2022 Fall General Physics B 2-nd midterm

[Wolfson Ch. 10-14 Ch. 32] 2022/12/02, ***8:20am – 09:50am***

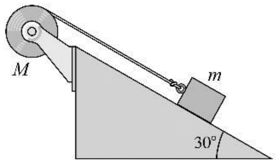
(i)依空格號碼順序在第二張**正面**寫下所有填充題答案，不要寫計算過程。

(ii)依計算題之題號順序在第二張**反面**以後寫下演算過程與答案，**每題從新的一頁寫起**。

Useful constant: The gravitational field on Earth g = 9.803 m/s^2. Moment of inertia of a thin rod about center is ML^2/12, disk or solid cylinder about its axis is MR^2/2, solid sphere about diameter is 2MR^2/5. 1 pN = 10^{-12} N. Speed of light c = 299792458 m/s.

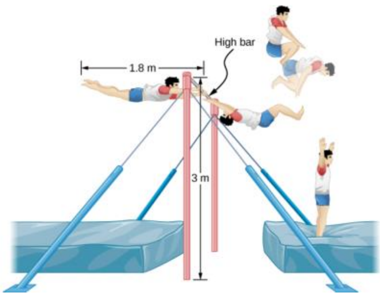
Part I. Filling the blank (total 75 points, 5 points per blank)

• A wheel with total mass 14.7 kg consists of a solid wooden disk with a thin metal rim bounded to the disk's edge. It starts from rest and rolls down an incline of height 1.00 m. At the bottom it's going at 3.38 m/s. Find the mass of the disk. 【01】 kg.

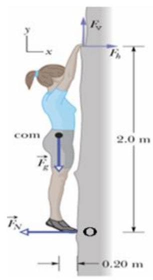


• A block of mass m = 4.0-kg rests on a slope and is attached by a string of zero mass to a pulley of mass M = 0.80 kg and radius R = 5.5 cm, as shown in right figure. Given the kinetic, as shown in right figure. Given the kinetic coefficient of friction (f_k) between block and slope is -0.26. When the block is released and slides down the slope, the acceleration of block is 【02】 m/s^2.

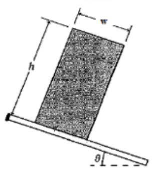
• An ice skater starts a spin with her arms stretched out to the sides. She balances on the tip of one skate to turn without friction. She then pulls her arms in so that her moment of inertia decreases by a factor of 4. In the process of her doing so, what happens of the factor of her kinetic energy? 【03】



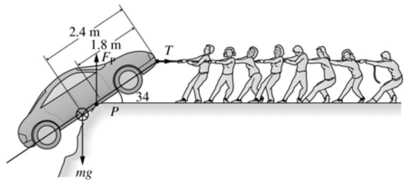
• An 83.5-kg gymnast dismounts from a high bar in the right figure. He starts the dismount at full extension, then tucks to complete a number of revolutions before landing. His moment of inertia when fully extended can be approximated as a rod of length 1.8 m and when in the tuck a rod of half that length. If his rotation rate at full extension is 2.0 rev/s and he enters the tuck when his center of mass is at 3.0 m height moving horizontally to the floor, how many revolutions can he execute if he comes out of the tuck at 1.8 m height? 【04】



• In the right figure, a 65 kg rock climber hangs by the crimp hold of one hand. Her feet touch the rock directly below her fingers. Assume that the force from the horizontal ledge supporting her fingers is equally shared by the four fingers. Calculate the horizontal and vertical components F_h and F_v of the force on each fingertip. 【05】



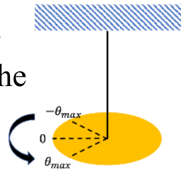
• A rectangular block four times as high as it is wide is resting on a board as the right figure. Let the coefficient of static friction between the block and the board be μ. Find the critical value of μ at which the block is equally likely to tip over or begin sliding as the tilt of the board increases. 【06】



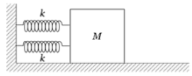
• The right figure shows a 1360-kg car that gas slipped over an embankment. People are trying to hold the car in place by pulling on a horizontal rope. The car's bottom is pivoted on the edge of the embankment, and its center of mass lies farther back, as shown in the right figure. If the car makes 34° angle with the horizontal, what force must the people apply to hold it in place? 【07】

• The cellular motor driving the flagellum (鞭毛) in *E. coli* (大腸桿菌) exerts a typical torque of 448 pN·nm on the flagellum. If this torque results from a force applied tangentially to the outside of the 16-nm-radius flagellum, what's the magnitude of that force? 【08】 pN.

• A torsional pendulum is oscillating as shown in the right figure with a frequency $f = 0.15\ s^{-1}$ and the max rotating angle $\theta_{max} = 10$ degrees. What is the maximum angular acceleration in the oscillation? 【09】 rad/s^2.



• A mass m is mounted to two identical springs with constant k as in the right figure. Find the angular frequency of the oscillation. 【10】



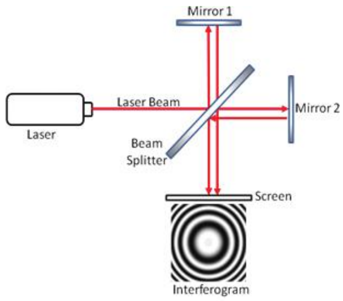
• A car's suspension system acts like a mass-spring system with $m = 1200\ kg$ and $k = 58000\ N/m$. Its absorber provides a damping constant $b = 10000\ kg/s$. As the car passes a bump, it bounces up and down. What is the period of the bounces? 【11】 s. (3 significant figures)

• When a laser beam fires at a moving atom (see the right figure), the laser frequency the atom sees will depend on its velocity. Assume the laser frequency is measured to be 335 THz (1 THz = 10^{12} Hz) in the rest frame of the atom. How much is the frequency change as the atom move toward the beam with a speed of 100 m/s? 【12】 Hz. (two significant figures)

• If a sound wave with an intensity of 0.2 W/m^2 is amplified (made stronger) by 33 dB. What is the intensity of the amplified wave? 【13】 W/m^2.

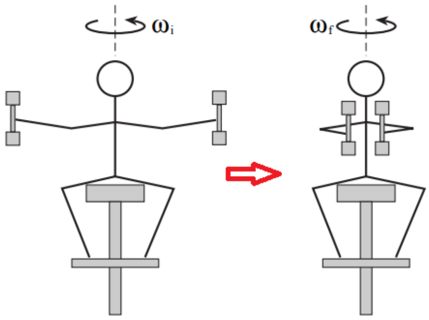
• The vibrating part of the G string of a certain violin is 330 mm long and has a fundamental (lowest) tone frequency of 196 Hz when under a tension of 50 N. What is the linear density of the string? 【14】 kg/m.

• Consider a Michelson interferometer as shown in the right figure. In the setup, the laser wavelength is 633 nm. As the Mirror 2 moves, 200 bright fringes appears at the center of the interference. How far did the Mirror 2 move? 【15】 μm. (three significant numbers)

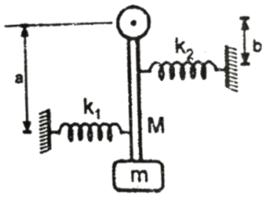


Part II Problems (total 30 points, 10 points per problem)

【1】 A student sits on a rotating stool holding two 1 kg objects as in the right figure. When his arms are extended horizontally, the objects are 1 m from the axis of rotation, and he rotates with angular speed of 0.82 rad/sec. The moment of inertia of the student plus the stool is 8.0 kg·m^2 and is assumed to be constant. The student then pulls the objects horizontally to a radius 0.32 m from the rotation axis. (a). Calculate the final angular speed of the student. (b). Calculate the change in kinetic energy of the system.

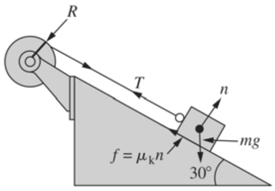


【2】A rod of mass ***M*** and length ***L*** is hinged at one end and carries a block with mass m at another end. A spring with spring constant ***k*₁** is installed at distance ***a*** form the hinge and another spring with constant ***k*₂** at a distance ***b***, as shown in the right figure. If the whole device is places on a smooth horizontal table. Find the frequency of vibrations.



【3】 The primary mirror of the Hubble Space Telescope has a diameter of 2.40 m.
(a) What is the angle between two stars that are just-resolvable? Assume an average light wavelength of 550 nm.
(b) If there are two stars at a distance of 1 million light year, how close together can they be and still be resolved? (A light year, or ly, is the distance that light travels in 1 year.)

Part I Answer Sheet, **Note: 有效位數錯誤者，扣 0.5 分。**

A 【01】	**8.35** $M = m_{disk} + m_{rim}$. Equating the initial and final mechanical energies we get $Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}(I_{disk} + I_{rim})\omega^2$, $2Mgh = Mv^2 + \left(\frac{1}{2}m_{disk} + m_{rim}\right)R^2\left(\frac{v}{R}\right)^2$, $(2gh - v^2)M = \left(\frac{1}{2}m_{disk} + M - m_{disk}\right)v^2$, $m_{disk} = 4M\left(1 - \frac{gh}{v^2}\right) = 8.345kg = 8.35kg$ (three significant figures)
A 【02】	**6.5** Draw a diagram of the situation. Applying Newton's second law to the mass gives $mgsin\theta - f_k - T = ma,\ n - mgcos\theta = 0,\ mgsin\theta - \mu_k mgcos\theta - T = ma,$ where we have used the force due to kinetic friction, $f_k = \mu_k n,$ Likewise, applying the rotational analog of Newton's second law to the wheel gives $\tau = I\alpha$, $TR = I\alpha$, because the tension is the only torsional force acting on the wheel, $I = (1/2)MR^2$ and $a = R\alpha$. The tension from the rotational application of Newton's second law gives  $T = \frac{I\alpha}{R} = \frac{\left(\frac{1}{2}MR^2\right)\left(\frac{a}{R}\right)}{R} = \frac{1}{2}Ma$, $\rightarrow mgsin\theta - \mu_k mgcos\theta - \frac{1}{2}Ma = ma,$ $\rightarrow mgsin\theta - \mu_k mgcos\theta = \frac{1}{2}Ma + ma = \left(\frac{2m+M}{2}\right)a,$ $\rightarrow mg\frac{1}{2} - \mu_k mg\frac{\sqrt{3}}{2} = \left(\frac{2m+M}{2}\right)a,$ $\rightarrow a = \frac{mg(1-\mu_k\sqrt{3})}{(2m+M)} = \frac{4kg \times 9.803m/s^2 \times (1+0.26\sqrt{3})}{2\times 4kg + 0.80kg} = 6.46255 = 6.5$ (two significant figures)
A 【03】	**increases by a factor of 4 (增加四倍)** KE=(1/2)Iω^2=(Iω)^2/2I=L^2/2I, I_f=(1/4)I_i, L_i=L_f,→ KE_f= $L_f^2/2I_f$= $L_i^2/2(1/4)I_i$→ KE_f= $4L_i^2/2I_i$=4 KE_i
A 【04】	**4 (four revolutions)** Using the equations of kinematics, we can find the time interval from a height of 3.0 m to 1.8 m. Since he is moving horizontally with respect to the ground, the equations of free fall simplify. This will allow the number of revolutions that can be executed to be calculated. Since we are using a ratio, we can keep the units as rev/s and don't need to convert to radians/s. The moment of inertia at full extension is I_0 = (1/12) mℓ_0^2 = (1/12)83.5kg(1.8m)^2 = 22.545 kg·m^2. The moment of inertia in the tuck is I_f = (1/12) mℓ_f^2 = (1/12)83.5kg(0.9m)^2 = 5.63625 kg·m^2. Conservation of angular momentum: $I_f\omega_f = I_0\omega_0 \Rightarrow \omega_f = I_0\omega_0/ I_f$ = 22.545 kg·m^2 (2.0 rev/s) / 5.63625 kg·m^2 = 8.0 rev/s. Time interval in the tuck: t = √2hg = √2(3.0-1.8)m/9.803m/s = 0.5s. In 0.5 s, he will be able to execute four revolutions at 8.0 rev/s.
A 【05】	**F_v=160N, F_h=16N (each 2.5 points)** $F_{net,x} = -F_N + 4F_h$, $F_{net,y} = 4F_v - mg = 0$, $F_v = \frac{mg}{4} = \frac{65kg \cdot 9.803m/s^2}{4} = 159.29875N = 160N$ (two significant figures) We calculate the net torque about an axis that is perpendicular to the page and passes through point O. $\tau_{net,z} = (0)F_N + (0.2)(mg) - (2.0)(4F_h) + (0)(4F_v) = 0$, $F_h = \frac{0.20m \times 65kg \times 9.803m/s^2}{4 \times 2.0m} = 15.929875N = 16N$ (two significant figures)
A 【06】	**0.25 (1/4)** **$\mu_c = \tan\theta_c$ = w/h = 1/4 = 0.25**
A 【07】	**6.6kN** For the car to remain in static equilibrium. Three forces act on the car: the tensile force of the rope, the force due to gravity, and the force exerted by the embankment. If we evaluate the torques about point *P*, the unknown force F_P exerted by the edge of the embankment does not contribute, so the tension necessary to keep the car in equilibrium can be found directly. Thus, the zero-torque condition for equilibrium gives $\left(\sum\vec{\tau}\right)_p = 0 = \vec{r}_{cm} \times m_{car}\vec{g} + \vec{r}_{rope} \times \vec{T} = m_{car}g(L-\ell)sin\left(\frac{\pi}{2} - \theta\right) - T\ell sin\theta$, where L = 2.4 m, ℓ = 1.8 m, and θ = 34°. Inserting the given quantities gives $T = m_{car}g\left(\frac{L-\ell}{\ell}\right)cot\theta = (1360kg)(9.803m/s^2)\left(\frac{2.4m - 1.8m}{1.8m}\right)cot34^o = 6588.54N = 6.6kN$ (two significant figures)
A 【08】	**28** We're told that the force is applied tangentially, so θ = 90°, and reduces to: $\tau = rF$. Solving for the motor's applied force: $F = \tau / r$ = 448 pN nm / 26 nm = 28 pN. (two significant figures)
A 【09】	**0.15**
A 【10】	$\sqrt{2k/m}$
A 【11】	**1.13**
A 【12】	**1.2×10^8**
A 【13】	**400**

A 【14】	2.99×10^{-3}
A 【15】	63.3

Part II Answer Sheet，**Note:** 有效位數錯誤者，扣 **0.5** 分。

A1

(a). 1.0 rad/s

(b). 0.74 J

(a).

m = 1 kg, R = 1 m, r = 0.32 m, ω_i = 0.82 rad/sec, and I_s = 8.0 kg·m^2, $\sum \vec{\mathbf{L}}$ = constant, $K_{rot} = (1/2)I\omega^2$.

The initial moment of inertia of the system is $I_i = I_s + 2mR^2$ = (8.0 kg·m^2) + 2 (1 kg) (1 m)^2 = 10 kg·m^2.
The final moment of inertia of the system is $I_f = I_s + 2mr^2$ = (8.0 kg·m^2) + 2 (1 kg) (0.32 m)^2 = 8.2048 kg·m^2.
From conservation of the angular momentum it follows that $L = I\omega = I_i\omega_i = I_f\omega_f$, $\omega_f = \omega_i I_i/I_f$ = (0.82 rad/sec) (10 kg·m^2 / 8.2048 kg·m^2 = 0.999415 rad/s = 1.0 rad/s. (two significant figures)

(b).
$\Delta K = K_f - K_i = (1/2)I_f\omega_f^2 - (1/2)I_i\omega_i^2$ = (1/2) (8.2048 kg m^2) (0.999415 rad/s)^2 - (1/2) (10 kg m^2) (0.82 rad/sec)^2 = (4.09760 J) - (3.362 J) = 0.7356 J = 0.74 J. (two significant figures)

A2

$$\tau = b\cdot k_2(b\cdot\theta) + a\cdot k_1(a\cdot\theta); \quad I\cdot\alpha = (MR^2 + \frac{m}{3}R^2)\ddot{\theta}$$

$$\tau = I\cdot\alpha \rightarrow (k_2b^2 + k_1a^2)\theta = (MR^2 + \frac{m}{3}R^2)\ddot{\theta}$$

$$\omega = \sqrt{\frac{(k_2b^2 + k_1a^2)}{(MR^2 + \frac{m}{3}R^2)}}$$

$$\rightarrow f = \frac{1}{2\pi}\sqrt{\frac{(k_2b^2 + k_1a^2)}{(MR^2 + \frac{m}{3}R^2)}}$$

A3 (5 points each)

(a) $\theta = 1.22\dfrac{550\text{ nm}}{2.40\text{ m}} = 2.80\times10^{-7}\text{rad}$

(b) $s = 10^6 \text{ ly} * 2.8 \times 10^{-7}\text{rad} = 0.28 \text{ ly}$