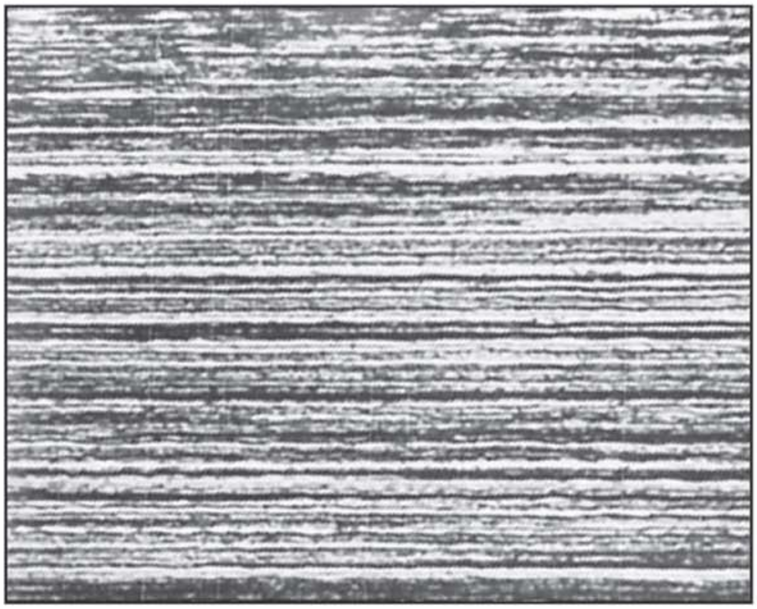


Figure 6.46(a)

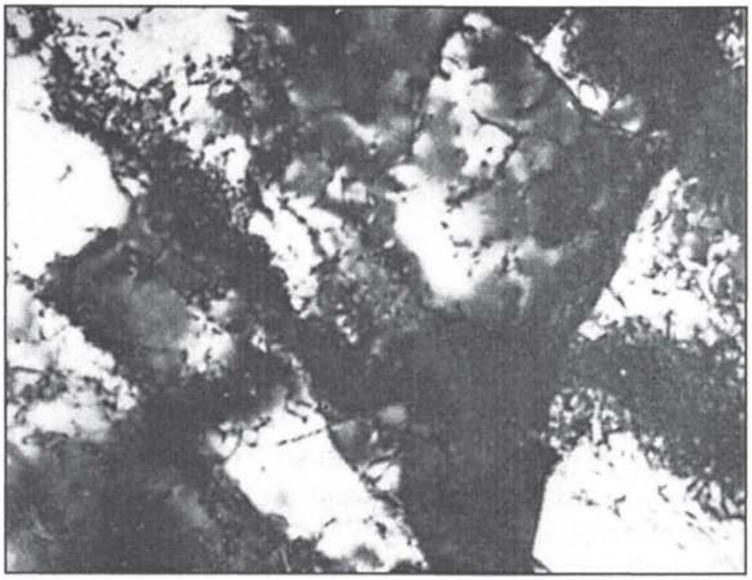


Figure 6.47(a)

Chapter 6, Solution 29

At 100×, one observes a highly elongated grain structure. Whereas at 20,000×, dislocation tangles and banded cells or *subgrains*, produced by extensive cold work, are evident within the microstructure.

Chapter 6, Problem 30

Describe what occurs microscopically when a cold-worked sheet of metal such as aluminum undergoes a recovery heat treatment.

Chapter 6, Solution 30

When a cold-worked sheet of metal such as aluminum is subjected to a recovery heat treatment, many dislocations are destroyed or move into lower energy configurations through the *polygonization* process. In this process, low-angle grain boundaries are formed and the metal ductility is significantly increased while the strength is only slightly reduced.

Chapter 6, Problem 31

Describe what occurs microscopically when a cold-worked sheet of metal such as aluminum undergoes a recrystallization heat treatment.

Chapter 6, Solution 31

During a recrystallization heat treatment, new strain-free grains are nucleated, and after sufficient time, grow until the grain structure is completely recrystallized.

Chapter 6, Problem 32

What generalizations can be made about the recrystallization temperature with respect to (*a*) the degree of deformation, (*b*) the temperature, (*c*) the time of heating at temperature, (*d*) the final grain size, and (*e*) the purity of the metal?

Chapter 6, Solution 32

(a) In order for recrystallization to occur, the metal must possess a minimum degree of deformation. The greater the extent of deformation above this required minima, the lower the temperature required for recrystallization.
(b) Increasing the temperature decreases the time required for complete recrystallization.
(c) Increasing the rate of heating raises the recrystallization temperature.
(d) The final grain size depends primarily upon the original extent of deformation; the greater the degree of deformation, the lower the annealing temperature required for recrystallization.
(e) With decreasing metal purity, the recrystallization temperature rises. Thus, solid-solution alloying additions increase the recrystallization temperature.

Chapter 6, Problem 33

Describe two principal mechanisms whereby primary recrystallization can occur.

Chapter 6, Solution 33

The two principal mechanisms of primary recrystallization are: the expansion of an isolated nucleus with a deformed grain; the migration of a high-angle grain boundary into a more highly deformed region of the metal.

Chapter 6, Problem 34

What are five important factors that affect the recrystallization process in metals?

Chapter 6, Solution 34

Five important factors affecting the metal recrystallization process are:

1. the extent of deformation of the metal prior to recrystallization;
2. the temperature used for the recrystallization process;
3. the length of time of the recrystallization process;
4. the initial grain size of the metal;
5. the composition of the metal, in terms of metal purity.

Chapter 6, Problem 35

Define superplasticity and list the conditions under which superplasticity can be achieved. Why is this an important behavior?

Chapter 6, Solution 35

Superplasticity is the ability of a material to sustain plastic deformation levels exceeding 1000% without fracture. This occurs in certain metals at specific elevated temperatures and under controlled and slow loading rates. This is an important behavior because we can take advantage of it to produce complex shapes that require heavy deformation out of metals. If we can produce heavy plastic deformation without fracture of metal we can produce complex shapes with a small number of operations and more economically.

Chapter 6, Problem 36

Discuss the major deformation mechanism that results in extensive plastic deformation in superplasticity.

Chapter 6, Solution 36

The deformation mechanism in superplasticity is not predominantly dislocations and their movements but rather mostly grain boundary sliding (grains slide on top of each other along the boundary) and grain boundary diffusion in which atoms from one grain diffuse across the grain boundary to the surrounding grains (see Figure 6.53).

Chapter 6, Problem 37

Why are nanocrystalline materials stronger? Answer based on dislocation activity.

Chapter 6, Solution 37

As grain size decreases, grain boundary density increases which can prevents slip and the ability of the metal to resist movement of dislocations increases. Any produced dislocations will quickly pile up at the boundaries and create dislocation entanglement resulting in increased strength of the metal.

Chapter 6, Problem 38

A 70% Cu–30% Zn brass sheet is 0.12 cm thick and is cold-rolled with a 20 percent reduction in thickness. What must be the final thickness of the sheet?

Chapter 6, Solution 38

$$\% \text{ cold reduction} = \left[\frac{t_0 - t_f}{t_0}\right] \times 100\%$$

$$0.20 = \left[\frac{0.12\text{ cm} - t_f}{0.12\text{ cm}}\right] \times 100\% \qquad t_f = \mathbf{0.096\text{ cm}}$$

Chapter 6, Problem 39

A 12.8-mm-diameter rod of an aluminum alloy is pulled to failure in a tension test. If the final diameter of the rod at the fractured surface is 10.8 mm, what is the percent reduction in area of the sample due to the test?

Chapter 6, Solution 39

$$\% \text{ cold reduction} = \left[\frac{\text{initial area} - \text{final area}}{\text{initial area}}\right] \times 100\%$$

$$= \left[\frac{(\pi/4)(12.8\text{ mm})^2 - (\pi/4)(10.8\text{ mm})^2}{(\pi/4)(12.8\text{ mm})^2}\right] \times 100\% = \mathbf{28.8\%}$$

Chapter 6, Problem 40

Calculate the percent cold reduction when an aluminum wire is cold-drawn from a diameter of 6.50 mm to a diameter of 4.25 mm.

Chapter 6, Solution 40

$$\% \text{ cold reduction} = \left[\frac{\text{initial area} - \text{final area}}{\text{initial area}}\right] \times 100\%$$

$$= \left[\frac{(\pi/4)(6.50\text{ mm})^2 - (\pi/4)(4.25\text{ mm})^2}{(\pi/4)(6.50\text{ mm})^2}\right] \times 100\% = \mathbf{57.2\%}$$

Chapter 6, Problem 41

What is the relationship between engineering strain and percent elongation?

Chapter 6, Solution 41

Engineering strain and percent elongation are related as,

% engineering strain = engineering strain × 100% = % elongation

Chapter 6, Problem 42

A sheet of aluminum alloy is cold-rolled 25 percent to a thickness of 0.2 cm. If the sheet is then cold-rolled to a final thickness of 0.16 cm, what is the total percent cold work done?

Chapter 6, Solution 42

$$\%\text{ cold reduction} = \left[\frac{t_0 - t_f}{t_0}\right] \times 100\%$$

$$0.25 = \left[\frac{t_0 - 0.2\text{ cm}}{t_0}\right] \times 100\% \qquad t_0 = 0.267\text{ cm}$$

The total cold work is therefore,

$$\text{total \% cold work} = \left[\frac{0.267\text{ cm} - 0.16\text{ cm}}{0.267\text{ cm}}\right] \times 100\% = \mathbf{40.1\%}$$

Chapter 6, Problem 43

A tensile specimen of cartridge brass sheet has a cross section of 10.0 mm × 4.0 mm and a gage length of 51 mm. Calculate the engineering strain that occurred during a test if the distance between gage markings is 63 mm after the test.

Chapter 6, Solution 43

$$\text{engineering strain } \varepsilon = \frac{l - l_0}{l_0} = \frac{63\text{ mm} - 51\text{ mm}}{51\text{ mm}} = \mathbf{0.235}$$

Chapter 6, Problem 44

A brass wire is cold-drawn 30 percent to a diameter of 0.90 mm. It is then further cold-drawn to 0.70 mm. What is the total percent cold reduction?

Chapter 6, Solution 44

$$\%\text{ cold reduction} = \left[\frac{A_0 - A_f}{A_0}\right] \times 100\%$$

$$\text{Initial reduction } 30.0 = \left[\frac{\frac{\pi}{4}d_0^2 - \frac{\pi}{4}(0.90\text{ mm})^2}{\frac{\pi}{4}(d_0)^2}\right] \times 100\% \qquad d_0 = 1.076\text{ mm}$$

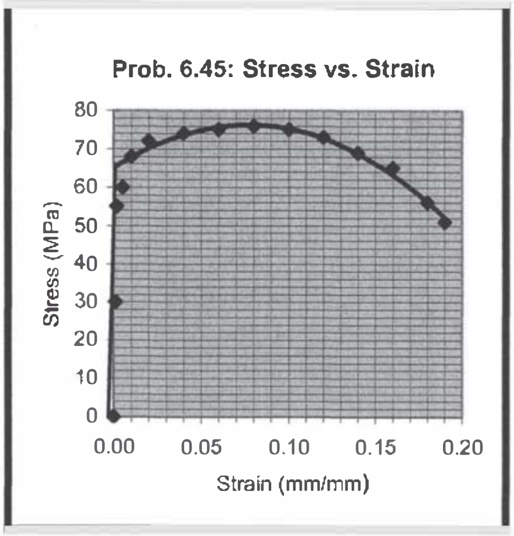
$$\text{total \% cold reduction} = \left[\frac{\frac{\pi}{4}(1.076\text{ mm})^2 - \frac{\pi}{4}(0.70\text{ mm})^2}{\frac{\pi}{4}(1.076\text{ mm})^2}\right] \times 100\% = \mathbf{57.7\%}$$

Chapter 6, Problem 45

The following engineering stress-strain data were obtained for a metal. (*a*) Plot the engineering stress-strain curve. (*b*) Determine the ultimate tensile strength of the alloy. (*c*) Determine the percent elongation at fracture.

Engineering stress (MPa)	Engineering strain (mm/mm)	Engineering stress (MPa)	Engineering strain (mm/mm)
0	0	76	0.08
30	0.001	75	0.10
55	0.002	73	0.12
60	0.005	69	0.14
68	0.01	65	0.16
72	0.02	56	0.18
74	0.04	51	(Fracture) 0.19
75	0.06		

Chapter 6, Solution 45



Engineering Stress (MPa)	Engineering Strain (mm/mm)	Engineering Stress (MPa)	Engineering Strain (mm/mm)
0	0	76	0.08
30	0.001	75	0.10
55	0.002	73	0.12
60	0.005	69	0.14
68	0.010	65	0.16
72	0.020	56	0.18
74	0.040	51	0.19
75	0.060		(Fracture)

(a) See stress-strain plot above.
(b) The ultimate tensile strength, based on the stress-strain curve, is **76 MPa**
(c) % elongation = engineering strain × 100% = 0.19×100% = **19%**.

Chapter 6, Problem 46

From the data of Prob. 6.45, estimate the yield strength of the metal.

Chapter 6, Solution 46

The yield strength is about 65 MPa.

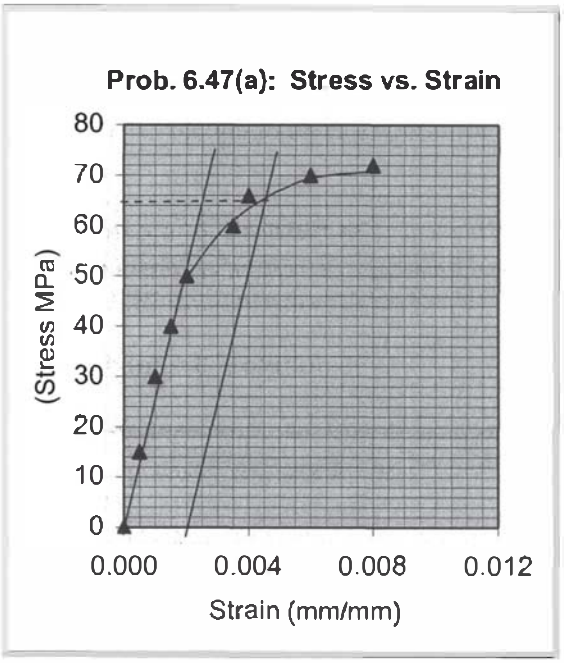
Chapter 6, Problem 47

The following engineering stress-strain data were obtained at the beginning of a tensile test for a metal. (*a*) Plot the engineering stress-strain curve for these data.(*b*) Determine the 0.2 percent offset yield stress for this steel. (*c*) Determine the tensile elastic modulus of this steel. (Note that these data only give the beginning part of the stress-strain curve.)

Engineering stress (MPa)	Engineering strain (mm/mm)	Engineering stress (MPa)	Engineering strain (mm/mm)
0	0	60	0.0035
15	0.0005	66	0.004
30	0.001	70	0.006
40	0.0015	72	0.008
50	0.0020		

Chapter 6, Solution 47

(a)



Engineering Stress (MPa)	Engineering Strain (mm/mm)	Engineering Stress (MPa)	Engineering Strain (mm/mm)
0	0	60	0.0035
15	0.0005	66	0.0040
30	0.0010	70	0.0060
40	0.0015	72	0.0080
50	0.0020		

(b) The 0.02 % offset yield stress was found graphically as **66 MPa.**

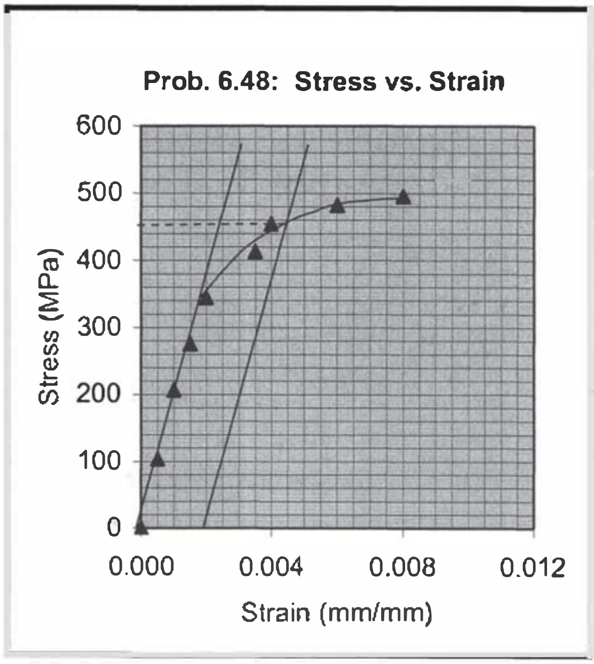
(c) The modulus of elasticity is found from the slope of the 0.2% offset curve as:

$$E = \frac{\sigma}{\varepsilon} = \frac{50 - 12}{0.002 - 0.0005} = \mathbf{25.3\ GPa}$$

Chapter 6, Problem 48

The following engineering stress-strain data were obtained at the beginning of a tensile test for a 0.2% C plain carbon steel. (*a*) Plot the engineering stress-strain curve for these data. (*b*) Determine the 0.2 percent offset yield stress for this steel. (*c*) Determine the tensile elastic modulus of this steel. (Note that these data only give the beginning part of the stress-strain curve.)

Chapter 6, Solution 48



Engineering Stress (MPa)	Engineering Strain (mm/mm)	Engineering Stress (MPa)	Engineering Strain (mm/mm)
0	0	413.4	0.0035
103.4	0.0005	454.7	0.0040
206.7	0.0010	482.3	0.0060
275.6	0.0015	496.1	0.0080
344.5	0.0020		

(b) The 0.02 % offset yield stress was found graphically as approximately **450 MPa**.
(c) The modulus of elasticity is found from the slope of the 0.2% offset curve as:

$$E = \frac{\sigma}{\varepsilon} = \frac{(450\times10^{3})-(0.0)}{0.0025-0.0} = \mathbf{180,000\,MPa = 180\,GPa}$$

Chapter 6, Problem 49

A tensile specimen of aluminum alloy is tested to fracture. At the fracture point, it has an engineering stress 180 MPa and engineering strain 34%. Calculate (*a*) the true stress at fracture, and (*b*) the true strain at fracture.

Chapter 6, Solution 49

(a) True stress σ_T:

$$\sigma_T = \sigma\,(1+\varepsilon) = 180(1+0.34) = 241.2\text{ MPa}$$

(b) True stress ε_T:

$$\varepsilon_T = \ln(1+\varepsilon) = \ln(1+0.34) = 0.293 = 29.3\%$$

Chapter 6, Problem 50

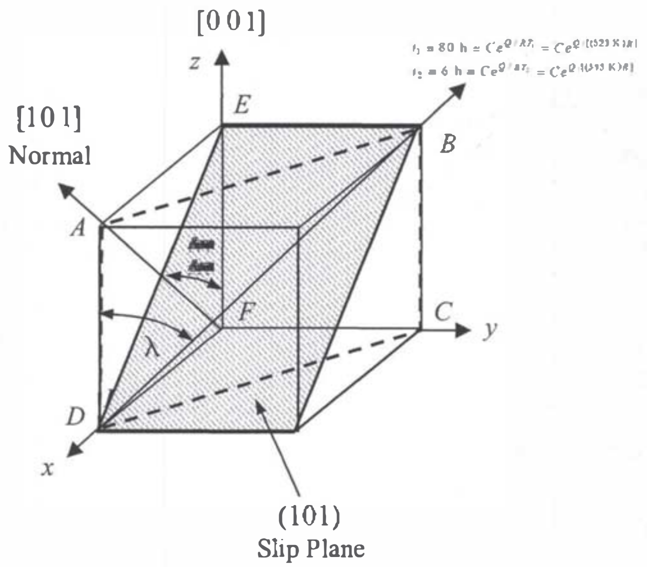
A stress of 55 MPa is applied in the [001] direction of a BCC single crystal. Calculate (*a*) the resolved shear stress acting on the (101) $[\bar{1}11]$ system and (*b*) the resolved shear stress acting on the (110) $[\bar{1}11]$ system.

Chapter 6, Solution 50

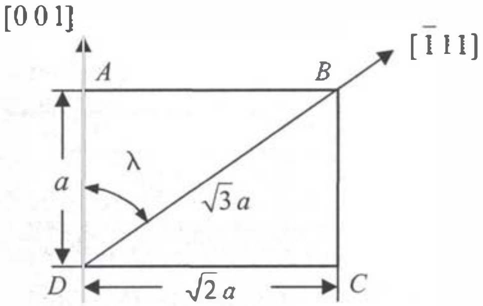
(a) First, the direction normal to the (101) plane is shown below in Fig. (a) to be the [101] direction. From Fig. (b), the value of angle λ, between the applied stress direction $[0\,0\,1]$ and the $[\bar{1}\,11]$ slip direction, is dictated by the geometry of rectangle ABCD:

$\cos\lambda = \cos a/\left[\sqrt{3}a\right] = 0.5774,\ \lambda = 54.7^{\circ}$. Finally, the angle ϕ lies between the $[0\,01]$ stress direction and the [101] normal and is thus equal to 45°. Substituting into Eq. (5.15), $\tau_r = (55\text{ MPa})(\cos 54.7^{\circ})(\cos 45^{\circ}) = \mathbf{22.5\ MPa}$

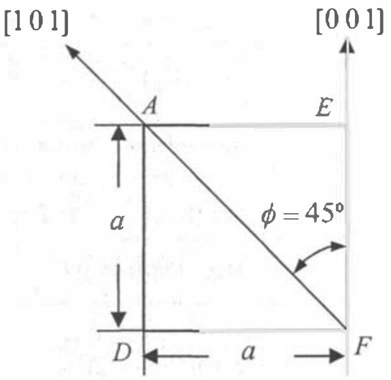
(a)



(b)

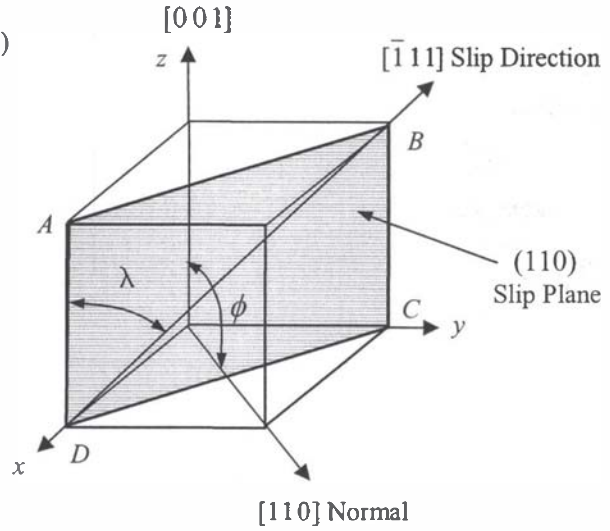


(c)

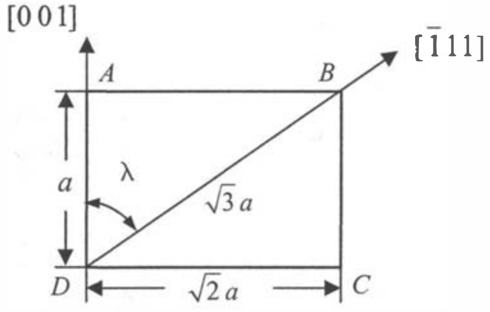


(b) The direction normal to the (110) slip plane, $[1\ 1\ 0]$, and the $[\bar{1}\ 1\ 1]$ slip direction are shown below in Fig. (d). From Fig. (e), the value of angle λ is calculated as: $\cos\lambda = \cos\ a/\left[\sqrt{3}a\right] = 0.5774,\ \lambda = 54.7^{\circ}$. Finally, the angle ϕ lies between the $[0\ 0\ 1]$ stress direction and the $[1\ 1\ 0]$ normal, and is thus equal to 90°. Substituting into Eq. (5.15), $\tau_r = (55\ \text{MPa})(\cos 54.7^{\circ})(\cos 90^{\circ}) = \mathbf{0}$.

(d)



(e)



Chapter 6, Problem 51

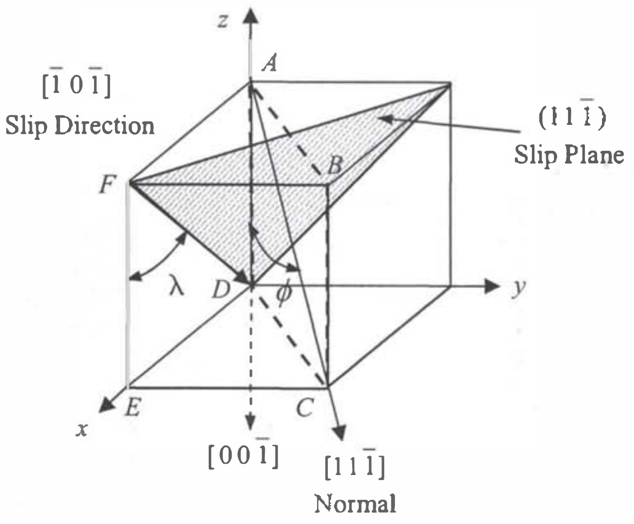
A stress of 4.75 MPa is applied in the $[0\,0\,\bar{1}]$ direction of a unit cell of an FCC copper single crystal. Calculate the resolved shear stress on the $(1\,1\,\bar{1})$ plane in the following directions: (a) $[\bar{1}\,0\,\bar{1}]$, (b) $[0\,\bar{1}\,\bar{1}]$, (c) $[\bar{1}\,1\,0]$.

Chapter 6, Solution 51

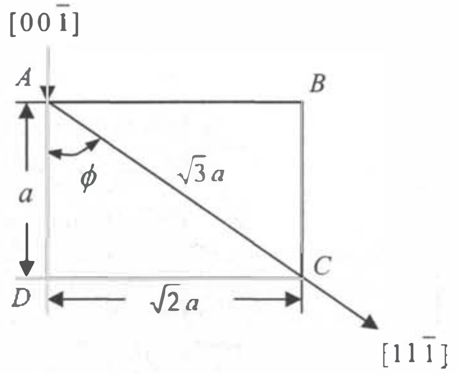
(a) For slip system $(1\,1\,\bar{1})\,[\bar{1}\,0\,\bar{1}]$, ϕ can be calculated based upon the geometry depicted within rectangle ABCD of Fig. (b): $\cos\phi = \cos a/\left[\sqrt{3}a\right] = 0.5774,\ \phi = 54.7^\circ$. From the (001) plane indicated in Fig. (c) as square ADEF, λ is equal to 45°. Thus, the resolved shear stress is:

$$\tau_r = \sigma \cos\lambda \cos\phi = (4.75\text{ MPa})(\cos 45^\circ)(\cos 54.7^\circ) = \mathbf{1.94\ MPa}$$

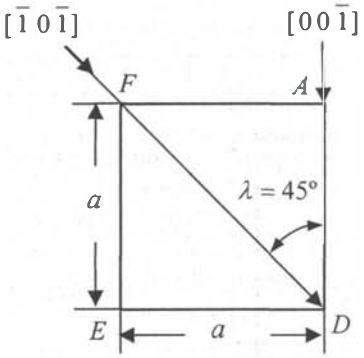
(a)



(b)



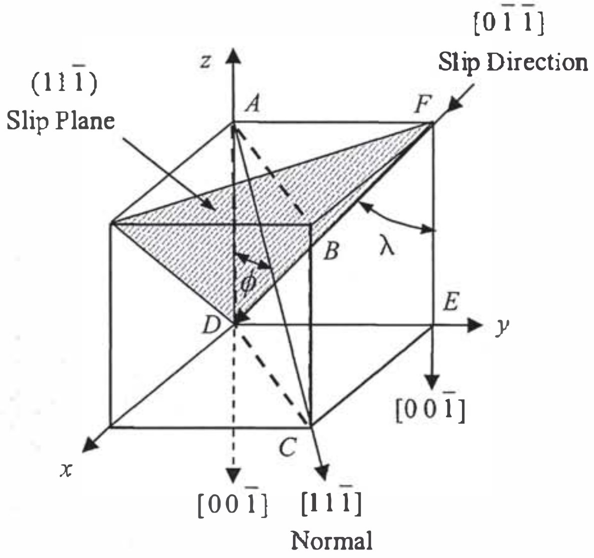
(c)



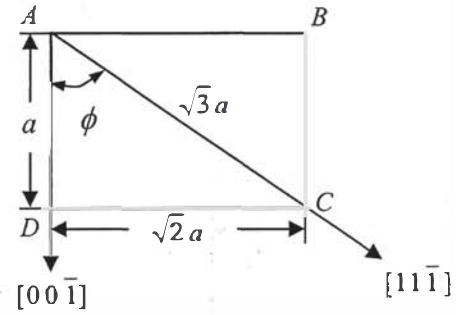
(b) For slip system $(1\,1\,\bar{1})\,[0\,\bar{1}\,\bar{1}]$, ϕ can be calculated based on the geometry depicted below in Fig. (e): $\cos\phi = \cos\ a/\left[\sqrt{3}a\right] = 0.5774$, $\phi = 54.7^\circ$. Referring to Fig. (f), by geometry λ is equal to 45°. Thus, $\tau_r = \sigma\cos\lambda\cos\phi = (4.75\text{ MPa})(\cos 45^\circ)(\cos 54.7^\circ) = \mathbf{1.94\ MPa}$.

(c) For slip system $(1\,1\,\bar{1})\,[\bar{1}\,1\,0]$, ϕ can be calculated based on the geometry depicted below in Fig. (h): $\cos\phi = \cos\ a/\left[\sqrt{3}a\right] = 0.5774$, $\phi = 54.7^\circ$. Referring to Fig. (i), by geometry λ is equal to 90°. Thus, $\tau_r = \sigma\cos\lambda\cos\phi = (4.75\text{ MPa})(\cos 90^\circ)(\cos 54.7^\circ) = \mathbf{0}$.

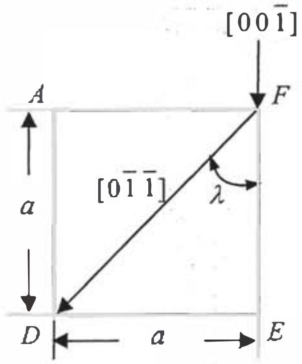
(d)



(e)



(f)



Chapter 6, Problem 52

A 20-cm-long rod with a diameter of 0.250 cm is loaded with a 5000 N weight. If the diameter decreases to 0.210 cm, determine (*a*) the engineering stress and strain at this load and (*b*) the true stress and strain at this load.

Chapter 6, Solution 52

$$\text{Area at start } A_0 = \frac{\pi d_0^2}{4} = \frac{\pi}{4}(0.25\text{ cm})^2 = 0.04909\text{ cm}^2 = 4.909\times 10^{-6}\text{ m}^2$$

$$\text{Area under load } A_i = \frac{\pi d_i^2}{4} = \frac{\pi}{4}(0.21\text{ cm})^2 = 0.03464\text{ cm}^2 = 3.464\times 10^{-6}\text{ m}^2$$

Assuming $A_0 l_0 = A_i l_i$ or $l_i / l_0 = A_0 / A_i$,

A $$\text{Engineering stress} = \frac{F}{A_0} = \frac{5000\text{ N}}{4.909\times 10^{-6}\text{ m}^2} = \mathbf{1019\times 10^6\ Pa = 1019\ MPa}$$

B $$\text{Engineering strain} = \varepsilon = \frac{l_i - l_0}{l_0} = \frac{A_0}{A_i} - 1 = \frac{4.909\times 10^{-6}\text{ m}^2}{3.464\times 10^{-6}\text{ m}^2} - 1 = 1.417 - 1 = \mathbf{0.417}$$

$$\text{True stress} = \sigma_T = \frac{F}{A_i} = \frac{5000\text{ N}}{3.464\times 10^{-6}\text{ m}^2} = \mathbf{1443\times 10^6\ Pa = 1443\ MPa}$$

$$\text{True strain} = \varepsilon_T = \ln\frac{l_i}{l_0} = \ln\frac{A_0}{A_i} = \ln\left[\frac{0.04909\text{ cm}^2}{0.03462\text{ cm}^2}\right] = \ln(1.418) = \mathbf{0.349}$$

Chapter 6, Problem 53

A stress of 75 MPa is applied in the $[0\,0\,1]$ direction on an FCC single crystal. Calculate (a) the resolved shear stress acting on the (111) $[\bar{1}\,0\,1]$ slip system, and (b) the resolved shear stress acting on the (111) $[\bar{1}\,1\,0]$ slip system.

Chapter 6, Solution 53

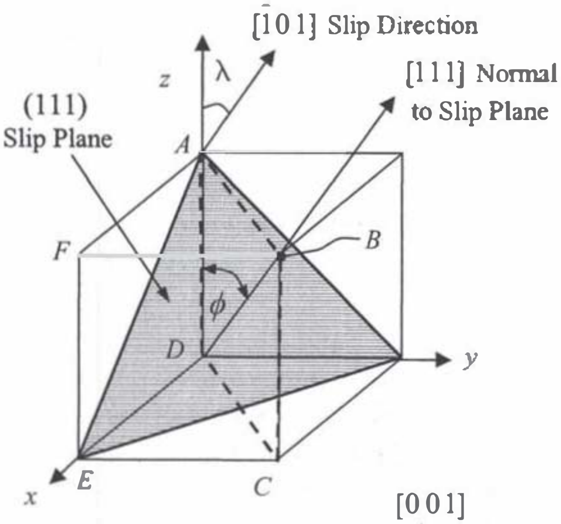
The resolved shear stress is calculated as: $\tau_r = \sigma\cos\lambda\cos\phi = 75\cos\lambda\cos\phi$. We must therefore determine the values of the angles for λ and ϕ.

(a) The slip plane and direction of the (111) $[\bar{1}\,0\,1]$ slip system are shown on the next page in Figures (a) through (c). Specifically, Fig. (a) illustrates the (111) slip plane, λ, the angle between the $[\bar{1}\,0\,1]$ slip direction and the applied axial stress direction, $[0\,0\,1]$. Referring to Fig. (b), the (001) plane is bisected by line AE and thus, $\lambda = \square\ EAD = \mathbf{45^o}$.

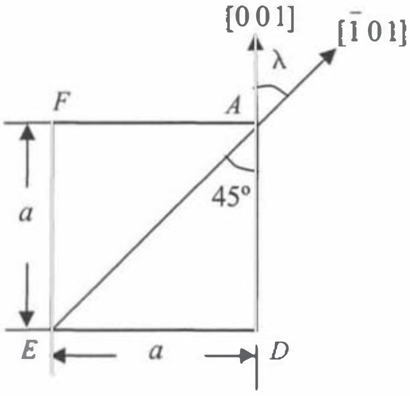
Since we are concerned with the cubic crystal system, the direction normal to the slip plane is simply the Miller indices of that plane. For the present case, the direction normal to the (111) is thus $[111]$. Referring to Fig. (c), rectangle ABCD shows ϕ between the $[1\,1\,1]$ normal and the applied axial stress direction, $[0\,0\,1]$. By geometry,

$$\cos\phi = \frac{a}{\sqrt{3}a} = \frac{1}{\sqrt{3}} = 0.5774, \quad \phi = \mathbf{54.7^o}$$

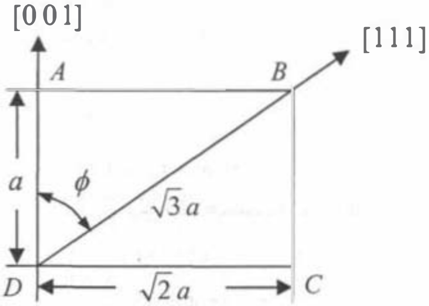
$$\tau_r = (75\text{ MPa})(\cos 45^\circ)(\cos 54.7^\circ) = \mathbf{30.6\ MPa}$$



(a) Slip System



(b) λ on (001) Plane

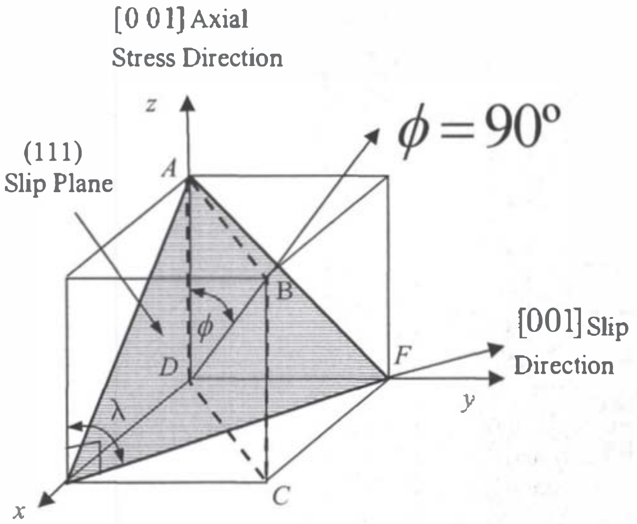


(c) ϕ on (110) Plane

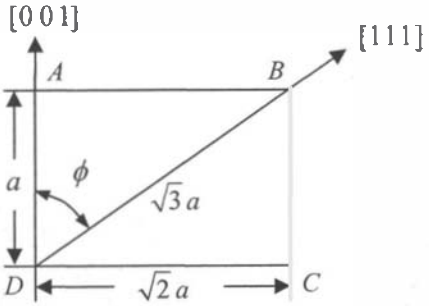
(b) Referring to Fig. (d), the slip plane and direction of the (111) $[\bar{1}\,1\,0]$ slip system are shown. As depicted by this sketch, the direction indices of the direction normal to the (111) plane are $[1\,1\,1]$, and the angle λ is 90°. The angle ϕ is shown below in Fig. (e) to lie between the axial $[0\,0\,1]$ direction and the normal to the slip plane, {110} and (111}, From rectangle ABCD, this angle is calculated as:

$$\cos\phi = a/\left[\sqrt{3}a\right] = 1/\sqrt{3} = 0.5774,$$

$[\bar{1}\ 1\ 0]$ Thus, $\tau_r = (75\text{ MPa})(\cos 90^\circ)(\cos 54.7^\circ) = \mathbf{0}.$



(d) Slip System



(e) ϕ on (110) Plane

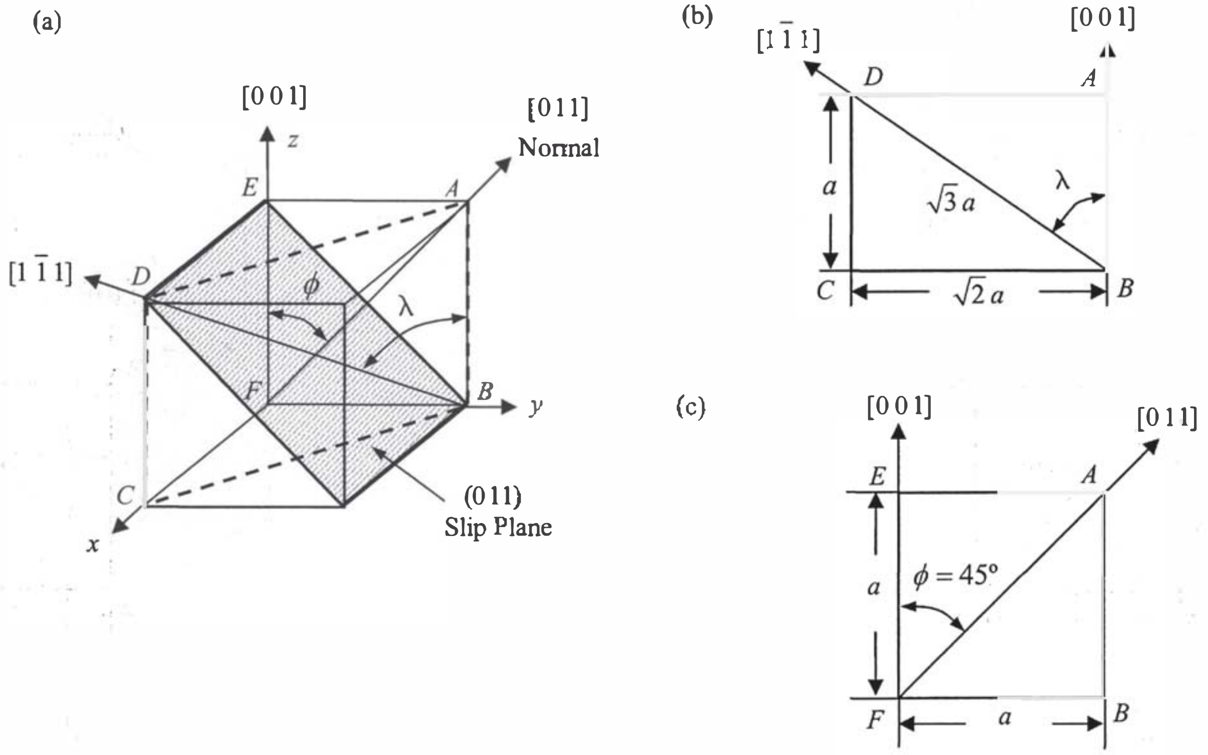
Chapter 6, Problem 54

A stress of 85 MPa is applied in the [001] direction of a unit cell of a BCC iron single crystal. Calculate the resolved shear stress for the following slip systems: (*a*) (01 1)$[1\bar{1}1]$, (*b*) (110)$[\bar{1}11]$, and (*c*) $(0\bar{1}1)$ [111].

Chapter 6, Solution 54

(a) For slip system (01 1) $[1\bar{1}1]$, λ can be calculated based upon the geometry depicted within rectangle ABCD of Fig. (b): $\cos\lambda = \cos\, a/\left[\sqrt{3}a\right] = 0.5774,\ \lambda = 54.7^{\circ}$. From the geometry depicted in Fig. (c) as square ABFE, ϕ is equal to 45°. Thus, the resolved shear stress is:

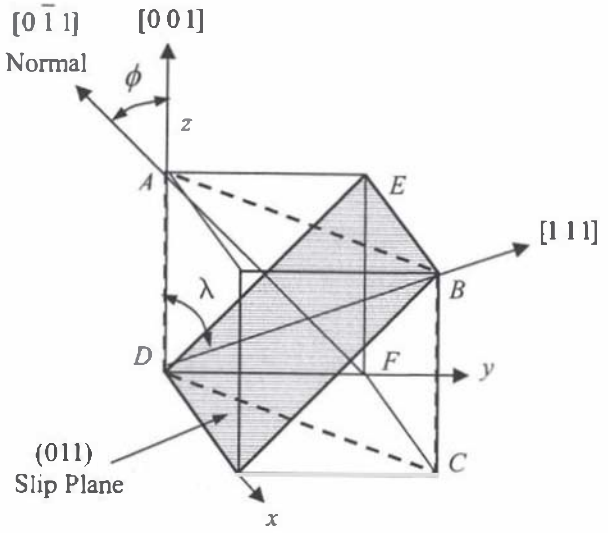
$$\tau_r = \sigma\cos\lambda\cos\phi = (85\text{ MPa})(\cos 54.7^{\circ})(\cos 45^{\circ}) = \mathbf{34.7\ MPa}$$



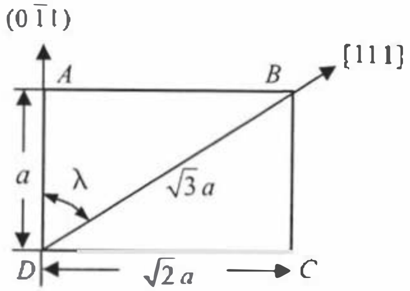
(b) For slip system (110) $[\bar{1}11]$, λ is calculated based on the geometry of rectangle EFGH in Fig. (e): $[\ 0\ \ 1\ \ 1\ ]$ $= 0.5774,\ \lambda = 54.7^{\circ}$. From the geometry of square ABCD in Fig. (f), ϕ equals 90°. Thus, $\tau_r = \sigma\cos\lambda\cos\phi = (85\text{ MPa})(\cos 54.7^{\circ})(\cos 90^{\circ}) = \mathbf{0}$.

(c) For slip system $(0\bar{1}1)[111]$, λ is calculated based on the geometry of rectangle ABCD in Fig. (h): $\cos\lambda = \cos\, a/\left[\sqrt{3}a\right] = 0.5774,\ \lambda = 54.7^{\circ}$. From the geometry of square AEFD in Fig. (i), ϕ equals 45°. Thus, $[\ \bar{1}\ \ 1\ \ 1\ ]$

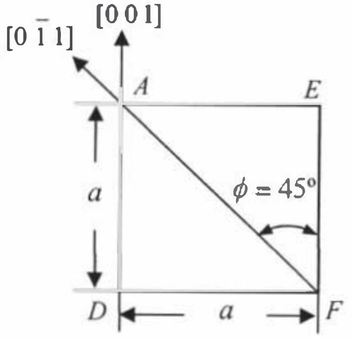
(g)


[0 1̄ 1]
Normal
[0 0 1]
ϕ
z
A
E
[1 1 1]
λ
B
D
F
y
(011)
Slip Plane
C
x

(h)


(0 1̄ 1)
A
B
[1 1 1]
a
λ
√3 a
D
√2 a
C

(i)


[0 1̄ 1]
[0 0 1]
A
E
a
ϕ = 45°
D
a
F

Chapter 6, Problem 55

The average grain diameter of an aluminum alloy is 14 μm with a strength of 185 MPa. The same alloy with an average grain diameter of 50 μm has a strength of 140 MPa. (a) Determine the constants for the Hall-Petch equation for this alloy. (b) How much more should you reduce the grain size if you desired a strength of 220 MPa?

Chapter 6, Solution 55

$$\sigma_Y = \sigma_0 + \frac{K}{\sqrt{d}}$$

(Problem refers to "yield" strength)

① $185\ \text{MPa} = \sigma_0 + \dfrac{k}{\sqrt{14\times10^{-6}\,\text{m}}}$

② $140\ \text{MPa} = \sigma_0 + \dfrac{k}{\sqrt{50\times10^{-6}\,\text{m}}}$

Two equations and Two unknowns

From equation 1, $\sigma_0 = 185 - \dfrac{k}{\sqrt{14\times10^{-6}}}$ (sub in 2)

$$\Rightarrow 140 = 185 - \frac{k}{\sqrt{14\times10^{-6}}} + \frac{k}{\sqrt{50\times10^{-6}}}$$

a) $\Rightarrow 45 - 126k \Rightarrow k = 0.36$

$\Rightarrow 185 = \sigma_0 + \dfrac{0.36}{\sqrt{14\times10^{-6}}} \Rightarrow \sigma_0 \simeq 89\ \text{MPa}$

b) $220\ \text{MPa} = 89\ \text{MPa} + \dfrac{0.36\ \text{MPa}\cdot\text{m}^{1/2}}{\sqrt{d}}$

$\sqrt{d} = \dfrac{0.36\ \cancel{\text{MPa}}\cdot\text{m}^{1/2}}{131\ \cancel{\text{MPa}}} = .0027 \Rightarrow d = 0.0027^2$

$\Rightarrow d = 7.5\times10^{-6}\ \text{m} \simeq 0.0075\ \text{mm}$

Chapter 6, Problem 56

An oxygen-free copper rod must have a tensile strength of 345 MPa (50 ksi) and a final diameter of 6.35 mm (0.250 in.) (a) What amount of cold work must the rod undergo (see Fig. 6.43)? (b) What must the initial diameter of the rod be?

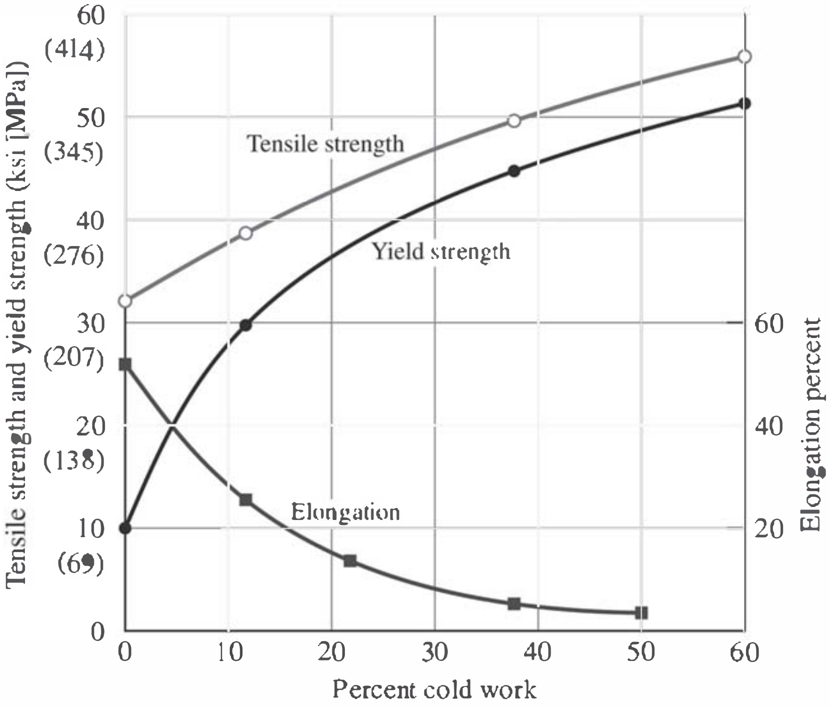


Figure 6.43

Chapter 6, Solution 56

(a) From Fig. 6.43, to attain a tensile strength of 345 MPa (50 ksi), the amount of cold work must be 40 percent.

(b) The initial diameter, based on 40 percent cold work is:

$$0.40 = \frac{\frac{\pi}{4}d_1^2 - \frac{\pi}{4}(6.35\text{ mm})^2}{\frac{\pi}{4}d_1^2}$$

$$d_1^2 - 0.40d_1^2 = 40.3225\text{ mm}^2, \qquad d_1 = \mathbf{8.20\ mm}$$

Chapter 6, Problem 57

A specimen of commercially pure titanium has a strength of 140 MPa. Estimate its average grain diameter using the Hall-Petch equation.

Chapter 6, Solution 57

6.57

$$\sigma_y = \sigma_0 + \frac{k}{\sqrt{d}}$$

(Problem refers to "yield" strength)

$$140 \text{ MPa} = 80 \text{ MPa} + \frac{0.40 \text{ MPa}\cdot\text{m}^{1/2}}{\sqrt{d}}$$

$$\Rightarrow \sqrt{d} = \frac{0.40 \cancel{\text{MPa}}\cdot\text{m}^{1/2}}{60 \cancel{\text{MPa}}} = 0.0066 \text{ m}^{1/2}$$

$$\Rightarrow d = 4.4 \times 10^{-5} \text{ m} = 0.0044 \text{ cm}$$

Chapter 6, Problem 58

Compare the strength of a copper specimen with an average grain diameter of 0.8 μm with another copper specimen with an average grain diameter of 80 nm using the Hall-Petch equation.

Chapter 6, Solution 58

6.58

$$\sigma_y = \sigma_0 + k/(d)^{1/2}$$

(Problem refers to "yield" strength)

$$\sigma_{Y(0.8\mu m)} = 25\ \text{MPa} + \frac{0.11\ \text{MPa}\cdot\text{m}^{1/2}}{\sqrt{0.8\times10^{-6}\ \text{m}}}$$

$$\sigma_{Y(0.8\mu m)} = 147.9\ \text{MPa}$$

$$\sigma_{Y(80nm)} = 25\ \text{MPa} + \frac{0.11\ \text{MPa}\cdot\text{m}^{1/2}}{\sqrt{80\times10^{-9}\ \text{m}}}$$

$$\sigma_{Y(80nm)} = 414\ \text{MPa}$$

Chapter 6, Problem 59

If it takes 115 h to 50 percent recrystallize an 1100-H18 aluminum alloy sheet at 250°C and 10 h at 285°C, calculate the activation energy in kilojoules per mole for this process. Assume an Arrhenius-type rate behavior.

Chapter 6, Solution 59

Assuming an Arrhenius-type rate behavior, $t = Ce^{Q/Rt}$, a system of two equations can be used to find the actuation energy:

$$t_1 = Ce^{Q/RT_1} \quad \text{where } t_1 = 115\text{ h} = 6900\text{ min};\ T_1 = 250^\circ\text{C} = 523\text{ K}$$
$$t_2 = Ce^{Q/RT_2} \quad \text{where } t_2 = 10\text{ h} = 600\text{ min};\ T_2 = 285^\circ\text{C} = 558\text{ K}$$

Dividing these equations, term by term,

$$\frac{t_1}{t_2} = \exp\left\{\frac{Q}{R}\left[\frac{1}{T_1} - \frac{1}{T_2}\right]\right\}$$
$$\frac{6900\text{ min}}{600\text{ min}} = \exp\left\{\frac{Q}{8.314\text{ J/mol}\cdot\text{K}}\left[\frac{1}{523\text{K}} - \frac{1}{558\text{ K}}\right]\right\}$$
$$11.5 = \exp\left\{(1.443\times10^{-5})Q\right\}$$
$$Q = \frac{\ln(11.5)}{1.443\times10^{-5}} = 169{,}255\text{ J/mol} = \mathbf{169.3\ kJ/mol}$$

Chapter 6, Problem 60

If it takes 12.0 min to 50 percent recrystallize a piece of high-purity copper sheet at 140°C and 200 min at 88°C, how many minutes are required to recrystallize the sheet 50 percent at 100°C? Assume an Arrhenius-type rate behavior.

Chapter 6, Solution 60

Since all three cases pertain to 50 percent recrystallization, the case data can be used to assess the time required for 100°C. First, Q must be calculated for the given data:

$$t_1 = 12\text{ min} = Ce^{Q/RT_1} = Ce^{Q/[(413\text{ K})R]}$$
$$t_2 = 200\text{ min} = Ce^{Q/RT_2} = Ce^{Q/[(361\text{ K})R]}$$

Dividing,

$$\frac{12\text{ min}}{200\text{ min}} = \exp\left\{\frac{Q}{8.314\text{ J/mol}\cdot\text{K}}\left[\frac{1}{413\text{K}} - \frac{1}{361\text{ K}}\right]\right\}$$

$$\ln(0.06) = \left[(-4.195\times10^{-5})Q\right]$$

$$Q = \frac{-2.813}{-4.195\times10^{-5}\text{ mol/J}} = 67,056\text{ J/mol}$$

Q may now be used to determine the time required at 100°C (373 K):

$$\frac{12\text{ min}}{t_2} = \exp\left\{\frac{67,056\text{ J/mol}}{8.314\text{ J/mol}\cdot\text{K}}\left[\frac{1}{413\text{ K}} - \frac{1}{373\text{ K}}\right]\right\}$$

$$t_2 = \frac{12\text{ min}}{e^{-2.094}} = \mathbf{97.4\ min}$$

Chapter 6, Problem 61

A 70% Cu–30% Zn brass wire is cold-drawn 20 percent to a diameter of 2.80 mm. The wire is then further cold-drawn to a diameter of 2.45 mm. (*a*) Calculate the total percent cold work that the wire undergoes. (*b*) Estimate the wire's tensile and yield strengths and elongation from Fig. 6.44.

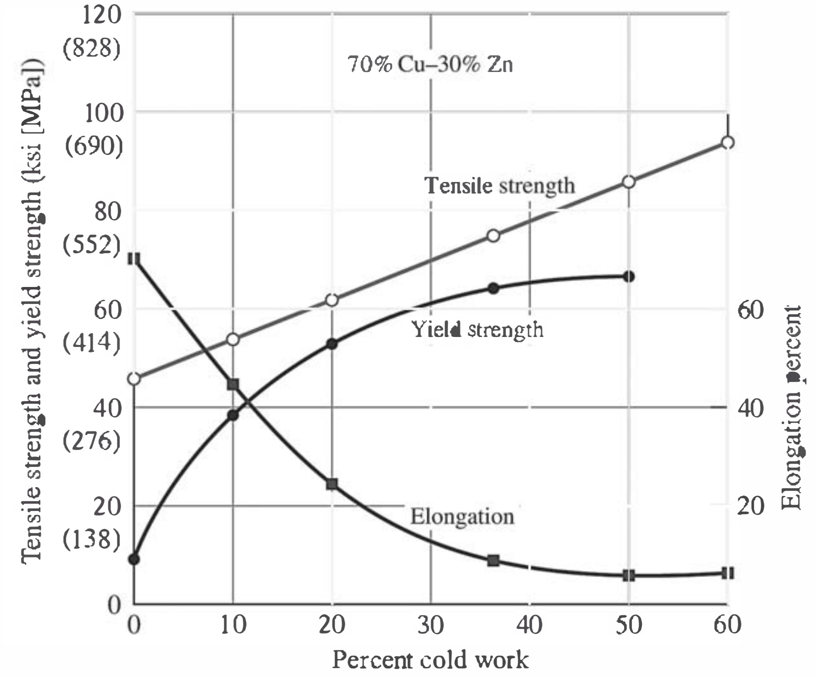


Figure 6.44

Chapter 6, Solution 61

(a) To calculate the total percent cold work, the initial wire diameter must be determined:

$$0.20 = \frac{\frac{\pi}{4}d_1^2 - \frac{\pi}{4}(2.80\text{ mm})^2}{\frac{\pi}{4}d_1^2}$$

$$d_1^2 - 0.20d_1^2 = 7.84\text{ mm}^2, \quad d_1 = 3.13\text{mm}$$

The total cold work is thus,

$$\text{total \% cold work} = \frac{(3.13\text{ mm})^2 - (2.45\text{ mm})^2}{(3.13\text{ mm})^2} \times 100\% = \frac{9.797 - 6.003}{9.797} \times 100\%$$
$$= \mathbf{38.7\,\%}$$

(b) From Fig. 6.44, for percent cold work of approximately 39%, read:

Ultimate Tensile Strength ≈**518 MPa**; Yield Strength ≈**428 MPa**; and Elongation ≈**5 %**.

Chapter 6, Problem 62

A 70% Cu–30% Zn brass sheet is to be cold-rolled from 1.78 to 1.02 mm. (a) Calculate the percent cold work,and (b) estimate the tensile strength,yield strength,and elongation from Fig. 6.44.

Chapter 6, Solution 62

(a) $\% \text{ cold work} = \dfrac{1.78 \text{ mm} - 1.02 \text{ mm}}{1.78 \text{ mm}} \times 100\% = \mathbf{42.9\%}$

(b) From Fig. 6.44, for percent cold work of approximately 43%, read:

Ultimate Tensile Strength ≈**551 MPa (80 ksi);** Yield Strength ≈**441 MPa (64 ksi);** and Elongation ≈**5 %**.

Chapter 6, Problem 63

If it takes 80 h to completely recrystallize an aluminum sheet at 250°C and 6 h at 300°C, calculate the activation energy in kilojoules per mole for this process. Assume an Arrhenius-type rate behavior.

Chapter 6, Solution 63

Assuming Arrhenius-type behavior,

$$t_1 = 80\ \text{h} = Ce^{Q/RT_1} = Ce^{Q/[(523\ \text{K})R]}$$

$$t_2 = 6\ \text{h} = Ce^{Q/RT_2} = Ce^{Q/[(573\ \text{K})R]}$$

Dividing,

$$\frac{80\ \text{h}}{6\ \text{h}} = \exp\left\{\frac{Q}{8.314\ \text{J/mol}\cdot\text{K}}\left[\frac{1}{523\ \text{K}} - \frac{1}{573\ \text{K}}\right]\right\}$$

$$\ln(13.33) = \left\{(2.007\times10^{-5})Q\right\}$$

$$Q = \frac{2.59}{2.007\times10^{-5}\ \text{mol/J}} = 129{,}048\ \text{J/mol} = \mathbf{129\ kJ/mol}$$

Chapter 6, Problem 64

If you were to make only two units of a certain component with a complicated geometry, what manufacturing process would you use?

6.64

Generally speaking, for specialized applications, or limited # of component with complex geometry, Casting would be an appropriate process. (Figure 6.2)

Forging can not always be used to create complex parts economically. Complex and expensive dies must be produced.

Machining would be very expensive as well.

Chapter 6, Problem 65

If you were to select a material for the construction of a robotic arm which would result in the smallest amount of elastic deformation (important for positional accuracy of the arm) and weight were not a critical criterion, which one of the metals given in Fig. 6.23 would you select? Why?

6.65

For robotic arm applications, to avoid elastic deformation of the arm, a material with very high stiffness (modulus of elasticity) must be used. Low density would also be important to keep the weight low.

If weight is not considered as a criterian, the best metal in Figure 6.23 would be SAE 1340 steel (Q&T) since it has the highest modulus of elasticity (note that the linear range has the steepest slope).

Chapter 6, Problem 66

How would you manufacture large propellers for large ships? What factors would influence the selection of material for this application?

6.66

a) Casting would be a good approach for objects that are large and have a complex contour. b) The material should be able to handle marine environment (salt water) and show resistance to corrosion. Also should have good fatigue failure resistance and high strength. Surface quality and hardness is also important to avoid wear.

Chapter 6, Problem 67

If you were to make a large number of components from gold, silver, or other precious metals, what metal forming process would use and why?

6.67

The main issue in production of coins from precious metals, is to minimize material waste. Even if a minute amount of material is wasted, if a large number of coins are produced, this could have considerable finanical loss. The steps in manufacturing precious metal coins are:

1- Casting to form a bar
2- Bar is rolled to the correct thickness
3- Round discs are produced using the blanking process
4- The surface is polished
5- The polished dics are compressed between two dies (called "coining") with the desired pattern in many strokes

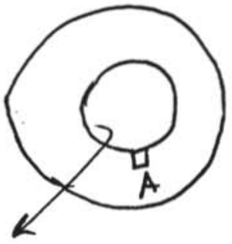
Important point: To avoid material waste, after each coin is struck, the dies are cleaned to remove precious metal residue.

Thus, the production process is much slower than the conventional coining process in the mint.

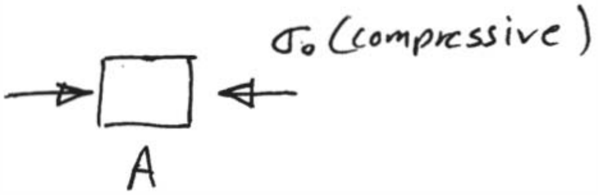
Chapter 6, Problem 68

Consider the casting of a thick cylindrical shell made of cast iron. If the casting process is controlled such that solidification takes place from the inner walls of the tube outward, as the outer layers solidify they shrink and compress the inner layers, what would be the advantage of developed compressive stresses?

6.68

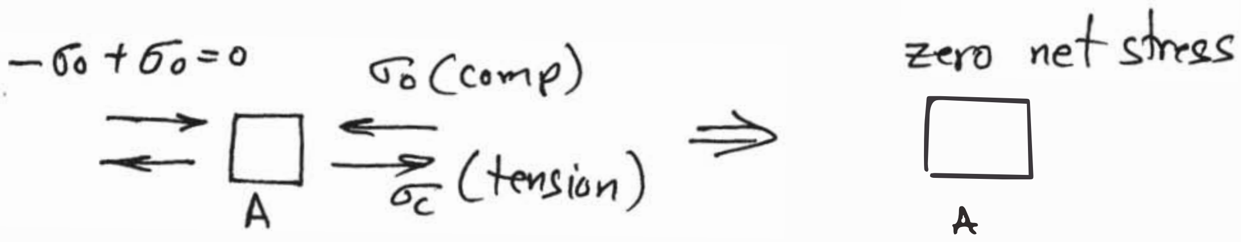


Inner core solidifies and shrink slightly. As each layer solidifies and shrinks, compressive residual stresses are produced. For instance, consider point A under compressive tangential stresses:



If internal loading creates a tensile stress of σ_0,

The total stress at the inner core at point A will be zero



The original compressive stress protects the cylinder againt tension.

Chapter 6, Problem 69

Consider casting a cube and a sphere on the same volume from the same metal. Which one would solidify faster? Why?

6.69

For a cube and a sphere of the same volume,

$$R = 0.62\ a$$

radius of the sphere

side of the cube

Surface area of the sphere $= 4\pi R^2 = 4.82 a^2$

Surface area of the cube $= 6a^2$

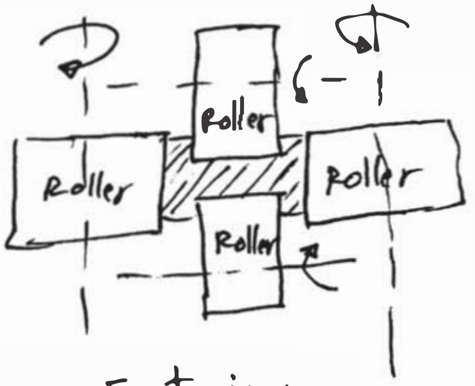
Cube has a larger surface area will lose heat more rapidly and will cool faster.

Chapter 6, Problem 70

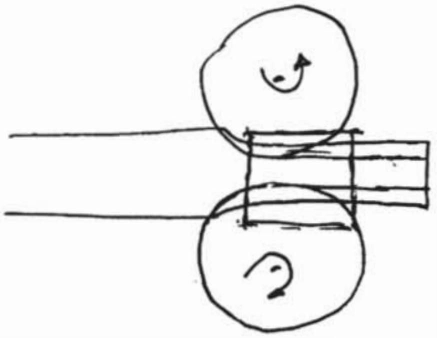
Design a process that produces long bars with an "H" cross-section from steel (indicate hot or cold if applicable). Draw schematics to show your procedure.

6.70

Rolling can produce an "H" cross-section at high temperatures (hot rolling).

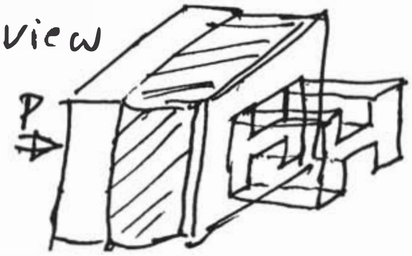


Front view



Side view

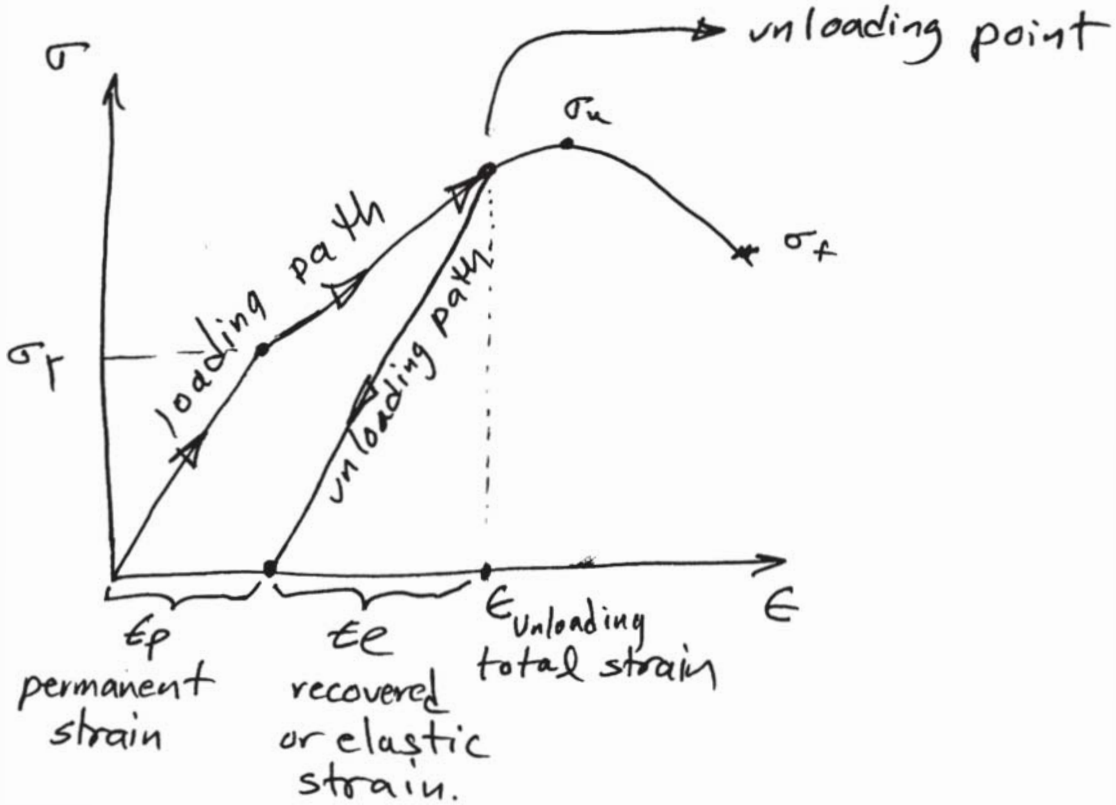
You may also use hot extrusion.



Chapter 6, Problem 71

(a) Draw a generic engineering stress-strain diagram for a ductile metal and highlight the key strength points (yield, ultimate, and fracture strength) on the curve. Schematically, show what happens if you load the specimen just below its ultimate tensile strength point and then unload to zero. (b) Will the specimen behave differently if you load it again? Explain.

6.71

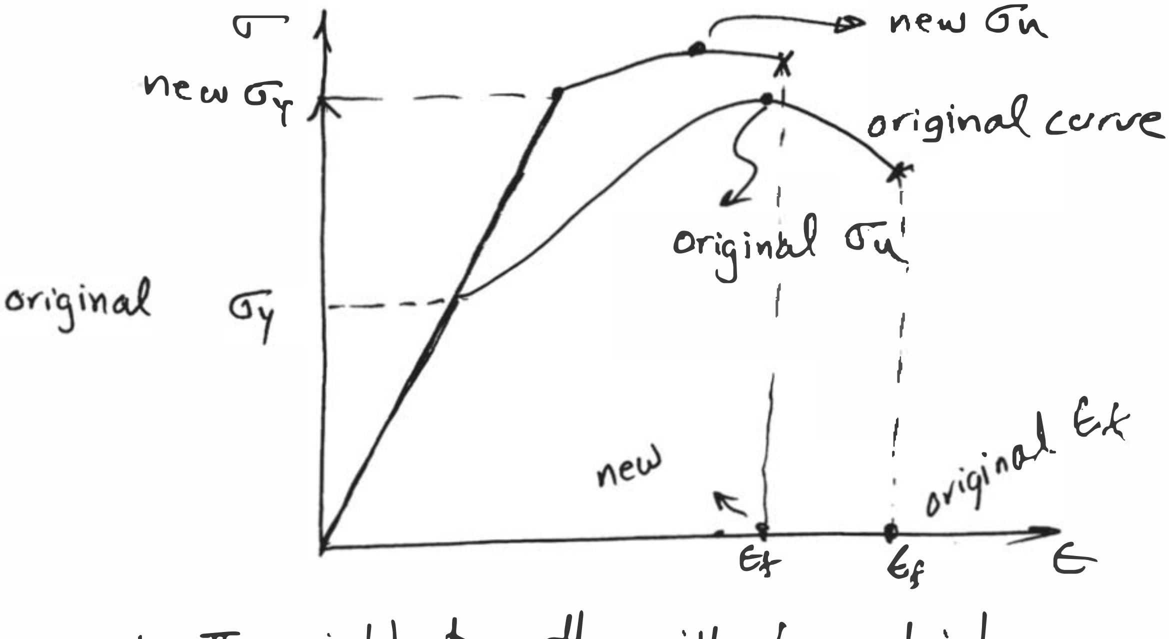


a) If the specimen is unloaded after the yield point has been passed, the un loading path will be paralel to the linear region. The curve will not return to zero and a significant portion of strain will remain in the form of plastic deformation. It is important to note that some strain will be recovered during unloading (elastic strain).

6.71 cont.

The strain just before unloading $= \epsilon_p + \epsilon_e = \epsilon_{total}$
After unloading ϵ_e is recovered and only ϵ_p remains. The permanent strain is due to breaking of bonds between atoms during the slip process.

b) If the unloaded specimen is loaded again, the σ-ϵ curve will be different as follows:



1- The yield strength will be higher
2- The ultimate strength will be slightly larger
3- The fracture strain will be lower.

cont.

6.71 cont.

Thus, as a result of the original loading and unloading, the material becomes stronger (strain hardens) and more brittle. This is due to dislocation formation.

Important point: the modulus of elasticity remains unchanged. stiffness is not affected significantly by formation of dislocations.

Chapter 6, Problem 72

6.72

(*a*) State the assumption behind the development of Eq. 6.14. (*b*) Is Eq. 6.14 (or its underlying assumption) valid throughout the engineering stress-strain curve?

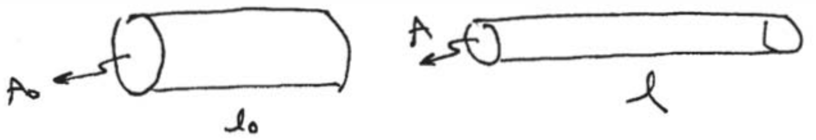
a) The assumption is that the volume of the gage-length remains constant. Thus as the length of cylinderical section increases, the diameter decreases.

$l > l_0 \Rightarrow A < A_0$ (tension)

$l < l_0 \Rightarrow A > A_0$ (compression)

Thus $l_0 A_0 = l A$

b) The assumption of constancy of volume is valid until the necking of the specimen is initiated. once the specimen forms a neck, this assumption is no longer valid ($l_0 A_0 \neq l A$) and equation 6.14 can not be used.



$A_0 l_0 = A l$

(befor a neck is formed)

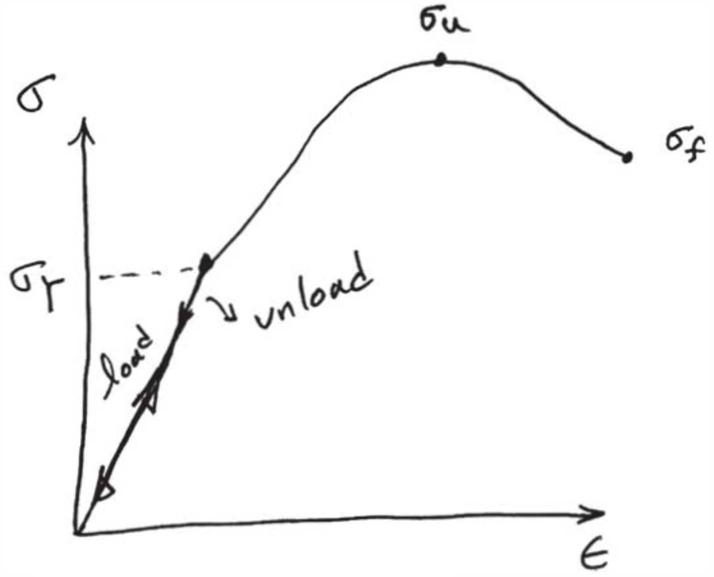
(after a neck is formed)

$A_0 l_0 \neq A l$

Chapter 6, Problem 73

Draw a generic engineering stress-strain diagram for a ductile metal and highlight the key strength points (yield, ultimate, and fracture strength) on the curve. (*a*) Schematically, show what happens if you load the specimen just below its yield point and then unload to zero. (*b*) Will the specimen behave differently if you load it again? Explain.

6.73

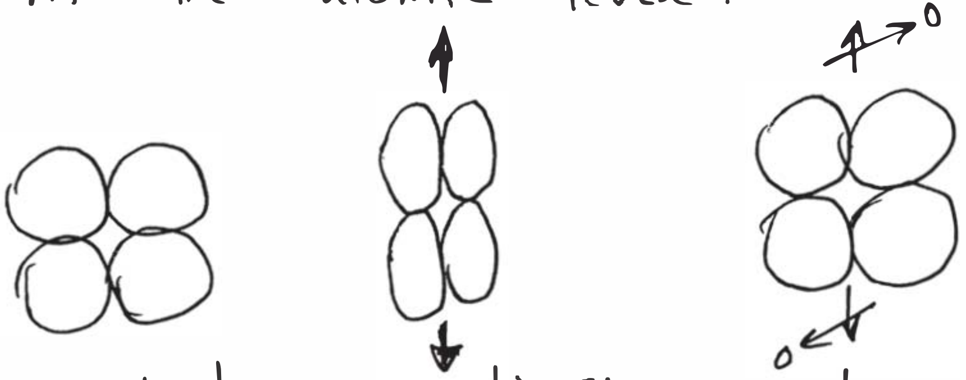


a) If the specimen is loaded below the yield point, removal of load will cause the specimen to regain its original shape and dimensions. The loading path is identical to the unloading path.

6.73 cont.

b) Loading of the specimen, again, will not produce a different σ-ε curve. The material is essentially unchanged.

At the atomic level:



no load

elongation of atoms due to tensile load.

return to original shape.

Chapter 6, Problem 74

In the rolling process, the selection of the roll material is critical. Based on your knowledge of both hot and cold rolling, what properties should the roller material have?

6.74

The roll material must have very high modulus of elasticity to avoid excessive elastic deformation. must have very high yield strenght not to undergo plastic deformation. High toughness is required so the rolls won't fracture.

For hot rolling, the material must have high temperature strength and must be creep resistant. Tool steel would be a good candidate for roll material.

Chapter 6, Problem 75

When manufacturing complex shapes using cold forging or shape rolling operations, the mechanical properties such as yield strength, tensile strength, and ductility measure differently dependent on the location and direction on the manufactured part. (*a*) How do you explain this from a micro point of view? (*b*) Will this happen during hot forging or rolling? Explain your answer.

6.75

a) During metal forming processes such as Cold rolling or forging, the grains stretch in the direction of flow (Figure 6.41). As a result, the component becomes stronger in this direction (direction of elongation of grains). In a direction transverse to the direction of flow, the material will be less strong. This is an example of anisotropic (direction-dependent) behavior. Similar changes occur, in hardness, yield strength and ductility.

<u>6.75 cont.</u>

b) During hot forming processes, as grains are elongated, the high temperature will cause continuous recrystallization and grain growth. As a result, the grains become more equiaxed. Directional changes in properties do not occur.

Chapter 6, Problem 76*

(a) Derive the relationship between true strain and engineering strain. (Hint: Start with expression for engineering strain.) (b) Derive a relationship between true stress and true strain. (Hint: Start with $\sigma_t = F/A_i = (F/A_o)(A_o/A_i)$.)

6.76

a)

$$\epsilon = \frac{l_i - l_o}{l_o} = \frac{l_i}{l_o} - \frac{l_o}{l_o} = \frac{l_i}{l_o} - 1$$

$$\Rightarrow \quad \epsilon + 1 = \frac{l_i}{l_o}$$

$$\epsilon_t = \ln \frac{l_i}{l_o} = \ln(\epsilon + 1) \Rightarrow \boxed{\epsilon_t = \ln(\epsilon + 1)}$$

b)

$$\sigma_t = \frac{F}{A_i} = \frac{F}{A_o}\left(\frac{A_o}{A_i}\right) \quad ; \quad \frac{A_o}{A_i} = \frac{l_i}{l_o}$$

$$\Rightarrow \sigma_t = \frac{F}{A_o}\left(\frac{l_i}{l_o}\right) \quad ; \quad \frac{l_i}{l_o} = (1+\epsilon) \text{ see part a}$$

$$\frac{F}{A_o} = \sigma$$

$$\Rightarrow \boxed{\sigma_t = \sigma(1 + \epsilon)}$$

Chapter 6, Problem 77

The engineering yield strength of a copper alloy is 165 MPa and the modulus of elasticity is 110 GPa. (*a*) Estimate the engineering strain just before yield. (*b*) What is the corresponding true strain? Are you surprised? Explain.

6.77

a) Just before yield, the specimen is in the elastic range. Linear relationship applies:

$$\sigma = E\epsilon \Rightarrow \epsilon = \frac{\sigma}{E} = \frac{165\,\text{MPa}}{110\,\text{GPa}} = 0.001 \quad (0.1\%)$$

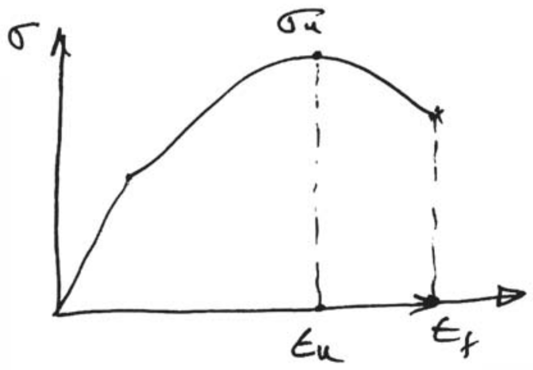
b) $\epsilon_t = \ln(1+\epsilon) = \ln(1.001) \cong 0.001$

The engineering and true strains are very close. This is not surprising because the specimen is still in the elastic range and any reduction in area is not significant. Refer to Figure 6.24 and note that the true and the engineering stress-strain curves are identical in the elastic region.

Chapter 6, Problem 78

For the alloy in problem 6.77, the engineering ultimate tensile strength is 267 MPa where the corresponding engineering strain 0.18. The reduction in area just before fracture is measured to be 34%. Determine (*a*) the true stress corresponding to the engineering ultimate tensile strength, and (*b*) the true strain just before fracture.

6.78



$\sigma_u = 267\,\text{MPa} \quad \epsilon_u = 0.18$

$$\frac{A_i}{A_o} = 0.66$$

a) $\sigma_t = \sigma(\epsilon + 1) = (267\,\text{MPa})(0.18 + 1)$

$\sigma_t = 315\,\text{MPa} \qquad \sigma_t > \sigma$

b) $\epsilon_t = \ln\left(\frac{A_o}{A_i}\right) = \ln\left(\frac{1}{0.66}\right) = 0.41 \quad (41\%)$

Chapter 6, Problem 79

The material for a rod of cross-sectional area 1742 mm^2 and length 1905 mm must be selected such that under an axial load of 533,333 N, it will not yield and the elongation in the bar will remain below 2.67 mm. (a) Provide a list of at least three different metals that would satisfy these conditions. (b) Narrow the list down if cost is an issue. (c) Narrow the list down if corrosion is an issue. Use Appendix I for properties and cost of common alloys only.

6.79

a) Since the rod is to remain elastic, Hooke's law applies:

$$\sigma = E\epsilon \Rightarrow E = \frac{\sigma}{\epsilon}$$

$$\sigma = \frac{533{,}333\,N}{1742\,mm^2} = 306\,MPa$$

$$\epsilon = \frac{2.67\,mm}{1905\,mm} = 0.0014$$

$$\Rightarrow E = \frac{\sigma}{\epsilon} = \frac{306\,MPa}{0.0014} = 218.6\,GPa$$

The modulus of elasticity must be al least 218.6 GPa. However this requirement is easily met as metals have significantly higher moduli of elasticity in the GPa range. (refer to Appendix I)

- any steel, aluminum, copper, ... alloy will be suitable.

cont.

6.79 cont.

b) low carbon steel alloys, Alloy 1006, 1020 are inexpensive. Al and Copper alloys are more expensive.

c) Al alloys are resistant to corrosion.

The dimensions in the x and y directions will reduce due to poisson effect.

$$\Delta l_x = \Delta l_y = l_{0x}\epsilon_x = l_{0y}\epsilon_y$$

$$\epsilon_x = \epsilon_y = -\nu\,\epsilon_z = -(0.27)(.0026) = -.0007$$

(compressive strain)

$$\Rightarrow l_x = l_y = 25.4\,\text{mm} - 25.4\,\text{mm}\,(-0.0007)$$

$$l_x = l_y = 25.38\,\text{mm}$$

b) 28% strain is a large amount of strain. Refer to Figure 6.3 and note that many metals fail below 24% strain. In our case, we know that the specimen has not failed but since the strain is large, it is better to use the true stress instead of the engineering stress.

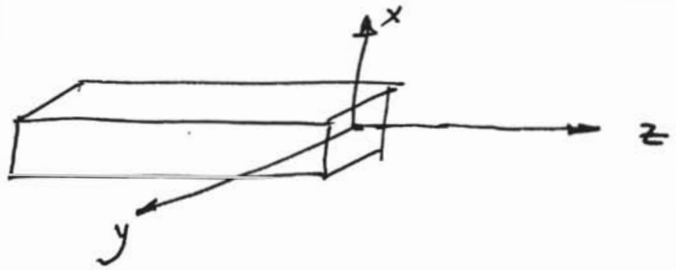
Also, 25% reduction in area is large and will cause a large difference between σ & σ_t.

Chapter 6, Problem 83

(*a*) Show, using the definition of the Poisson's ratio, that it would be impossible to have a negative Poisson's ratio for isotropic materials. (*b*) What would it mean for a material to have a negative Poisson's ratio?

6.83

$$\nu = -\frac{\epsilon_x}{\epsilon_z}$$



if ν is negative, then both ϵ_x & ϵ_z must be positive since the equation has a negative sign to begin with. For both ϵ_x and ϵ_z to be positive would mean ~~that~~ under tension in the z-direction, the specimen becomes longer ($\epsilon_z > 0$) and thicker ($\epsilon_x > 0$). This is not possible.

(some anisotropic fiber reinforced composite materials may have a negative ν but not isotropic materials).

b) The yield strength of Al 2024 is 345 MPa. The applied stress of 414 MPa exceeds the yield strength under uniaxial loading condition. Thus the component has yielded. As a result, the Hooke's Law does not apply and is not valid.

We can estimate the strain at 414 MPa for Al 2024-T81 from Figure 6-23 to be approximately 0.008. Note that the cube will be permanently deformed. ($\epsilon_z = 0.008$; $\epsilon_x = \epsilon_y \simeq -.3(0.008) \simeq .00024$

Similar to part a, $l_z = 1.008$

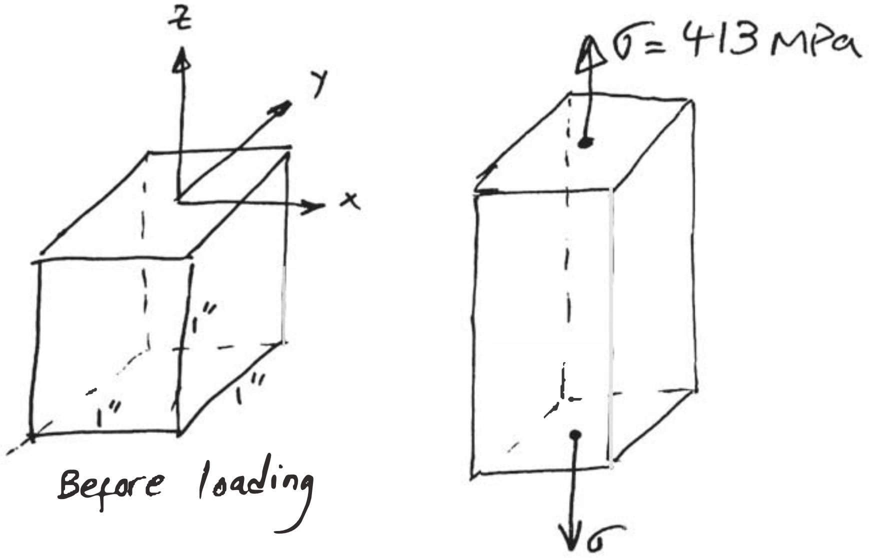
$l_x = l_y \cong .99976$

(Please note that these measurement are highly approximated)

Chapter 6, Problem 84

A 25.4 mm cube of tempered stainless steel (alloy 316) is loaded along its zdirection under a tensile stress of 413 MPa. (a) Draw a schematic of the cube before and after loading showing the changes in dimension. (b) Repeat the problem assuming the cube is made of tempered aluminum (alloy 2024). Use Fig. 6.15b and Appendix 1 for relevant data.

6.84



a) At 413 MPa of uniaxial stress, the component will not yield ($\sigma_y = 509.9$ MPa (74 ksi); Appendix I). The component is elastic and Hooke's Law applies.

$$\sigma_z = E\epsilon_z \quad \Rightarrow \quad \epsilon_z = \frac{\sigma_z}{E} = \frac{509.9\ \text{MPa}}{192.7\ \text{GPa}} = 0.0026$$

Length in the z direction after loading will be

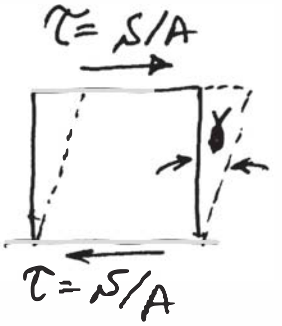
$$\Delta l_z = l_{0z}\,\epsilon_z = 25.4\ \text{mm}\,(0.0026) = 0.067\ \text{mm}$$

$$l_z = 25.467\ \text{mm}$$

Chapter 6, Problem 85

A 25.4-mm cube tempered stainless steel (alloy 316) is loaded on the same face with a shear stress of 413 MPa. Draw a schematic of the cube before and after loading showing any changes in the shape. (G=75.86 GPa; use Fig. 6.17c)

6-85



$$\tau = \frac{S}{A} = 206.5\,\text{MPa}$$

for elastic region $\tau = G\gamma$.

At a stress level of 206.5 MPa, the material is elastic. Hooke's law for shear can be applied.

$$\gamma = \frac{\tau}{G} = \frac{206.5\,\text{MPa}}{75.86\,\text{GPa}} = 0.0027 \text{ rad}$$

$(\sim 0.15^\circ)$

Note that shear stress causes angular distortions or shear strains. In contrast normal stress cause length changes or normal strain.

Chapter 6, Problem 86*

Three different metal alloys are tested for their hardness using the Rockwell scale. Metal 1 was rated at 60 R_B, metal 2 at 60 R_C, and metal 3 at 60 R_F. What do these ratings tell about these metals? Give an example of a component that is made of a metal that has a hardness of around 60R_C.

6-86

a) The metal with 60 R_C hardness is the hardest followed by 60 R_B followed by 60 R_f. It is important to know the relative position of these scales to each other.

b) The hardness 60 R_C is very high. Steel cutting tools are usually made of steels with such high hardness. The high hardness protects the tool from surface wear and damage.

Chapter 6, Problem 87

A fellow student asks you "What is the modulus of elasticity of plain carbon steel?" Can you answer this question? Explain.

6.87

yes, we can answer that question. There are only minor variations in modulus of elasticity within each alloy system due to composition. For istance most steels regardless of composition have a modulus of elasticity of 200 - 205 GPa. stainless steels are a little lower at 195 GPa. Aluminum alloys are between 69 and 73 GPa. So, generally speaking that question can be answered in a range that is acceptable.

Chapter 6, Problem 88

A fellow student asks you " What is the yield strength of titanium?" Can you answer this question? Explain.

6.88

No, it is impossible to answer this question. One must know, the composition of the alloy, heat treatment condition, and cold work status to be able to answer the question accurately. (Range: 140GPa – 710GPa)

Chapter 6, Problem 90

Why do BCC metals in general require a higher value of τ_c than FCC metals when they both have the same number of slip systems?

6.90

Examination of Table 6.4 shows that indeed BCC metals have in general a higher τ_c than both FCC and HCP metals. Recall from chapter 3 that although both FCC and HCP metals have close-packed planes, BCC metals do not. This causes the slip process to be more demanding and thus higher τ_c.

Chapter 6, Problem 93

(*a*) In loading of a single crystal, how would you orient the crystal with respect to the loading axis to cause a resolved shear stress of zero? (*b*) What is the physical significance of this, i.e., under these conditions, what happens to the crystal as σ increases?

6.93

a) If the loading axis is perpendicular to the slip plane, $\phi = 0°$, or $\lambda = 90°$. Alternatively, if the loading axis is parallel to the slip plane, $\phi = 90°$, $\lambda = 0$.

Under the above conditions, τ will be zero.

b) under these conditions slip will not occur. As σ_0 increases and reaches a critical level, the crystal ruptures or fractures without slip.

Chapter 6, Problem 94

Starting with a 51-mm (2") diameter rod of brass,we would like to process 5.1-mm (0.2") diameter rods that possess minimum yield strength of 276 MPa (40 ksi) and a minimum elongation to fracture of 40%, (see Fig. 6.44). Design aprocess that achieves that. Hint:Reduction of the diameter directly from 51 mm (2") to 5.1 mm (0.2") is not possible, why?

6.94

According to Figure 6.44, to achieve a yield strength of 276 MPa (40 ksi) and % elongation to fracture of 40%, the workpiece must be extruded or drawn to no more than 12% CW.

However, the problem requires that we reduce the diameter from 51mm(2") to 5.1mm(0.2").

$$\%CW = \frac{51^2 - 5.1^2}{51^2} = 0.99 \qquad 99\ \%\ CW.$$

Clearly, it is not possible to achieve 99% CW in one pass, as the metal becomes very brittle around 50 to 60% CW (see Figure 6.44).

We must reduce the diameter by 30 to 40%, anneal to soften the metal, and then reduce again. (repeat) The final pass in reduction of the diameter should be such that we achieve ~ 12% CW.

cont.

6.94 cont.

$$0.12 = \frac{d^2 - 5.1^2}{d^2}$$

$$\Rightarrow 0.12d^2 - d^2 = -5.1^2 \qquad \text{or } 0.88d^2 = 26.01$$

$$d^2 = \frac{26.01}{0.88} = 29.56$$

$\Rightarrow d \cong 5.44\text{mm}$ (diameter before the last reduction)

Chapter 6, Problem 97

The cupro-nickel substitutional solid solution alloys Cu-40 wt% Ni and Ni-10 wt% Cu have similar tensile strengths. For a given application that only tensile strength is important, which one would you select?

6-97

If the two alloys have similar strengths and strength is the main criteria, one would want to minimize cost. Because Ni is significantly more expensive than Cu, one should select the alloy with less Ni.

Chapter 6, Problem 98

Without referring to tensile strength data or tables, which of the following substitutional solid solutions would you select if higher tensile strength was the selection criterion: Cu-30 wt% Zinc or Cu-30 wt% Ni? Hint: Compare melt temperatures of Cu, Ni, and Zn.

6.98

The melt temperature of nickel is 1452°C and that of zinc is 419°C. This means the strength of the bonds between Ni atoms is significantly higher than zinc atoms. Thus, it will be harder to break Ni bonds. As aresult, the alloy that contain Ni will be stronger. (recall from chapter2 that Ni forms a mixture of metallic and covalent bonds).