2022 Fall General Physics B final exam

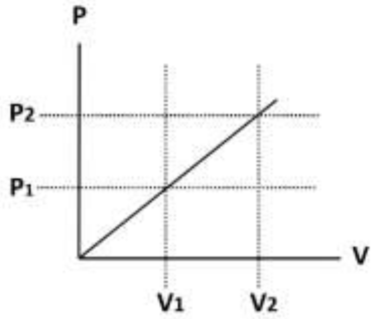
[Wolfson Ch. 15-19] **2023/01/10**, ***8:20am – 09:50am***

B

(i)依空格號碼順序在第二張**正面**寫下所有填充題答案，不要寫計算過程。

(ii)依計算題之題號順序在第二張**反面**以後寫下演算過程與答案，**每題從新的一頁寫起**。

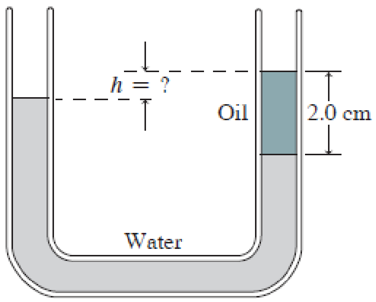
R = 8.314 J / K mol = Universal gas constant, *k* = 1.38×10^{-23} J / K = Boltzmann's constant



Part I. Filling the blank (total 75 points, 5 points per blank)

- A quantity of an ideal monatomic gas consists of N atoms, initially at temperature T_1. The pressure and volume are then slowly doubled, in such a way as to trace out a straight line on the P-V diagram as in the right figure. In terms of N, k = Boltzmann's constant, and T_1, find the work done by the gas. 【01】. ($P_2 = 2\ P_1$, $V_2 = 2\ V_1$)
- What should be the approximate specific-heat ratio of a gas consisting of 45% CO_2 molecules ($\gamma = 1.29$), 25% N_2 ($\gamma = 1.41$), and 30% Ne ($\gamma = 1.66$)? 【02】 .
- An ideal gas with pressure P and temperature T adiabatically free-expands from V to 4V. The entropy change is 【03】. (Give your answer in terms of R, T, V).
- A cyclic heat engine operates between a source temperature of 900ºC and a sink temperature of 20ºC. The mechanical work done by this engine is 4 kW. What is the least rate of heat rejection per kW net output of the engine? 【04】 (five significant figures) (ºK = ºC + 273)
- One mole of a monatomic gas is heated in such a way that its molar specific heat is 2R. During the heating, the volume of the gas is doubled. By what factor does the temperature of the gas change? 【05】.
- A refrigeration system with a COP of 3.5 is used to make ice from water at 0°C at a rate of 1000 lbs per day. What is the minimum power required by the refrigerator? 【06】 W. (heat of fusion of water = 334 kJ/kg) (1 lb = 0.454 kg) (three significant figures)
- Find the change in entropy if 500 g of water at 80 ºC is added to 300 g of water at 20 ºC. 【07】 cal/K. (three significant figures)
- Mars's atmospheric pressure is about 1% that of Earth, and its average temperature is around 215 K. Find the volume 【08】 L of 1 mol of the Martian atmosphere. (five significant figures)
- A 100.0 g piece of metal at 300.0°C is dropped into a cup containing 500.0 g of water at 15.0°C. The final temperature of the system is measured to be 40.0°C. What is the specific heat of the metal? 【09】 J/(kg·K) Assuming no heat is exchanged with the surroundings or the cup. The specific heat of water is 4190 J/(kg·K). (five significant figures)
- A power line wire spans between two support towers. The wire is made of aluminum, and on a winter day when the temperature is 0˚C the wire's actual length is 300.0 m. By how much does its length increase 【10】cm on a summer day when it's 40˚C ? (The linear thermal expansion coefficient for Aluminum is 24×10^{-6} 1/K) (three significant figures)
- The lamp heater is to heat up the room mainly by thermal radiation. It takes 1000 W by setting the surface temperature at 900 K. If you want to reduce the energy cost to 800 W, the surface temperature should become 【11】 K. (four significant figures)
- You're having your home's heating system replaced, and the heating contractor has specified a new system that supplies energy at the maximum rate of 40 kW. You know that your house loses energy at the rate of 1.2 kW per˚C temperature difference between interior and exterior, and the minimum winter temperature in your area is 10 ºC. The maximum temperature should be 【12】 ºC to maintain indoors. (three significant figures)

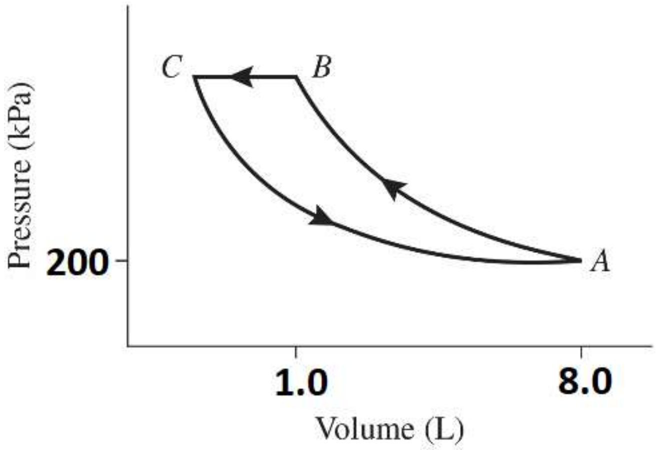
• An incompressible fluid flows steadily with a fluid speed of 3.5 m/s, through a pipe with a diameter of 2.0 cm. What is the flux? 【13】 L/s. (two significant figures) Then the pipe is narrowed to a diameter 1.00 cm with the pipe center 1m above. What does the fluid speed become? 【14】 m/s. (three significant figures) (Gravitational acceleration, g=10 m/s^2)



• A U-shaped tube open at both ends contains water and a quantity of oil occupying a 2.0-cm length of the tube, as shown in the right figure. If the oil's density is 70% of water's, what's the height difference h? 【15】 cm. (one significant figures)_

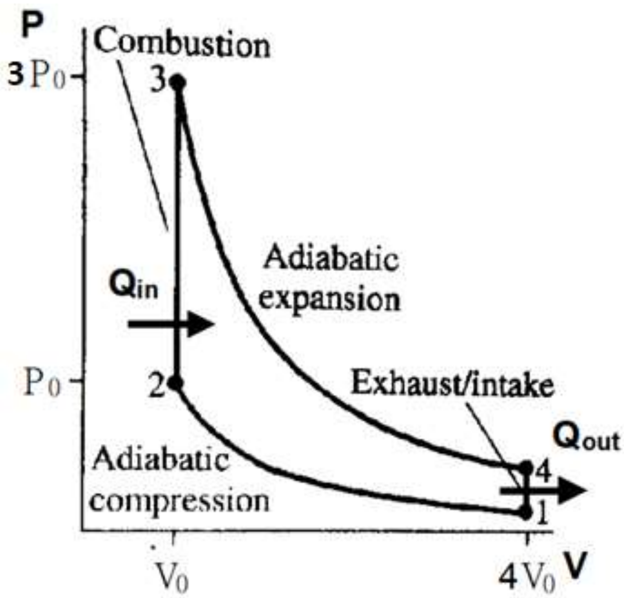
Part II Problems (total 30 points, 10 points per problem)

【B1】A 8.0-L ideal gas with $\gamma = 5/3$ is at 300 K and 200 kPa. The gas is compressed adiabatically until its volume becomes 1.0-L, then cooled at constant pressure back to 300 K, and finally allowed to expand isothermally to its original state as shown in the right figure.



(a). What is the pressure (kPa) at B? (2 points)
(b). How much work (J) we need to compress it from A to B? (2 points)
(c). What is the volume (L) at C? (3 points)
(d). What is the net work (J) done on the gas during one cycle? (five significant figures) (3 points)

【B2】 The Otto cycle consists of two adiabatic and two constant-volume processes, as shown in the figure. The gas in the engine has a specific heat ratio $\gamma = 1.8$.



(a). Find the pressure at point 1 and point 4 in terms of P_0. (three significant figures) (2 points)
(b). Find the engine's efficiency, assuming all processes are reversible. (two significant figures) (3 points)
(c). Find the maximum temperature in terms of the minimum temperature
T_{max} / T_{min}. (two significant figures) (3 points)
(d). What is the efficiency difference compared with that of a Carnot engine operating between the same temperature extremes? $e_{Carnot} - e_{Otto}$ =? (two significant figures) (2 points)

【B3】 On planet X, the absolute pressure at a depth of 2.00 m below the surface of a liquid nitrogen lake is 5.0×10^5 N/m^2. At a depth 10.0m below the surface, the absolute pressure is 15.0×10^5 N/m^2. The density of liquid nitrogen is 808.0 kg/m^3.
(a). What is the atmospheric pressure on planet X? (two significant figures)
(b). What is the acceleration due to gravity on planet X? (four significant figures)

Part I Answer Sheet, Note: 有效位數錯誤者，扣 0.5 分。

B 【01】	**(3/2)NkT$_1$** Given: $P/V = P_1/V_1$, $P(V) = (P_1/V_1)V$, $P_2 = 2P_1$, $V_2 = 2V_1$ Ideal gas law: $PV = NkT$, $T_2/T_1 = P_2V_2/(P_1V_1) = 4$, $T_2 = 4T_1$. $W = \int_{V1}^{V2} PdV = (P_1/V_1)\int_{V1}^{V2} VdV = ½(P_1/V_1)(V_2^2 - V_1^2) = (3/2)P_1V_1 = (3/2)NkT_1$.
B 【02】	**1.38** By generalizing the result of the previous problem, the molar specific heat of a mixture of three gases is $C_V = f_1 C_{V1} + f_2 C_{V2} + f_3 C_{V3}$. For each gas and the mixture, use $C_V = R(\gamma-1)^{-1}$ to obtain $(\gamma-1)^{-1} = f_1(\gamma_1-1)^{-1} + f_2(\gamma_2-1)^{-1} + f_3(\gamma_3-1)^{-1} = 0.45/0.29 + 0.25/0.41 + 0.3/0.66 = 1.55172 + 0.60976 + 0.45455 = 2.61603$, $\gamma = 1.38226 = 1.38$
B 【03】	**nRln(4) or 1.386nR**
B 【04】	**1.3318** $e_{\max} = e_{\text{rev}} = 1 - \frac{T_C}{T_H} = 1 - \frac{20+273}{900+273} = 1 - \frac{293}{1173} = 1 - 0.24979 = 0.75021$ $\frac{W_{net}}{Q_H} = e_{\max} = 0.75021$ $Q_H = \frac{W_{net}}{0.75021} = \frac{4}{0.75021} = 5.33184$ $Q_C = Q_H - W_{net} = 5.33184 - 4 = 1.33184 = 1.3318$ kW (five significant figures)
B 【05】	**4 (four)** Concepts: First law of thermodynamics Reasoning: $\Delta E_{int} = \Delta Q - P\Delta V$, where E_{int} is the internal energy of the system, P is the pressure, V is the molar volume, and Q is the heat transferred to the gas by the surroundings. For an ideal gas, ΔE_{int} depends on the temperature only, $\Delta E_{int} = nC_v\Delta T = 3R\Delta T/2$. Given $C_v = \Delta Q/\Delta T = 2R$, $\Delta Q = 2R\Delta T$. Details of the calculation: Using the ideal gas law, PV = RT, we have $\Delta E_{int} = \Delta Q - P\Delta V = 2RdT - (RT/V)\Delta V = (3R/2)\Delta T$. Therefore $(RT/V)\Delta V = (R/2)\Delta T$, or $2\Delta V/V = \Delta T/T$, $2\ln V = \ln T$, $\ln V^2 = \ln T$, $\ln V^2 - \ln T = 0$, $\ln(V^2/T) = 0$ or V^2/T = const. Therefore, doubling the volume of the gas will increase its temperature by a factor of four.
B 【06】	**501 W** The first bit of information we need for this question is the ΔH_{fusion} of water at 1 atm and 0°C; this is not a number that you need memorize, rather you may find it easily in many references under "heat of fusion of water." ΔH_{fusion} = 334 kJ/kg. Next, we need the definition of the Coefficient of Performance (COP) for a refrigeration system; COP = Q/W, where Q is the heat removed and W is the work expended; you should familiarize yourself with this definition. So for this question, W = Q/COP = 1000 lb/day (1 day/24 hours) (1 hour /3600 sec) (0.454 kg/lb)(334 kJ/kg) (1/3.5) = 501 W.
B 【07】	**3.20** The total amount of water is 800g, so the final temperature of the system is given by (5/8)353K + (3/8)293K = 330.5K. For m_1 = 500 g and m_2 = 300 g, the entropy change is given by $\Delta S = \int \frac{dQ}{T} = \int_{353}^{330.5} c_w m_1 \frac{dT}{T} + \int_{293}^{330.5} c_w m_2 \frac{dT}{T} = c_w\left(m_1 ln\frac{330.5}{353} + m_2 ln\frac{330.5}{293}\right) = 3.20\ cal/K$
B 【08】	**1764.1** nR = PV/T = (1 atm) × 22.4/273 = (0.01 atom) × V/215, V=1764.1
B 【09】	**2014.4** $0.1 \times (300 - 40) \times C = 0.5 \times (40 - 15) \times 4190, C = 2014.4$
B 【10】	**28.8** $\Delta L = 300 \times 24 \times 10^{-6} \times (40 - 0) = 0.288m = 28.8cm$

B **【11】**	**851.2** $1000 \times (900)^4 = 800 \times (T)^4, \quad \mathrm{T} = 851.2\mathrm{K}$
B **【12】**	**43.3** $40 = (\mathrm{T} - 10) \times 1.2, \mathrm{T} = 43.3$
B **【13】**	**1.1** $3.5 \times (0.01)^2\pi = 10.99 \times 10^{-4}\ [m^3/s] = 1.1\ [L/s]$
B **【14】**	**14.0** $3.5 \times (0.01)^2 = v_2 \times (0.005)^2, \quad \mathrm{v}_2 = 14.0$
B **【15】**	**0.6** $(2.0 - \mathrm{h}) \times 1 = 2.0 \times 0.7, \quad \mathrm{h} = 0.6$

Part II Answer Sheet, **Note:** 有效位數錯誤者，扣 **0.5** 分。

【B1】

(a). 6400 kPa (2 points)

(b). 7200 J (2 points)

(c). 0.25 L (3 points)

(d). 6454.8 J (five significant figures) (3 points)

Given a 8.0 L ideal gas with γ = 5/3 is at 300 K and 200 kPa (A). The gas is compressed adiabatically until its volume becomes 1.0 L (B), then cooled at constant pressure back to 300 K (C), and finally allowed to expand isothermally to its original state (A)

(a). From PV^γ = constant for adiabatic process, $P_B = P_A\left(\frac{V_A}{V_B}\right)^\gamma = 200 \times \left(\frac{8}{1}\right)^{5/3}$ kPa $= 200 \times (8)^{5/3}$ kPa $=$ 6400 kPa

(b). From $W_{A\to B} = \frac{P_BV_B - P_AV_A}{\gamma - 1}$ for adiabatic process, $W_{A\to B} = \frac{3}{2}(6400 \times 1 - 200 \times 8)\,J = \frac{3}{2}(6400 - 1600)\,J = 7200\,J$

(c). According to $PV = nRT$ and $\langle K\rangle = \frac{3}{2}nRT$, $\Delta E_{\mathrm{int}} = 0$ and PV = constant for isothermal process.

$$V_C = V_A\left(\frac{P_A}{P_C}\right) = 8 \times \left(\frac{200}{6400}\right)\ L = 0.25\ L$$

(d). For a cyclic process, $\Delta E_{\mathrm{int}} = 0$ and which implies that $\mathrm{Q} = -\mathrm{W}$ from $\Delta E_{\mathrm{int}} = \mathrm{Q} + \mathrm{W}$

$$-Q = W = W_{A\to B} + W_{B\to C} + W_{C\to A} = 7200 + 6400 \times (1 - 0.25) - 1600 \times \int_{0.25}^{8} \frac{dV}{V}$$

$$= 7200 + 4800 - 1600 ln\left(\frac{8}{0.25}\right) = 12000 - 1600 ln\left(\frac{8}{0.25}\right) = 12000 - 5545.2 = 6454.8\,J$$

【B2】

(a) $P_1 = 0.0825P_0$, $P_4 = 0.247P_0$ (three significant figures) (2 points)

(b) 0.67 (two significant figures) (3 points)

(c) 9.1 (two significant figures) (3 points)

(d) 0.22 (two significant figures) (2 points)

The engine absorbs heat (Q_h) during combustion, and expels heat to the environment (Q_c) during the exhaust segment. Both these processes are at constant volume, so $Q = nC_V\Delta T$, and the efficiency is:
$e_{Otto} = W / Q_h = 1 - Q_c / Q_h = 1 - \Delta T_c / \Delta T_h$. We can find the respective temperature changes assuming the gas mixture in the engine is ideal: $T = pV / nR$.
(a) Using PV^γ = const, $P_1 = P_2(V_2/V_1)^\gamma = P_0(1/4)^{1.8} = 0.08247P_0$, $P_4 = P_3(V_3/V_4)^\gamma = 3P_0(1/4)^{1.8} = 0.24741P_0$
(b) The hot temperature change is between point 2 and point 3 in Figure:

$$\Delta T_h = T_3 - T_2 = \frac{1}{nR}[3PV_0 - P_0V_0] = \frac{2P_0V_0}{nR}$$

where we use the values for the pressure and volume given in the figure. The cold temperature change is between point 1 and point 4 in the figure, but the pressures aren't given in this case. Therefore, the cold temperature change can be written:

$$\Delta T_c = T_4 - T_1 = \frac{1}{nR}[0.24741P_0 - 0.0824P_0]4V_0 = \frac{0.66P_0V_0}{nR}$$

$$e_{Otto} = 1 - \frac{\Delta T_c}{\Delta T_h} = 1 - \frac{\frac{0.66P_0V_0}{nR}}{\frac{2P_0V_0}{nR}} = 1 - \frac{0.66}{2} = 1 - 0.33 = 0.67$$

(c) $T_{min} = T_1 = \frac{1}{nR}[0.0824P_0]4V_0 = \frac{0.3296P_0V_0}{nR}$

$$T_{max} = T_3 = \frac{1}{nR}[3PV_0] = \frac{3P_0V_0}{nR} = 9.1T_{min}$$

(d) $e_{Carnot} = 1 - \frac{T_{min}}{T_{max}} = 1 - \frac{T_{min}}{9.1T_{max}} = 1 - \frac{1}{9.1} = 1 - 0.11 = 0.89$

$$e_{Carnot} - e_{Otto} = 0.89 - 0.67 = 0.22$$

【B3】

(a). $2.5 \times 10^5 N/m^2$ (two significant figures)
(b). 154.7 m^2/s (four significant figures)

$P = P_0 + \rho g h$

$5.0 \times 10^5 = P_0 + 808 \times g \times 2$
$15.0 \times 10^5 = P_0 + 808 \times g \times 10$

$P_0 = 2.5 \times 10^5 N/m^2$ $g = 154.7$ m/s^2