Problems and Solutions to Smith/Hashemi
Foundations of Materials Science and Engineering 5/e

Chapter 4, Problem 1

Define the homogeneous nucleation process for the solidification of a pure metal.

Chapter 4, Solution 1

In a homogeneous nucleation process, the liquid metal is sufficiently undercooled and thus able to provide the atoms to form nuclei.

Chapter 4, Problem 2

In the solidification of a metal, what is the difference between an embryo and a nucleus? What is the critical radius of a solidifying particle?

Chapter 4, Solution 2

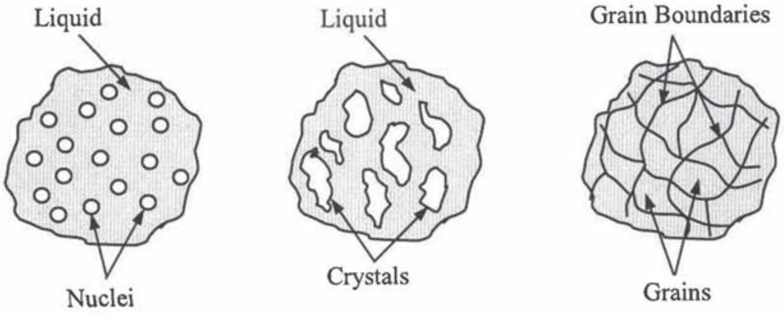
An embryo refers to a solidified cluster of atoms which is unstable and can thus redissolve. A nucleus is a comparatively larger stable cluster which can grow into a crystal. The dimension defining the boundary between an embryo and a nucleus is the critical radius.

Chapter 4, Problem 3

The solidification process of a pure metal liquid involves 3 stages. Describe and illustrate this process.

Chapter 4, Solution 3

In general, the solidification of a pure metal involves: the formation of stable nuclei in the liquid melt; the growth of these nuclei into stable nuclei in the liquid melt; and the formation of a solidified structure containing grains formed from the crystals. These three stages are illustrated below.



Chapter 4, Problem 4

In the solidification of a pure metal, what are the two energies involved in the transformation? Write the equation for the total free-energy change involved in the transformation of liquid to produce a strain-free solid nucleus by homogeneous nucleation. Also illustrate graphically the energy changes associated with the formation of a nucleus during solidification.

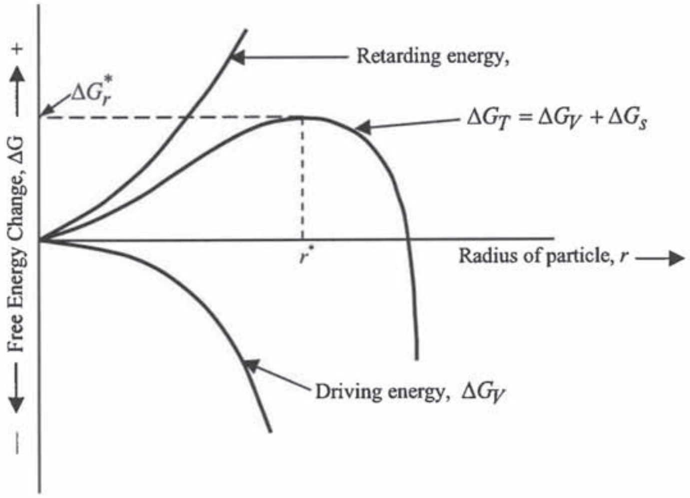
Chapter 4, Solution 4

The two energies involved in the solidification of a pure metal are surface free energy and volume free energy. These energies contribute to the total free energy, ΔG_T:

$$\Delta G_T = \Delta G_V + \Delta G_s$$

$$= \frac{3}{4}\pi r^2 \Delta G_\upsilon + 4\pi r^2 \gamma$$

= Volume free energy + Surface free energy



Chapter 4, Problem 5

Distinguish between homogeneous and heterogeneous nucleation for the solidification of a pure metal.

Chapter 4, Solution 5

In homogeneous nucleation of a liquid metal, the nuclei are formed from atoms of the metal solidifying. In heterogeneous nucleation, impurity atoms or container surfaces act as nucleating agents.

Chapter 4, Problem 6

Distinguish between equiaxed and columnar grains in a solidified metal structure.

Chapter 4, Solution 6

Equiaxed grain dimensions are approximately equal in all directions whereas columnar grains are elongated in a particular direction.

Chapter 4, Problem 7

During solidification, how does the degree of undercooling affect the critical nucleus size? Assume homogeneous nucleation.

Chapter 4, Solution 7

In general, the greater the degree of undercooling of a liquid melt, the smaller the critical radius of the nuclei formed.

Chapter 4, Problem 8

How can the grain size of a cast ingot be refined? How is grain refining accomplished industrially for aluminum alloy ingots?

Chapter 4, Solution 8

The grain size of a cast ingot can be refined by: solidifying the metal at a rapid rate; and adding grain refining agents (heterogeneous nucleating agents). Grain refining of aluminum ingots is accomplished through chill casting and by adding grain refining agents such as titanium and/or boron.

Chapter 4, Problem 9

Describe the grain structure of a metal ingot that was produced by slow-cooling the metal in a stationary open mold.

Chapter 4, Solution 9

In general, equiaxed grains are formed adjacent to the cold mold wall where rapid cooling occurs during solidification. Elongated columnar grains are formed in the metal ingot interior, in the direction of thermal gradients, due to slow cooling of the metal in the mold interior.

Chapter 4, Problem 10

How are large silicon single crystals for the semiconductor industry produced?

Chapter 4, Solution 10

Large single crystals of silicon are produced using a pure silicon seed crystal with a pure silicon melt (Czochralski process).

Chapter 4, Problem 11

What special techniques must be used to produce single crystals?

Chapter 4, Solution 11

Single crystals can be produced by introducing a single crystal as a seed crystal. The seed continuously rotates as it is slowly lowered and then withdrawn from the melt.

Chapter 4, Problem 12

Distinguish between a substitutional solid solution and an interstitial solid solution.

Chapter 4, Solution 12

A substitutional solid solution is one in which the solute atoms of the elements replace those of the solvent atoms in the crystal lattice. An interstitial solid solution is one in which the solute atoms of the elements are positioned in the interstitial spaces between the solvent atoms of the crystal lattice.

Chapter 4, Problem 13

What is a metal alloy? What is a solid solution?

Chapter 4, Solution 13

A metal alloy is a mixture of two or more metals or of a metal (metals) and a non-metal (nonmetals). A solid solution is a type of alloy which is solid and consists of two or more elements atomically dispersed in a single phase structure.

Chapter 4, Problem 14

Define the formation of the interstitial solid solutions. Explain why.

Chapter 4, Solution 14

One atom is much larger than another can form interstitial solid solution. Small-sized atoms, e.g. H, C, N, fix into the empty spaces available among the solvent or parent atoms.

Chapter 4, Problem 15

What are the conditions that are favorable for extensive solid solubility of one element in another (Hume-Rothery rules)?

Chapter 4, Solution 15

Four conditions favor extensive solid solubility:

1. less than 15% difference between the atomic diameters of the elements forming the solid solution;
2. identical valence of the elements;
3. similar electronegativities;
4. common crystal structure of the elements.

Chapter 4, Problem 16

Describe and illustrate the following imperfections that can exist in crystal lattices: (a) Frenkel imperfection, (b) Schottky imperfection, and (c) Is it possible to have a Frenkel defect in the covalent crystal?

Chapter 4, Solution 16

a) A Frenkel imperfection is a vacancy-interstitialcy pair which sometimes occurs in ionic crystals.
b) A Schottky imperfection is a cation-anion divacancy which sometimes occurs in ionic crystals.

Frenkel Imperfection

Schottky Imperfection

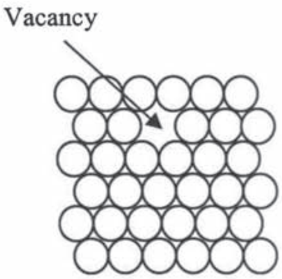
c) No, impossible. Because there is no ion in the covalent crystal.

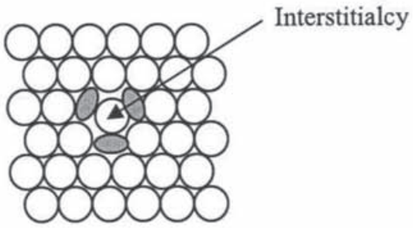
Chapter 4, Problem 17

Describe and illustrate the following types of point imperfections that can be present in metal lattices: (a) vacancy, (b) divacancy, and (c) interstitialcy.

Chapter 4, Solution 17

a) A vacancy, a point defect, is an atomic site which is missing an atom.
b) A divacancy is a defect in a crystal lattice where two atoms are missing from adjoining atomic sites.
c) An interstitialcy is a point defect where an atom occupies an interstitial site between surrounding atoms in normal sites.





Chapter 4, Problem 18

Describe the concept of volume or three dimensional defects.

Chapter 4, Solution 18

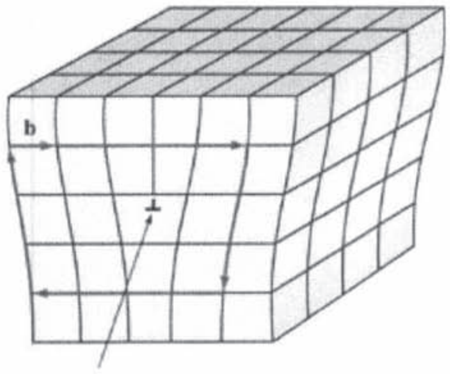
Volume or three-dimensional defects form when a cluster of point defects join to form a 3-D void ore a pore. The size of a volume defect may range from a few nanometers to centimeters or sometimes larger.

Chapter 4, Problem 19

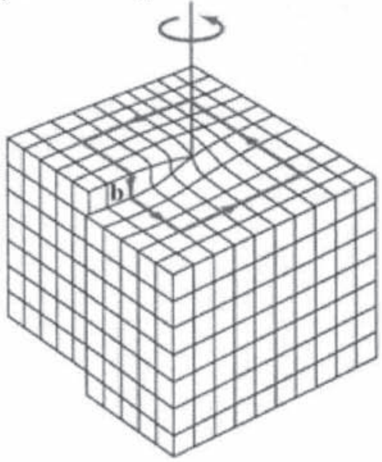
Describe and illustrate the edge- and screw-type dislocations. What type of strain fields surround both types of dislocations?

Chapter 4, Solution 19

An edge dislocation is a line imperfection caused by an extra half plane of atoms between two normal planes of atoms. Whereas a screw dislocation is a line imperfection created by applying upward and downward shear stress to regions of a perfect crystal separated by a common plane.

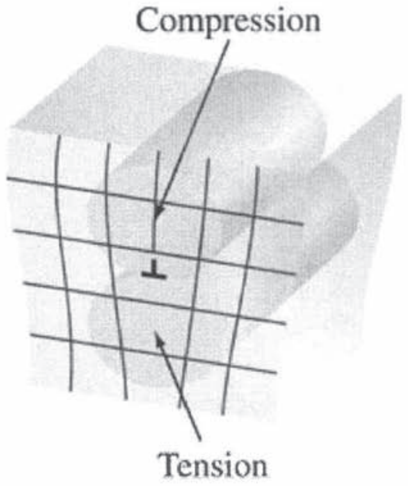


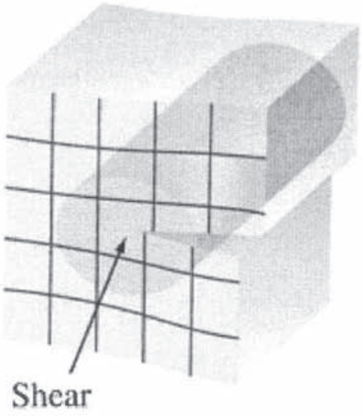
Dislocation Line



Screw Dislocation

The strain fields associated with the edge and screw dislocations are shown below:





Chapter 4, Problem 20

Describe the structure of a grain boundary. Why are grain boundaries favorable sites for the nucleation and growth of precipitates?

Chapter 4, Solution 20

Grain boundaries are surface imperfections that separate grains of different orientations. The grain boundary, a narrow region between two grains, is approximately two to five atomic diameters in width and contains mismatched atoms from adjacent grains. Grain boundaries are favorable sites for the nucleation and growth of precipitates because the irregular atom arrangement at grain boundaries provides lower atomic packing and high energy. Atoms are thus able to diffuse more rapidly to form precipitates.

Chapter 4, Problem 21

Describe and illustrate for the following planar defects: (a) twins, (b) low-angle tilt boundaries, (c) small-angle twist boundaries, (d) external surfaces, and (e) stacking faults. For each deflect, express the impact on the properties of the material.

Chapter 4, Solution 21

(a) twins:

Twins or twin boundaries are another example of a two-dimensional defect. A twin is defined as a region in which a mirror image of the structure exists across a plane or a boundary.

(b) small angle tilt boundaries : When an array of edge dislocations are oriented in a crystal in a manner that seems to misorient or tilt two regions of a crystal, Fig. 4.24a, a two-dimensional defect called a small angle tilt boundary is formed. The misorientation angle θ for a small angle boundary is generally less than 10°.

(d) small angle twist boundaries: When an array of screw dislocations are oriented in a crystal in a manner that seems to misorient two regions of a crystal, Fig. 4.24b, a two-dimensional defect called a small angle twist boundary is formed.

(e) external surfaces : all surfaces are defects because the atoms on the surface are only bonded to other atoms on one side and as a result possess a higher state of energy.

(f) stacking faults: when one or more planes of atoms in the stacking series are missing

Twins, small angle boundaries, and stacking faults are regions of high energy due to local lattice distortions and tend to strengthen a metal.

Chapter 4, Problem 22

Why are grain boundaries easily observed in the optical microscope?

Chapter 4, Solution 22

Grain boundaries can be easily observed under an optical microscope because they etch more rapidly than grains. Chemical etching thus produces tiny grooves along grain boundaries which appear as dark lines under an optical microscope because they reflect light less intensely.

Chapter 4, Problem 23

Explain how one can measure the average grain size of a metal using a micrograph at a known magnification.

Chapter 4, Solution 23

Once a photomicrograph is prepared at a specific magnification, a random line of known length is drawn on the photomicrograph. The number of grains intersected by this line are then determined and the ratio of the number of grains to the actual length of the line is determined, n_L. The average grain diameter is then determined using Eq. 4.8. C is a constant (C=1.5 for typical microstructures), and M is the magnification.

Chapter 4, Problem 24

Describe various ranges of grain size. What do these ranges tell you about a metal?

Chapter 4, Solution 24

Generally speaking, a material may be classified as coarse-grained, $n<3$; medium-grained, $4<n<6$; fine-grained, $7<n<9$, and ultrafine-grained, $n>10$. The finer the grain size, the stronger the material will be at room temperature.

Chapter 4, Problem 25

How is the grain size of polycrystalline materials measured by the ASTM method?

Chapter 4, Solution 25

In the ASTM method of measuring grain size of polycrystalline materials, the grain size number, *n*, is defined by the equation $N = 2^{n-1}$, where N is the number of grains per square inch, measured on a polished and etched surface at a magnification of 100x.

Chapter 4, Problem 26

Describe the optical metallography technique. What qualitative and quantitative information can this technique provide? What levels of magnification can be achieved by this technique?

Chapter 4, Solution 26

Optical metallography techniques are used to study the features and internal make-up of materials at micro-meter level (magnification level of around 2000X). Qualitative and quantitative information pertaining to grains size, grain boundary, existence of various phases, internal damage, and some defects may be extracted using optical metallography techniques.

Chapter 4, Problem 27

Describe the atomic force microscope (AFM). What are the modes of operation (draw a schematic)? What is the dimensional resolution that it can achieve? What information can it provide?

Chapter 4, Solution 27

The AFM uses a similar approach to STM in that it uses a tip to probe the surface. However, in this case, the tip is attached to a small cantilever beam. As the tip interacts with the surface of the sample, the forces (Van der Waals forces) acting on the tip deflect the cantilever beam. The interaction may be a short range repulsive force (contact mode AFM) or a long-range attractive force (non-contact mode AFM). The deflection of the beam is monitored using a laser and a photodetector set-up. The surface topography is determined from these small displacements. Unlike STM, the AFM approach does not rely on a current tunneling through the tip and can therefore be applied to all materials even nonconductors. The resolution is similar to STM.

Chapter 4, Problem 28

What is a transmission electron microscope (TEM)? How does it work (draw a schematic). What is the dimensional resolution that it can achieve? What information can it provide?

Chapter 4, Solution 28

TEM, Fig. 33, is an important technique for studying defects and precipitates (second phases) in materials. Specimens to be analyzed using a TEM must have a thickness of several hundred nanometers or less depending on the operating voltage of the instrument. Nano-scale features can be resolved by TEM. In the TEM, an electron beam is produced by a heated tungsten filament at the top of an evacuated column and is accelerated down the column by high voltage (usually from 100 to 300 kV). Electromagnetic coils are used to condense the electron beam, and then it is passed through the thin specimen placed on the specimen stage. As the electrons pass through the specimen, some are absorbed and some are scattered so that they change direction. After the electron beam has passed through the specimen, it is focused with the objective coil (magnetic lens) and then enlarged and projected on a fluorescent screen, Fig 4.34. An image can be formed either by collecting the direct electrons or the scattered electrons

Chapter 4, Problem 29

Describe the scanning tunneling microscope (STM). What are its modes of operation (draw a schematic)? What is the dimensional resolution that they can achieve? What information can it provide?

Chapter 4, Solution 29

Scanning probe microscopy includes both Scanning Tunneling Microscopy (STM) and Atomic Force Microscopy (AFM). In STM, an extremely sharp tip (Fig. 4.36), traditionally made of metals such as tungsten, nickel, platinum-iridium, or gold, and more recently out of carbon nanotubes, is used to probe the surface of a sample. The tip is first positioned a distance in the order of an atom diameter (~ 0.1 to 0.2 nm) from the surface of the sample. At such proximity, the electron clouds of the atoms in the tip of the probe interact with the electron clouds of the atoms on the surface of the sample. If at this point a small voltage is applied across the tip and the surface, the electrons will "tunnel" the gap and, therefore, produce a small current that may be detected and monitored. The magnitude of current will be measured when the tip is positioned directly above an atom (its electron cloud). This current will be maintained at the same level as the tip moves over the atoms and valleys between atoms. This will be accomplished by adjusting the vertical position of the tip. The small movements required to adjust and maintain the current through the tip will then be used to map the surface. The system can also operate at constant height mode where the changes in current are monitored. The dimensional resolution is at the atomic level. SPM is used to examine the surface of materials at the atomic level. Only used for conducting materials.

Chapter 4, Problem 30

What is a high resolution transmission electron microscope (HRTEM)? What is the dimensional resolution that it can achieve? What information can it provide?

Chapter 4, Solution 30

The technique is very similar to TEM. However, the prepared sample must be significantly thinner in the order of 10-15 nm. The instrument allows for a resolution of about 0.1 nm which in turn allows viewing of the crystal structure and defects at the atomic level. To grasp what this degree of resolution may reveal about a structure, consider the lattice constant of the silicon unit cell is five times larger than the resolution offered by HRTEM at approximately 0.543 nm. It can provide information about defects at the atomic level.

Chapter 4, Problem 31

What is a scanning electron microscope (SEM)? What magnifications can it achieve? How does it work (draw a schematic). What information can it provide?

Chapter 4, Solution 31

SEM is an important tool in materials science and engineering used for microscopic feature measurement, fracture characterization, microstructure studies, thin coating evaluations, surface contamination examination, and failure analysis of materials. It uses incident flow of electrons instead of light to help map the image. The dimensional resolution is around 5nm. Magnification ranges from 15X to 100,000X.

Chapter 4, Problem 32

Calculate the size (radius) of the critical nucleus for pure iron when nucleation takes place homogeneously.

Chapter 4, Solution 32

For iron, $\gamma = 204\times10^{-3}\ \text{J/m}^2$, $\Delta H_f = -2098\times10^6\,\text{J/m}^3$, $T_m = 1808\ \text{K}$

The amount of undercooling is then $\Delta T = 0.2T_m = 0.2(1808\ \text{K}) = 361.6\ \text{K}$ and the critical radius becomes,

$$r^* = \frac{-2\gamma T_m}{\Delta H_f \Delta T} = \frac{-2(204\times10^{-3}\ \text{J/m}^2)(1808\ \text{K})}{(-2098\times10^6\,\text{J/m}^3)(361.6\ \text{K})} = \mathbf{9.72\times10^{-10}\,m = 0.972\ nm}$$

Chapter 4, Problem 33

Calculate the number of atoms in a critically sized nucleus for the homogeneous nucleation of pure iron.

Chapter 4, Solution 33

The relevant volumes, based on the solution of Problem 4.8, are:

$$\text{Vol. of critical-sized nucleus} = \frac{4}{3}\pi r^{*3} = \frac{4}{3}\pi(0.972\ \text{nm})^3 = 3.85\times10^{-27}\text{m}^3$$

$$\text{Vol. of unit cell of Pt} = a^3 = (0.28664\ \text{nm})^3 = 2.355\times10^{-29}\ \text{m}^3$$

$$\text{Vol. per atom} = \frac{2.355\times10^{-29}\text{m}^3}{2\ \text{atoms/BCC unit cell}} = 1.178\times10^{-29}\text{m}^3\text{/atom}$$

Thus, the number of atoms in a critically sized nucleus is:

$$\frac{\text{Volume of nucleus}}{\text{Volume per atom}} = \frac{3.85\times10^{-27}\text{m}^3}{1.178\times10^{-29}\,\text{m}^3\text{/atom}} = \mathbf{327\ atoms}$$

Chapter 4, Problem 34

Calculate the size (radius) of the critically sized nucleus for pure platinum when homogeneous nucleation takes place.

Chapter 4, Solution 34

The critical radius of nucleus is calculated as,

$$r^* = \frac{-2\gamma T_m}{\Delta H_f \Delta T}$$

where $\gamma = 240\times10^{-3}\text{ J/m}^2$, $\Delta H_f = -2160\times10^6\text{ J/m}^3$, $T_m = 2045\text{ K}$

and the undercooling $\Delta T = 0.2T_m = 0.2(2045\text{ K}) = 409\text{ K}$. Substituting,

$$r^* = \frac{-2(240\times10^{-3}\text{ J/m}^2)(2045\text{ K})}{(-2160\times10^6\text{ J/m}^3)(409\text{ K})} = 1.11\text{ nm}$$

Chapter 4, Problem 35

Calculate the number of atoms in a critically sized nucleus for the homogeneous nucleation of pure platinum.

Chapter 4, Solution 35

The number of atoms in a critically sized nucleus is found as,

$$\frac{\text{Volume of nucleus}}{\text{Volume per atom}} = \frac{\text{Volume of nucleus}}{(\text{Vol. of unit cell})(\text{No. atoms per unit cell})}$$

Using the critical radius result of Problem 4.9,

$$\text{Vol. of critical-sized nucleus} = \frac{4}{3}\pi r^{*3} = \frac{4}{3}\pi(1.11\text{ nm})^3 = 5.73\times10^{-27}\text{m}^3$$

$$\text{Vol. of unit cell of Pt} = a^3 = (0.32939\text{nm})^3 = 3.574\times10^{-29}\text{ m}^3$$

$$\text{Vol. per atom} = \frac{3.574\times10^{-29}\text{m}^3}{4\text{ atoms/FCC unit cell}} = 8.935\times10^{-30}\text{m}^3\text{/atom}$$

Substituting,

$$\frac{\text{Volume of nucleus}}{\text{Volume per atom}} = \frac{5.73\times10^{-27}\text{m}^3}{8.935\times10^{-30}\text{m}^3\text{/atom}} = \mathbf{641\ atoms}$$

Chapter 4, Problem 36

In Example Problem 4.3, if a carbon atom occupies the interstitial void, how may iron neighbors will it have, or in other words, what will its coordination number be?

Chapter 4, Solution 36

The carbon atoms will have 6 neighbors (two at the corners and 4 at the center of the faces of the 4 neighboring cells). Such interstitial sites are octahedral sites and have a coordination number of 6.

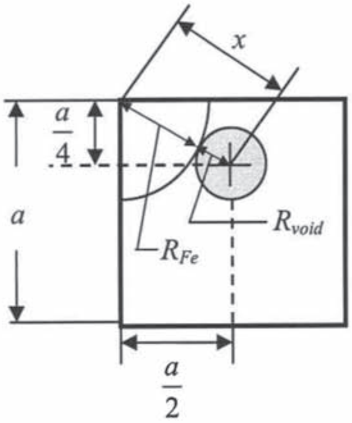
Chapter 4, Problem 37

(a) Calculate the radius of the largest interstitial void in the BCC α iron lattice. The atomic radius of the iron atom in this lattice is 0.124 nm, and the largest interstitial voids occur at the (¼, ½, 0); (½, ¾, 0); (¾, ½, 0); (½, ¼, 0), etc., type positions. (b) If an iron atom occupies this interstitial void, how many iron atom neighbors will it have, or in other words, what will its coordination number be?

Chapter 4, Solution 37

(a) For BCC crystal structure,

$$a = \frac{4R}{\sqrt{3}} = \frac{4(0.124\text{ nm})}{\sqrt{3}} = 0.286\text{ nm}$$



Letting x = Fe atom radius + Interstitial void radius,

$$x^2 = \frac{1}{16}a^2 + \frac{1}{4}a^2 = \frac{5}{16}a^2$$

$$x = \sqrt{\frac{5}{16}}a = 0.559a = (0.559)(0.286\text{ nm}) = 0.160\text{ nm}$$

The interstitial void radius is thus,

$$R_{void} = x - R_{Fe} = 0.160\text{ nm} - 0.124\text{ nm} = \mathbf{0.036\text{ nm}}$$

(b) The site will have four iron neighbors (two corner atoms and two atoms at the centers of the neighboring cubes). The coordination # is 4. Such interstitial sites are called tetrahedral sites.

Chapter 4, Problem 38

Consider a copper lattice with an excessive average of one vacancy in every 100 unit cells. What will be its density? Compare this to the theoretical density of copper.

Chapter 4, Solution 38

4.38

Assume equivalent of one atom is missing in 100 copper unit cells.

$\# \text{ of atoms} = 400 - 1 = 399 \text{ atoms}$

$$\text{mass of 100 unit cells} = 399 \times \frac{63.55 \times 10^{-3}\ \text{kg}}{6.02 \times 10^{23}\ \text{atoms}} = 4.21 \times 10^{-23}\ \text{kg}$$

$$\text{for Copper,} \quad a = \frac{4R}{\sqrt{2}} = \frac{4(0.1278\ \text{nm})}{\sqrt{2}} = 0.3610 \times 10^{-9}\ \text{m}$$

$$a = 3.610 \times 10^{-10}\ \text{m}$$

$$\text{volume of one cell} = a^3 = (3.610 \times 10^{-10}\ \text{m})^3$$

$$a^3 = 4.704 \times 10^{-29}\ \text{m}^3$$

$$\text{volume of 100 cells} = 4.704 \times 10^{-27}\ \text{m}^3$$

$$\rho = \frac{m_{100\ \text{cells}}}{V_{100\ \text{cells}}} = \frac{4.21 \times 10^{-23}\ \text{kg}}{4.704 \times 10^{-27}\ \text{m}^3} = 8.94 \times 10^{3}\ \text{kg/m}^3$$

$$\rho_{\text{theoretical}} = 8.98 \times 10^{3}\ \text{kg/m}^3$$

Chapter 4, Problem 39

Assume that Figure 4.14 shows a representative image of the overall atomic composition of a substitutional solid solution of an alloy of nickel (dark spheres) and copper (light spheres) and estimate (a) the a/o of each element in overall crystal, and (b) the wt/o of each metal?

Chapter 4, Solution 39

4.39

The figure shows space for 36 atoms

7 Nickel atoms
28 copper atoms
1 defect

If the image represents the overall composition,

a)

$$\text{Ni a/o} = \frac{7}{36} = 0.194 \times 100 = 19.4$$

$$\text{Cu a/o} = \frac{28}{36} \times 100 = 77.7$$

$$\text{defect a/o} = \frac{1}{36} \times 100 = 2.78$$

b)

$$\text{Ni} = \frac{58.69 \times 10^{-3}\,\text{kg}}{6.02 \times 10^{23}\ \text{atoms}} \times 0.194 = 1.891 \times 10^{-26}\,\text{kg}$$

$$\text{Cu} = \frac{63.55 \times 10^{-3}\,\text{kg}}{6.02 \times 10^{23}} \times 0.777 = 8.202 \times 10^{-26}\,\text{kg}$$

$$\text{total} = 1.009 \times 10^{-25}\,\text{kg}$$

$$\text{wt\% Ni} = \frac{1.891 \times 10^{-26}\,\text{kg}}{1.009 \times 10^{-25}\,\text{kg}} \times 100 = 18.7\%$$

$$\text{wt\% Cu} = \frac{8.202 \times 10^{-26}\,\text{kg}}{1.009 \times 10^{-25}\,\text{kg}} \times 100 = 81.3\%$$

Chapter 4, Problem 40

The alloy used for the gold and medal awarded to the first place winner in Salt Lake City Olympics had a composition of 92.0 a/o silver, 7.5 a/o copper, and 0.5 a/o gold (the medal is gold-plated). Determine the absolute mass of each metal in a 0.253-kg metal. Repeat the same calculation for (b) the silver medal: 92.5 a/o silver and 7.5 a/o copper, and (c) the bronze medal: 90 a/o copper and 10 a/o tin.

Chapter 4, Solution 40

4.40 a) gold medal : mass $= 0.253\,kg$ comp: 92% Ag, 7.5% Cu, 0.5% Au

$$\text{Relative mass, Ag} = \frac{107.9\times10^{-3}\,kg}{6.02\times10^{23}\,\text{atoms}}\times 0.92 = 1.649\times10^{-25}\,kg$$

$$\text{Relative mass, Cu} = \frac{63.55\times10^{-3}\,kg}{6.02\times10^{23}\,\text{atoms}}\times 0.075 = 7.974\times10^{-27}\,kg$$

$$\text{Relative mass, Au} = \frac{197.0\times10^{-3}\,kg}{6.02\times10^{23}\,\text{atoms}}\times 0.005 = 1.636\times10^{-27}\,kg$$

total $\overline{1.745\times10^{-25}\,kg}$

$$\%\,Ag = \frac{1.649\times10^{-25}\,kg}{1.745\times10^{-25}\,kg}\times100\% = 94.5\%$$

$$\%\,Cu = \frac{7.974\times10^{-27}\,kg}{1.745\times10^{-25}\,kg}\times100\% = 4.57\%$$

$$\%\,Au = \frac{1.636\times10^{-27}\,kg}{1.745\times10^{-25}\,kg}\times100\% = 0.94\%$$

Ag in medal $= (0.253\,kg)(.945) = 0.239\,kg$

Cu in medal $= (0.253\,kg)(0.0457) = 0.012\,kg$

Au in medal $= (0.253\,kg)(.00940) = 0.002\,kg$

b) Silver medal : mass = 0.253 kg Comp: 92.5% Ag, 7.5% Cu

$$\text{Relative mass Ag} = \frac{107.9\times10^{-3}\,\text{kg}}{6.02\times10^{23}\,\text{atoms}} \times 0.925 = 1.658\times10^{-25}\,\text{kg}$$

$$\text{Relative mass Cu} = \frac{63.55\times10^{-3}\,\text{kg}}{6.02\times10^{23}\,\text{atoms}} \times 0.075 = 7.917\times10^{-27}\,\text{kg}$$

$$\overline{1.737\times10^{-25}\,\text{kg}}$$

$$\%\,\text{Ag} = \frac{1.658\times10^{-25}\,\text{kg}}{1.737\times10^{-25}\,\text{kg}} \times 100\% = 95.4\%$$

$$\%\,\text{Cu} = \frac{7.917\times10^{-27}\,\text{kg}}{1.737\times10^{-25}\,\text{kg}} \times 100\% = 4.6\%$$

Ag in medal = 0.253 kg (0.954) = 0.241 kg
Cu in medal = 0.253 kg (0.046) = 0.012 kg

c) Bronze medal : mass = 0.253 kg Comp: 90% Cu 10% Sn

$$\text{Relative mass, Cu} = \frac{107.9\times10^{-3}\,\text{kg}}{6.02\times10^{23}\,\text{atoms}} \times 0.9 = 1.613\times10^{-25}\,\text{kg}$$

$$\text{Relative mass, Sn} = \frac{118.7\times10^{-3}\,\text{kg}}{6.02\times10^{23}\,\text{atoms}} \times 0.1 = 1.972\times10^{-26}\,\text{kg}$$

$$\text{total} = 1.810\times10^{-25}\,\text{kg}$$

$$\%\,\text{Cu in medal} = \frac{1.613\times10^{-25}\,\text{kg}}{1.81\times10^{-25}\,\text{kg}} \times 100\% = 89.1\%$$

$$\%\,\text{Sn in medal} = \frac{1.972\times10^{-26}\,\text{kg}}{1.810\times10^{-25}\,\text{kg}} \times 100\% = 10.9\%$$

Cu in medal = (0.253 kg)(0.891) = 0.225 kg
Sn in medal = (0.253 kg)(0.109) = 0.028 kg

Chapter 4, Problem 41

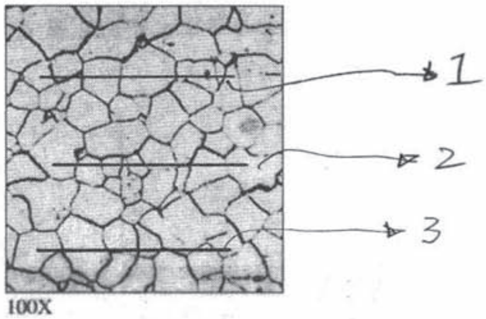
For the grain structure in Problem 4.46, estimate the average grain diameter.

Chapter 4, Solution 41

4.41

Draw a line of length 2.54×10^{-2} m on the figure and count (estimate) the # of grains intersected by the line. Repeat this process 3 times in different regions and average the results:

$$d = \frac{C}{n_L M} \quad ; \quad n_L = \frac{\text{\# grains intersecting the line}}{\text{length of the line}}$$



$n_{L1} = 9/2.54 \times 10^{-2}$ m $C = 1.5$

$n_{L2} = 9/2.54 \times 10^{-2}$ m $M = 100$

$n_{L3} = 8/2.54 \times 10^{-2}$ m

$$\underset{9}{d_1} = \frac{1.5}{\frac{9}{2.54\times10^{-2}\,m}(100)} = 4.23\times10^{-5}m = 42.3\,\mu m$$

$$d_2 = \frac{1.5}{\frac{8}{2.54\times10^{-2}\,m}(100)} = 4.76\times10^{-5}\,m = 47.6\,\mu m$$

$$d_1 = d_2$$

$$d_{avg} = \frac{42.3\,\mu m + 47.6\,\mu m}{2} \approx 45\,\mu m$$

Chapter 4, Problem 42

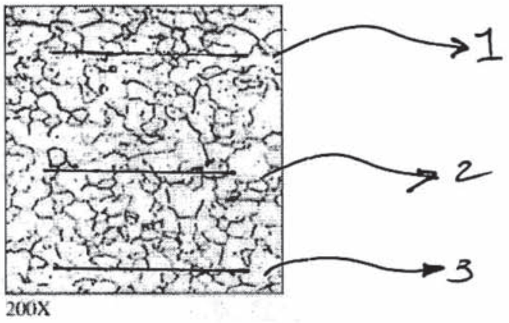
For the grain structure in Problem 4.45, estimate the average grain diameter.

Chapter 4, Solution 42

4.42

Draw a line of length $2.54\times10^{-2}\,m$ on the figure and count (estimate) the # of grains intersected by the line. Repeat the process 3 times in different regions of the picture. Average the results.

$$d = \frac{C}{n_L M} \quad ; \quad n_L = \frac{\#\text{grains intersecting the line}}{\text{length of the line}}$$



$$n_{L1} = \frac{14}{2.54\times10^{-2}\,m} \qquad C = 1.5$$

$$n_{L2} = \frac{13}{2.54\times10^{-2}\,m} \qquad M = 200$$

$$n_{L3} = \frac{13}{2.54\times10^{-2}\,m}$$

$$d_1 = \frac{1.5}{\frac{14}{2.54\times10^{-2}\,m}(200)} = 1.36\times10^{-5}\,m = 13.6\,\mu m$$

$$d_2 = d_3 = \frac{1.5}{\frac{13}{2.54\times10^{-2}\,m}(200)} = 1.47\times10^{-5}\,m = 14.7\,\mu m$$

$$d_{avg} = \frac{13.6\,\mu m + 14.7\,\mu m}{2} \simeq 14\,\mu m$$

Chapter 4, Problem 43

If there are 200 grains per square 2.54×10^{-2} m on a photomicrograph of a metal at 100x, what is its ASTM grain-size number?

Chapter 4, Solution 43

$$N = 200 = 2^{n-1}$$

$$\ln 200 = (n-1)(\ln 2)$$

$$5.298 = (n-1)(0.693)$$

$$n = 7.65 + 1 = \mathbf{8.65}$$

Chapter 4, Problem 44

If there are 400 grains per square 2.54×10^{-2} m on a photomicrograph of a ceramic material at 200×, what is the ASTM grain-size number of the material?

Chapter 4, Solution 44

$$N = \left(\frac{200}{100}\right)^2 400 \text{ grains / } 2.54\times10^{-2}\text{ m}$$

$$= 1600 \text{ grains / } 2.54\times10^{-2}\text{ m}$$

$$N = 1600 = 2^{n-1}$$

$$\ln 1600 = (n-1)(\ln 2)$$

$$7.378 = (n-1)(0.693)$$

$$n = 10.64 + 1 = \mathbf{11.64}$$

Chapter 4, Problem 45

Determine the ASTM grain-size number of the type 430 stainless steel micrograph shown in Fig. P4.45 This micrograph is at 200x. Classify the grain size according to the value of n, i.e., coarse, medium, fine or ultrafine grain size. Measure the area of the image for your calculations.

Figure P4.45

Chapter 4, Solution 45

Estimating 100 grains/$(2.54\times10^{-2}\text{ m})^2$ from the micrograph

$$\text{No. of grains at }100\times = \frac{200^2}{100^2}(100)$$

$$= 400\text{ grains}/(2.54\times10^{-2}\text{ m})^2$$

$$N = 400 = 2^{n-1}$$

$$\ln 400 = (n-1)(\ln 2)$$

$$5.99 = (n-1)(0.693)$$

$$n = 8.64 + 1 = \mathbf{9.64}$$

(c) According to our scale on page 166, this metal is between fine grained and ultrafine grained.

Chapter 4, Problem 46

Determine, by counting, the ASTM grain-size number of the low-carbon sheet steel shown in Fig. P4.46. This micrograph is at 100x. Classify the grain size according to the value of n, i.e., coarse, medium, fine, or ultrafine grain size. Measure the area of the image for your calculations.

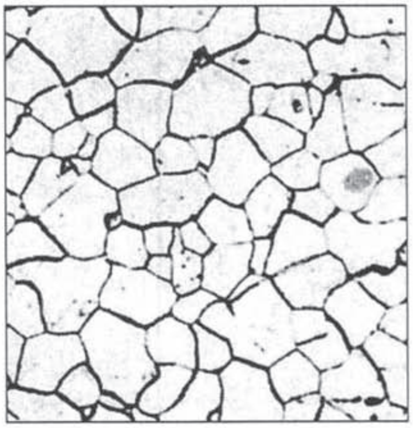
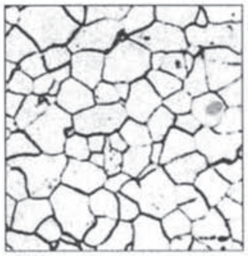


Figure P4.46

Chapter 4, Solution 46

(a) Estimating 40 grains/$(2.54\times10^{-2}\ \text{m})^2$ from the micrograph



$$N = 40 = 2^{n-1}$$
$$\ln 40 = (n-1)(\ln 2)$$
$$3.689 = (n-1)(0.693)$$
$$n = 5.3 + 1 = \mathbf{6.3}$$

(b) According to our scale on page 166, this metal is between medium gained and fine grained.

Problem 4.47
Iron oxide, FeO, is an ionic compound made up of Fe^{2+} cations and O^{2-} anions. However, when available, a small number of Fe^{3+} cations may replace Fe^{2+} cations. How will this substitution affect the atomic structure of the compound if at all? (Consult section 2.5.1 related to packing of ionic compounds.)

4.47

According to Sec 2.5.1, the electrical neutrality of a compand must be maintained. Thus, in FeO, if a Fe^{++} ion is replaced by a Fe^{+++} ion, there will be one extra + charge and electrical neutrality is denied. To remedy this, the compound will accept one more Fe^{+++} to subtitude Fe^{++}. Now, there are two additional + charges that can be neutralized by an additional O^{--}. In this process the total # of Fe ions remains the same but O ions increase by one. Overall, depending on the availability of Fe^{+++}, the O a/o will increase to about 51 a/o instead of 50%.

Synthesis and Evaluation Problems:

Problem 4.48
It is easier for the iron lattice to house carbon atoms at temperatures slightly higher than 1185 K than slightly lower temperatures (Consult section 3.10 on polymorphism). Use the results in example problem 4.3 and problem 4.37 (solve this problem first if you have not already done so) to explain why.

4.48

From example 4.3 (FCC iron):

The radius of the interstitial site is 0.053 nm

From problem 4.37: (BCC iron)

The radius of the interstitial site is 0.036 nm

From Figure 2.11, the radius of carbon atom is 0.077 nm

Note that the carbon atom is bigger than both interstitial sites. However, the interstitial site in FCC iron is bigger and carbon can fit there with less difficulty.

Problem 4.49

The gamma iron and silver both possess FCC crystal structures. The location for interstitial voids will be the same for both. Will the size of the interstitial voids be different? If yes, what will be the interstitial void size for silver? (hint: use example 4.3)

4.49

The equation for determining the interstitial void radius for FCC unit cells is developed in example 4.3 :

$$r_{void} = (\sqrt{2}-1)R$$

since the equation is dependent on R, the radius of atom, the interstitial site radius in γ iron will be different than that in silver although both metals are FCC.

$$r_{silver} = (\sqrt{2}-1)(0.144) = 0.0596 \text{ nm}$$

For comparison,

$$r_{\gamma \text{Iron}} = (\sqrt{2}-1)(0.129) = 0.0530 \text{ nm}$$

$$r_{silver} > r_{\gamma \text{Iron}}$$

Problem 4.50

The chemical formula for an intermetallics compound of Cu and Al is Cu_2Al. According to the formula, the atom percent of Al should be exactly 33.33% (one out of three atoms should be aluminum). However, in practice, one can find a range of 31% to 37% for Al. How do you explain this discrepancy?

4.50

Because of the relative similarity in atomic radius and electronic structure of Cu and Al, some of the Cu atoms may replace Al atoms thus producing Al a/o of less than 33.33 a/o. If Al atoms replace Cu atoms, Al a/o will be above 33.33 a/o. Such intermetallic compounds are non-stoichiometric.

Problem 4.51
Comment, based on calculations and comparisons, on the extent of solubility of copper in nickel based on the Hume-Rothery rules.

4·51

$r_{Ni} = 0{\cdot}125$ nm $r_{Cu} = 0{\cdot}128$ nm

2·4 % difference is size (a small amount)

Both are FCC.

Both have +2 Valence.

Both have 1.8 electronegativity.

These two metals have unlimited solubility in each other.

Problem 4.52
In chapter 3 (example 3.11), we calculated the theoretical density of copper to be 8.98×10^3kg/m^3. Determine the experimentally measured density of copper by referring to Appendix II. To what do you attribute this difference?

4.52

From the Appendix II, the density of copper is 8.89×10^3 kg/m^3. This is lower than the theoritical density of 8.98×10^3 kg/m^3. This difference is due to defects in the lattice and along the grain boundaries. In addition impurities are always present. The theoretical calculation does not include defects or impurities.

Chapter 4, Problem 4.53

Using the data in the following table, predict the relative degree of solid solubility of the following elements in aluminum:

(a) Copper
(b) Manganese
(c) Magnesium
(d) Zinc
(e) Silicon

Use the scale: very high, 70%–100%; high, 30%–70%; moderate, 10%–30%, low, 1%–10%; and very low, <1%.

Element	Atom radius (nm)	Crystal structure	Electro-negativity	Valence
Aluminum	0.143	FCC	1.5	+3
Copper	0.128	FCC	1.8	+2
Manganese	0.112	Cubic	1.6	+2, +3, +6, +7
Magnesium	0.160	HCP	1.3	+2
Zinc	0.133	HCP	1.7	+2
Silicon	0.117	Diamond cubic	1.8	+4

Chapter 4, Solution 4.53

(a) **low** (b) **very low** (c) **moderate** (d) **high** (e) **low**

Chapter 4, Problem 4.54

Using the data in the following table, predict the relative degree of atomic solid solubility of the following elements in iron:

(a) Nickel
(b) Chromium
(c) Molybdenum
(d) Titanium
(e) Manganese

Use the scale: very high, 70%–100%; high, 30%–70%; moderate, 10%–30%, low, 1%–10%; and very low, <1%.

Element	Atom radius (nm)	Crystal structure	Electro-negativity	Valence
Iron	0.124	BCC	1.7	+2, +3
Nickel	0.125	FCC	1.8	+2
Chromium	0.125	BCC	1.6	+2, +3, +6
Molybdenum	0.136	BCC	1.3	+3, +4, +6
Titanium	0.147	HCP	1.3	+2, +3, +4
Manganese	0.112	Cubic	1.6	+2, +3, +6, +7

Chapter 4, Solution 4.54

(a) **high** (b) **very high** (c) **moderate** (d) **low** (e) **moderate**

Problem 4.55

The following pairs of elements can form solid solution alloys. Predict which ones will form substitutional and which ones interstitial alloys. Justify your answers.

(a) Copper and tin (bronze)
(b) Aluminum and silicon
(c) Iron and nitrogen
(d)Titanium and hydrogen

Solution 4.55

(a) atom radius Cu = 0,128 nm
atom radius Sn = 0,158 nm

$$\therefore \frac{0{,}158\,\text{nm} - 0{,}128\,\text{nm}}{0{,}128\,\text{nm}} \times 100\% = 23\% \text{ difference}$$

will form substitutional alloy

(b) atom radius Al = 0,143 nm
atom radius Si = 0,117 nm

will form substitutional alloy

$$\therefore \frac{0{,}143 - 0{,}117}{0{,}143} \times 100\% = 18\% \text{ difference}$$

(c) atom radius Fe = 0,124 nm
atom radius N = 0.071 nm

$$\therefore \frac{0{,}124 - 0{,}071}{0{,}124} \times 100\% = 43\% \text{ difference}$$

will form interstitial alloy

(d) atom radius Ti = 0,149 nm
atom radius H = 0,046 nm

$$\therefore \frac{0{,}147 - 0{,}046}{0.147} \times 100\% = 69\% \text{ Difference}$$

will form interstitial alloy

Problem 4.56

The angle θ of a small-angle tilt boundary in Fig. 4.24 has a relationship of sin (θ/2) = b/2D, if the distance D between the dislocations is 600 nm and the magnitude of burger vector b = 0.3 nm, determine the angle θ.

4.56

$D = 600\,nm \;;\; b = 0.3\,nm$

$$\sin\left(\frac{\theta}{2}\right) = \frac{b}{2D} = \frac{0.3\,nm}{(2)600\,nm} = 0.00025$$

$$\frac{\theta}{2} = \sin^{-1}(0.00025) = 0.0143^\circ$$

$$\therefore \theta = 2(0.0143^\circ) = 0.0286^\circ$$

Problem 4.57

(a) Estimate the density of a 75 wt% copper-25 wt% Ni alloy (use Appendix II for density data). (b) What will be the most probable crystal structure for this alloy? (c) Determine the a/o of each element in this alloy. (d) Determine the mass in grams of the atoms inside a unit cell of this alloy. (e) Determine the lattice constant for this alloy.

4.57

a) $\rho_{copper} = 8.96 \times 10^3 \text{ kg/m}^3$

$\rho_{nickel} = 8.9 \times 10^3 \text{ kg/m}^3 \Rightarrow \rho_{alloy} = 0.75(8.96\times10^3) + (0.25)(8.9\times10^3)$

$\rho_{alloy} = 8.94 \times 10^3 \text{ kg/m}^3$

b) The structure will be FCC since both Ni and Cu are FCC.

c) Consider 1 kg of the alloy:

Copper : $0.75\text{ kg}\left(\frac{6.02\times10^{23}\text{ atoms}}{63.55\times10^{-3}\text{ kg}}\right) = 7.10\times10^{24}\text{ atoms}$

Ni : $0.25\text{ kg}\left(\frac{6.02\times10^{23}\text{ atoms}}{58.69\times10^{-3}\text{ kg}}\right) = 2.56\times10^{24}\text{ atoms}$

$\frac{7.10\times10^{24}\text{ atoms}}{7.10\times10^{24}\text{ atoms} + 2.56\times10^{24}\text{ atoms}} \times 100\% = 73.5\% \Leftarrow \text{Cu}$

$\frac{2.56\times10^{24}\text{ atoms}}{7.10\times10^{24}\text{ atoms} + 2.56\times10^{24}\text{ atoms}} \times 100\% = 26.5\% \Leftarrow \text{Ni}$

~ 73.5% Cu atoms to ~ 26.5% Ni atoms

d) There will be 4 atoms per unit cell (FCC).
based on the atom % calculations above
~ 3 copper atoms to 1 nickel atom in each cell.

Thus, the mass of the unit cell will be,

$$m_{cell} = 3\left(\frac{63.55\times10^{-3}\,kg}{6.02\times10^{23}\,atoms}\right) + 1\left(\frac{58.69\times10^{-3}\,kg}{6.02\times10^{23}\,atoms}\right)$$

$$m_{cell} = 4.14\times10^{-25}\,kg$$

e) $V = a^3 = \frac{m}{\rho} \Rightarrow a^3 = \frac{4.14\times10^{-25}\,kg}{8.94\times10^{3}\,kg/m^3}$

$= 3.59\times10^{-10}\,m$

$= 0.359\ nm$

Problem 4.58
What is the significance or impact of schottky and/or frenkel imperfection of the properties and behavior of ionic materials?

4.58

Once Schottky and and Frenkel imperfections (vacancies) are formed, they are the free to move across the material as their own entities. These movements will help the electrical conduction process. Also, the defects will reduce the density of the material in case of schottlcy imperfections.

Problem 4.59
(a) What is the theoretical atom% of each element in the compound FeO? (b) What is the corresponding wt% of each element in the compound? (c) What is your conclusion?

4.59

a) The atom % for each element is 50 %.

b) For each Fe atom there is one O atom.

$$\text{mass of one Fe atom} = \frac{55.85 \times 10^{-3}\ \text{kg}}{6.02 \times 10^{23}\ \text{atoms}} = 9.27 \times 10^{-26}\ \text{kg}$$

$$\text{mass of one O atom} = \frac{16.0 \times 10^{-3}\ \text{kg}}{6.02 \times 10^{23}\ \text{atoms}} = 2.66 \times 10^{-26}\ \text{kg}$$

$$\text{wt\% Fe} = \frac{9.27 \times 10^{-26}\ \text{kg} \times 100}{(9.27 \times 10^{-26} + 2.66 \times 10^{-26})\ \text{g}} = 77.7\ \%$$

$$\text{wt\% O} = 23.3\ \%$$

c) The conclusion is not to confuse atom % with wt %.

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Problem 4.60
The image below is an optical micrograph of 1018 steel (200X) made of mostly iron and a small amount of carbon (only 0.18 wt%). Describe all the features that you observe in this micrograph. What do you think the different colors represent?

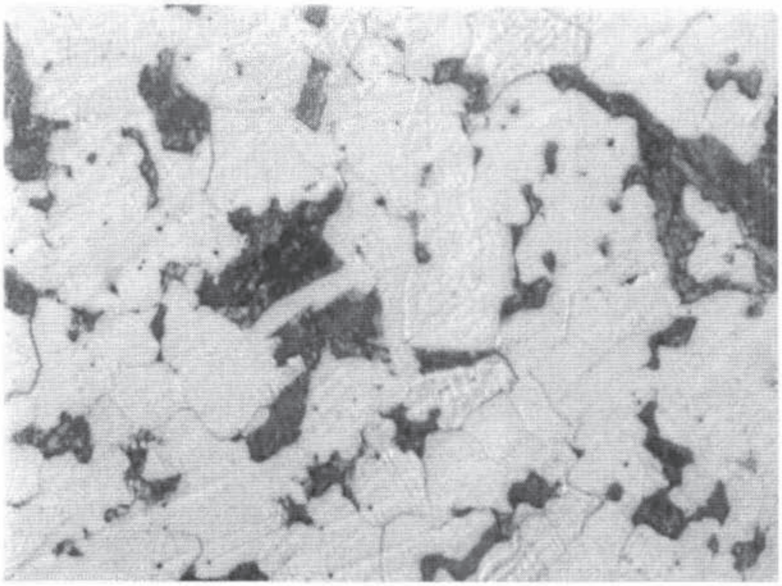


Figure P4.60

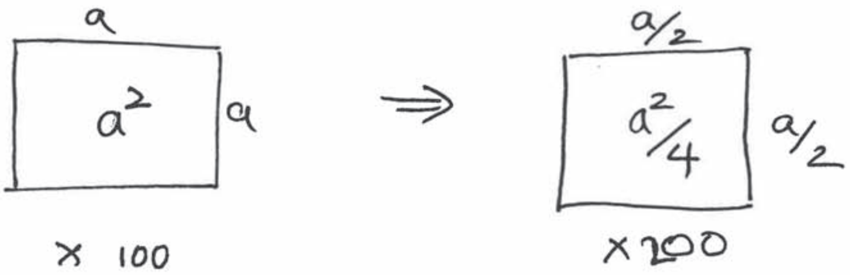
4.60

You can clearly observe two different constituents: the dark area and the lighter area. There are two different phases (materials) that reacted differently to the etchant. You can also clearly observe the grain boundaries. It is also clear from light reflection that grains are oriented at different directions relative to each other.

Problem 4.61
When the magnification in a metallurgical microscope is increased by a factor of 2, what happens to the size of the area that you are observing (area magnification)?

4.61

When the magnification is increased by a factor of 2, the area will be magnified by a factor of 4. You will be looking at an area that is



four times smaller but with the same visual dimensions of the scope.

Problem 4.62

For a given application, you would need to select the metal with larger grain size between copper (n=7) and mild steel (n=4). (a) Which one would you pick? (b) If strength is an important consideration, which alloy would you pick and why? (c) What if the application was at elevated temperatures: would your answer in part b change and why?

4.62

a) larger "n" means smaller grain size, pick copper.

b) The two metals are different and we can not compare their strengths based on grain size.

Although Fe has a larger grain size than copper, its original strength is significantly higher than copper (mild steel 283 MPa, copper 69 MPa)

Grain size comparison with respect to strength are best when the metal composition is the same.

c) At elevated temperatures, the grain boundaries melt first so the metal with ~~more~~ smaller grain size will be more susceptible to deformation. However, the temperature at which grain boundary melt dow will occur will be different for different metals.

Problem 4.63
The image below is an SEM micrograph (500X) and shows the fracture surface of a gear. Describe all the features that you are observing in this micrograph. Can you estimate the average grain diameter (assume C=1.5)?

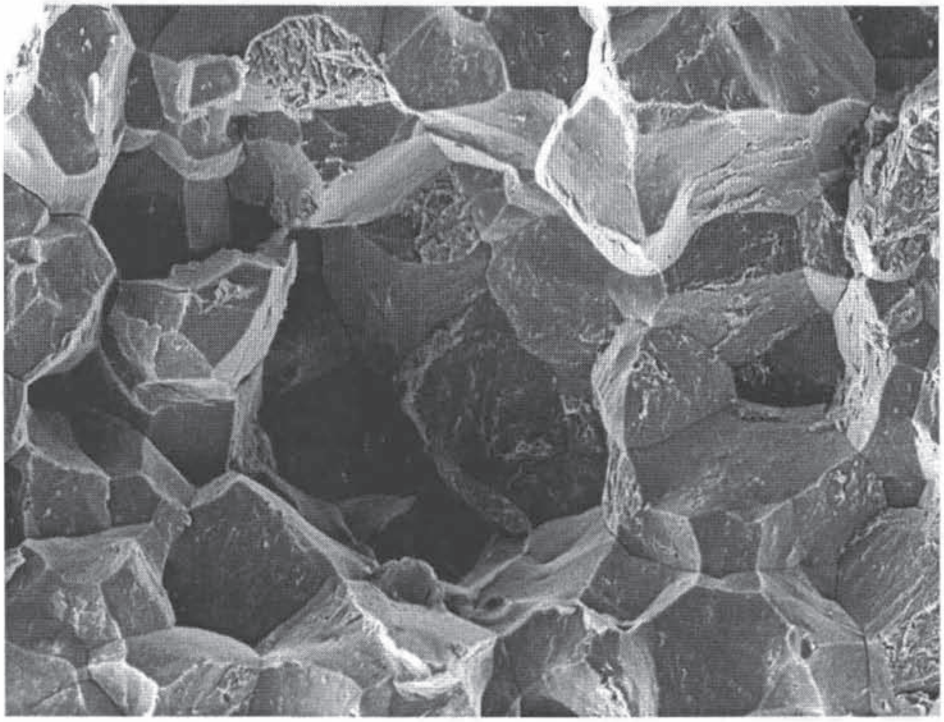


Figure P4.63 (Courtesy of met-tech)

4.63

The image shows 3-D grains on the fracture surface of the material. It is clear that some grains are fractured along their boundaries as you can see sharp corners. But some grains have fractured in the middle and not along the boundary (see top left, and top right grains). The fracture propagated along the grain boundaries.

If you draw a $2.54\times10^{-2}m$ line at the bottom of the figure, you will notice that it will intersect ~ four grains, $n_L = 4/2.54\times10^{-2}m$.

$$\Rightarrow d = \frac{C}{n_L(M)} = \frac{1.5}{\frac{4}{2.54\times10^{-2}m}(500)}$$

$$= 1.9\times10^{-5}m$$

$$= 19\ um$$

Problem 4.64
Describe a (a) Schottky imperfection, and (b) Frenkel imperfection for an ionic compound Na_2O.

4.64

a) A Schottky imperfection: $2Na^+$ cation vacancies and one O^{2-} anion vacancy.

b) Ionic radius: Na^+ 0.098nm; O^{2-} 0.132nm

It is likely the smaller positive Na^+ cation moves into an interstitial site to form a Frenkel imperfection.

Problem 4.65

The image below is a TEM image and shows the structure of a cold-worked aluminum alloy. Describe all the features that you are observing in this micrograph. Speculate as to what happened.

4.65

In the TEM image, one can observe grains, grain boundaries, dislocation lines, pile up of dislocations along the boundaries, and low angle boundaries.

Cold working is a process of applying large forces (stresses) to the material at room temperature. The large stresses will cause dislocations and movements. Formation and movement of dislocations will cause the material to deform heavilly.

We can conclude that the component was severely deformed.

Problem 4.66
In Figure 4.34, the HRTEM image of AIM is presented. In the figure, the dislocations are highlighted with arrows and the letter "d". Can you verify that what the scientist designates as an edge dislocation is in fact an edge dislocation (hint: compare Figure 4.34 to Figure 4.18a)? Also, discuss how the scientist knows that a stacking fault exists at the top of the image.

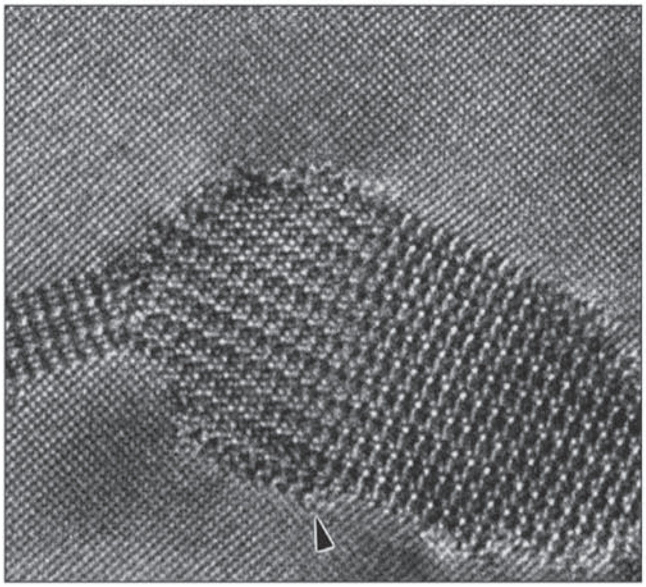
4.66

If you follow the line of atoms (light lines) with a pencil, you will notice that anywhere you see the letter "d", there is an extra (unmatched) line of atoms, a dislocation similar to figure 4.18 a. Keep tracing the line of atoms one-by-one and you will see the extra plane of atoms.

In the top region, between the two arrows, you will notice that the atomic spacing is more compacted and different than the rest of the image. This is generally the sign of a stacking fault (see section 4.4.3).

Problem 4.67

The image below is a HRTEM image of an Al-Cu-Mg-Li alloy. Describe all the features that you are observing in this micrograph. Speculate as to what you are observing.



4.67

The HRTEM image shows two structurally distinct regions in the Aluminum alloy.

The outer region is the Aluminum matrix and the central region is a completely different material, an intermetallic compound. It is clear that atom size and distribution is different in the two areas.