

Course announcement

- The 5th homework set solution has been posted.
- 1/10 will be Final exam.

Final Exam

- Exams will be started at **8:00AM and ends at 9:50AM.**
- **Please bring student ID and calculator.**
- You can bring one A4 information sheet for the exam.
- Cheating will result in 0 points for the whole exam and will be reported to university.
- There will be 6 problem sets. Range: 16~19 chapter of Essential University Physics by Richard Wolfson.
- All the problems will be related to materials covered in class, problem discussed in class, and homework problems.

Policy for COVID-19

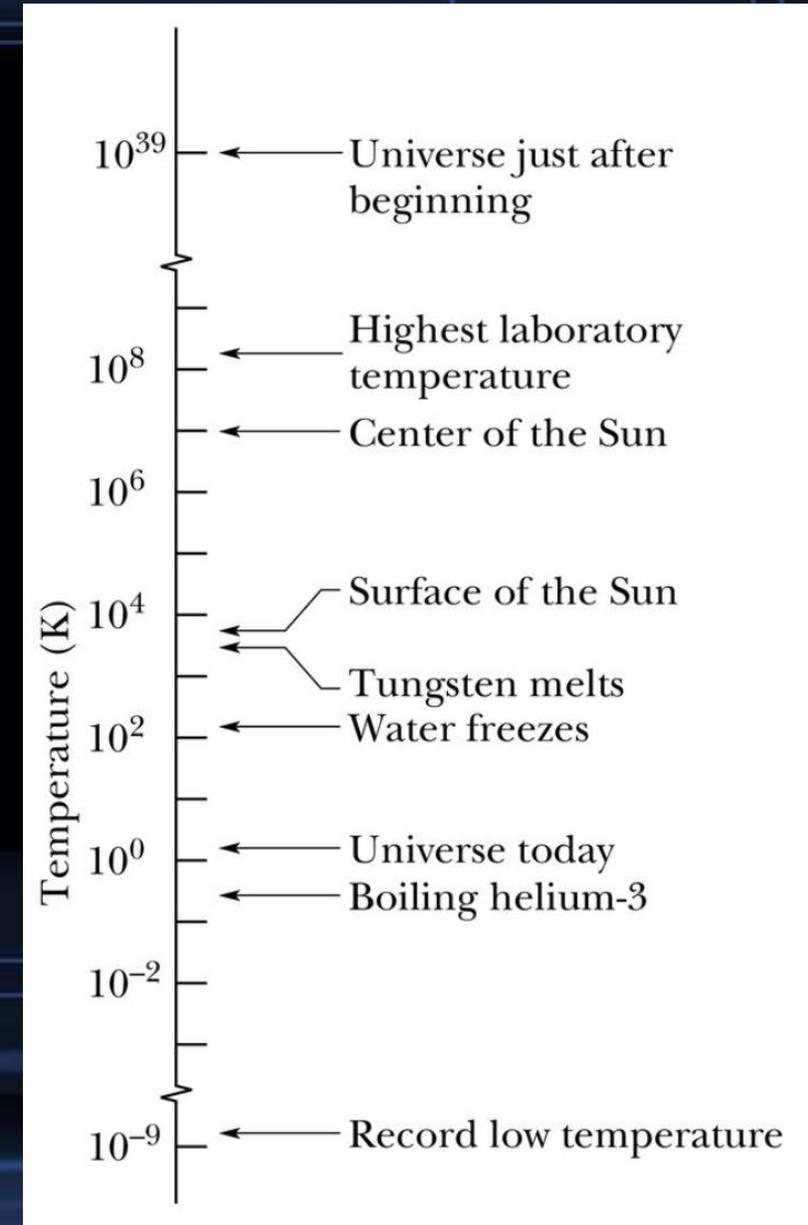
- We follow university guideline about course under COVID-19.
- Please have facial mask with you
- For students who cannot attend exam due to COVID-19, they can have **test remotely with monitor of web camera.** (<https://teams.live.com/meet/9570955571789>). The problem will be posted on eLearn and can be handed via eLearn. **Only students inform me in advance can have test via eLearn.**

GENERAL PHYSICS B1

REVIEW III

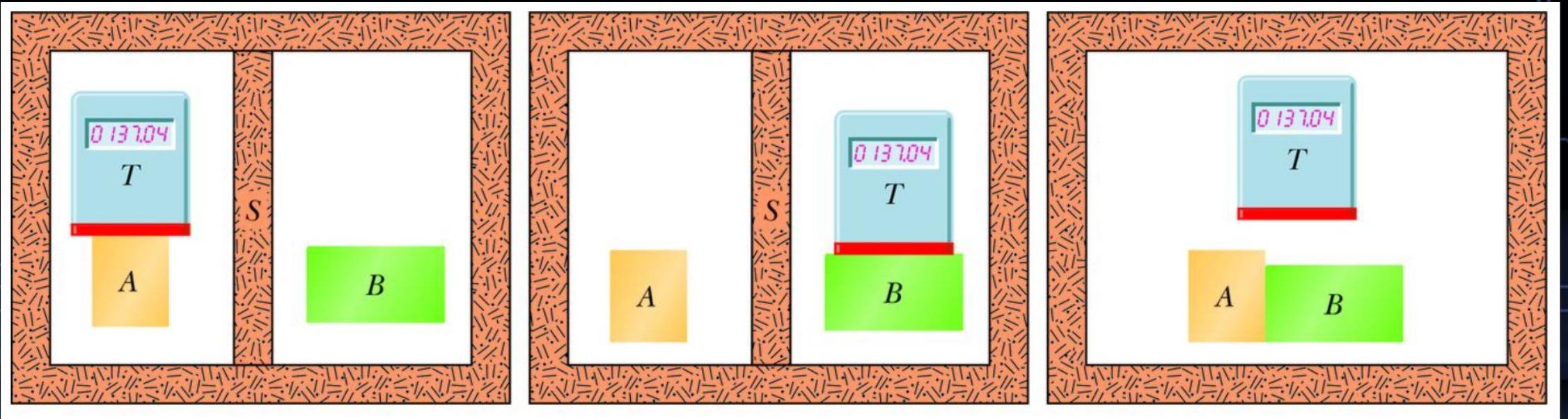
Temperature

- Temperature is one of the seven **SI base quantities**. Physicists measure temperature on the **Kelvin scale**, which is marked in units called kelvins. The definition of kelvins is given by Boltzmann constant $k=1.3806505 \times 10^{-23}$ J/K.
- The temperature of **a natural system** apparently has no upper limit, it does have a lower limit; this limiting low temperature is taken as the zero of the Kelvin temperature scale.



The Zeroth Law of Thermodynamics

- If bodies A and B are each in **thermal equilibrium** with a third body T, then A and B are in thermal equilibrium with each other.



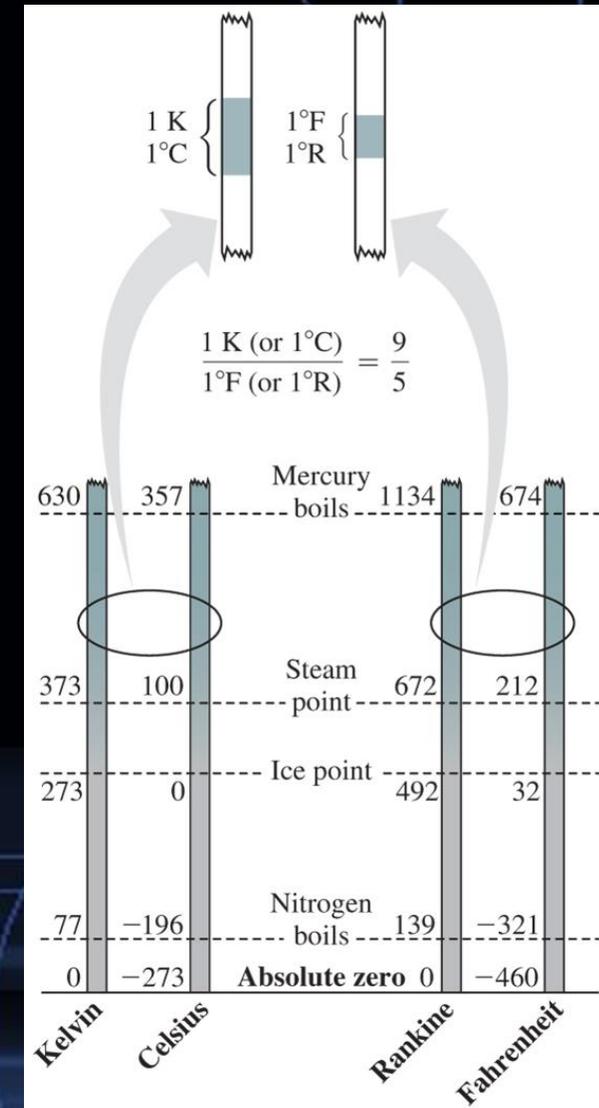
The Celsius and Fahrenheit scales

- Other frequently used temperature scale: Celsius, Fahrenheit, and Rankine scale.
- Conversion between scales:

$$T_C = T - 273.15^\circ$$

$$T_F = \frac{9}{5}T_C + 32^\circ$$

Temperature	°C	°F
Boiling point of water ^a	100	212
Normal body temperature	37.0	98.6
Accepted comfort level	20	68
Freezing point of water ^a	0	32
Zero of Fahrenheit scale	≈ -18	0
Scales coincide	-40	-40



Heat

- Heat is the energy transferred between a system and its environment because of a temperature difference that exists between them.
- Before scientists realized that heat is transferred energy, heat was measured in terms of its ability to raise the temperature of water. Thus, the calorie (cal) was defined as the amount of heat that would raise the temperature of 1 g of water from 14.5°C to 15.5°C.
- In 1948, the scientific community decided that since heat (like work) is transferred energy, the SI unit for heat should be the one we use for energy—namely, the joule. **1 cal = 4.1868 J**

The Absorption of Heat by Solids and Liquids

- The heat capacity C of an object is the proportionality constant between the heat Q that the object absorbs or loses and the resulting temperature change ΔT of the object:

$$Q = C \Delta T = C(T_f - T_i)$$

- Specific Heat: A “heat capacity per unit mass” or specific heat c that refers not to an object but to a unit mass of the material of which the object is made.

$$Q = cm \Delta T = cm(T_f - T_i)$$

Heat Transfer Mechanisms

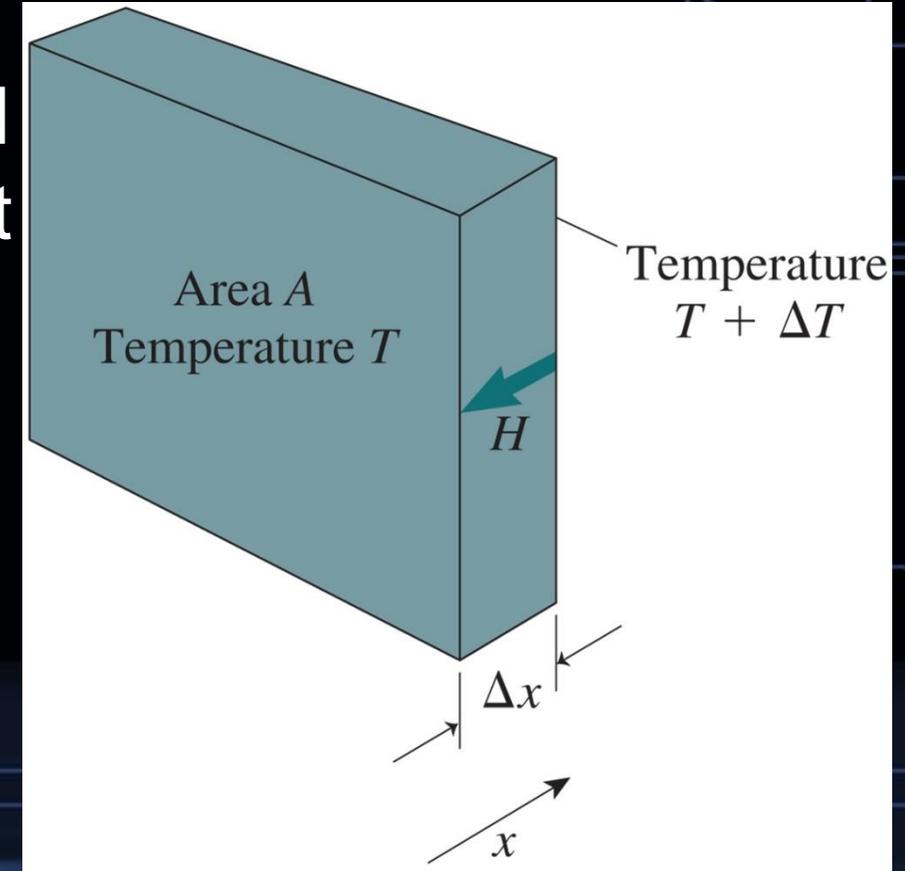
- We have discussed the transfer of energy as heat between a system and its environment, but we have not yet described how that transfer takes place. There are three transfer mechanisms: **conduction, convection, and radiation.**

Conduction

- Consider a slab of face area A and thickness L , whose faces are maintained at temperatures T and $T + \Delta T$, Experiment shows that the heat conduction rate H (the amount of heat Q transferred per unit time t) is:

$$H = \frac{dQ}{dt} = -kA \frac{\Delta T}{\Delta x}$$

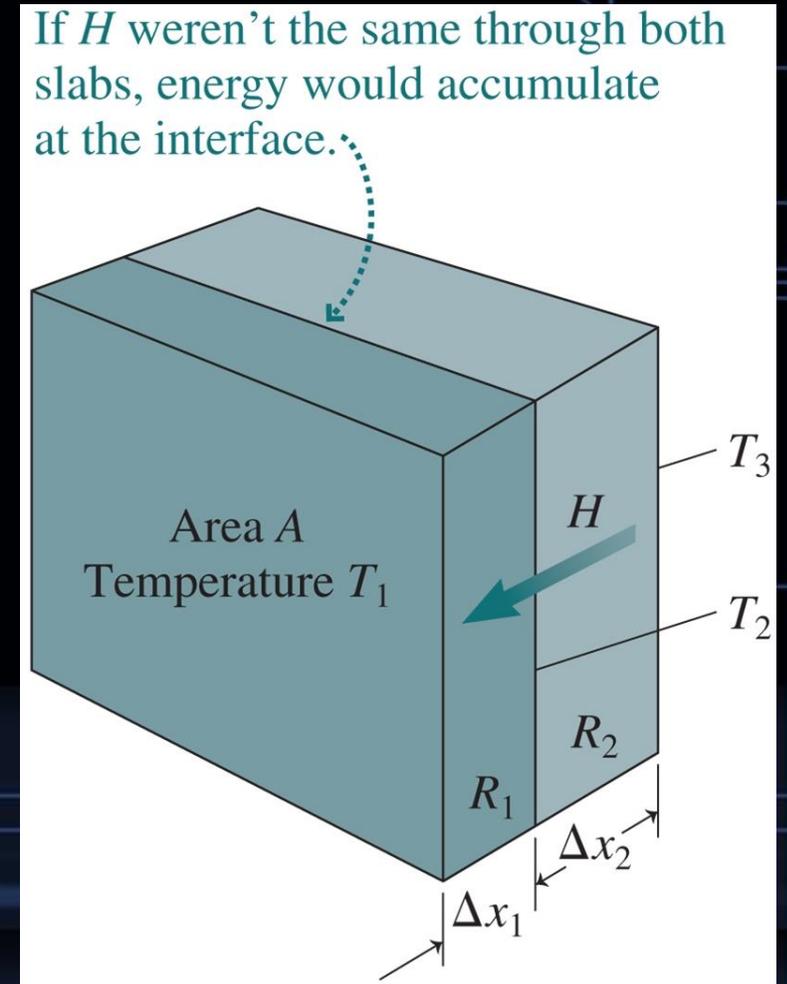
Where k is the thermal conductivity with SI unit of W/mK



Conduction of a composite slab

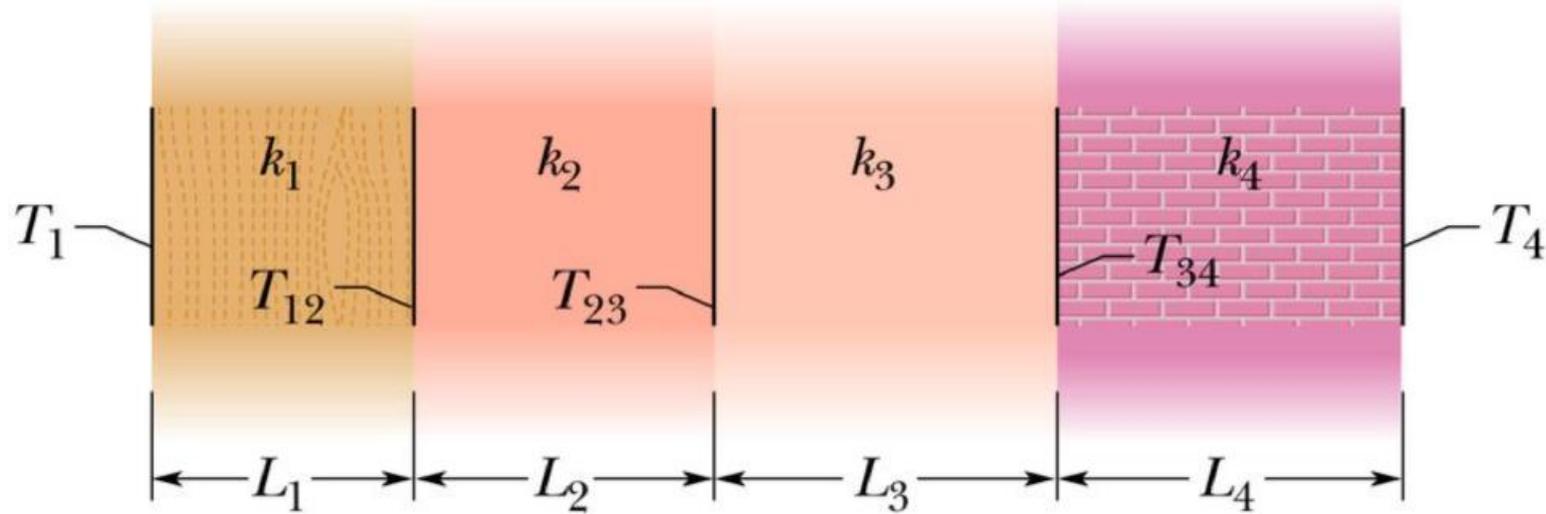
- If we consider a composite slab as shown in the figure that contains two different materials with thermal conductivity k_1 and k_2 .
- The heat flow H must be the same through both slabs, so energy doesn't accumulate at the interface:

$$H = -k_1 A \frac{T_2 - T_1}{\Delta x_1} = -k_2 A \frac{T_3 - T_2}{\Delta x_2}$$



2. Thermal Conduction of Multilayer Insulation

As in the figure, a wall consisting of four layers, with thermal conductivities $k_1 = 0.060 \text{ W/m}\cdot\text{K}$, $k_3 = 0.040 \text{ W/m}\cdot\text{K}$, and $k_4 = 0.12 \text{ W/m}\cdot\text{K}$ (k_2 is not known). The layer thicknesses are $L_1 = 1.5 \text{ cm}$, $L_3 = 2.8 \text{ cm}$, and $L_4 = 3.5 \text{ cm}$ (L_2 is not known). The known temperatures are $T_1 = 30^\circ\text{C}$, $T_{12} = 25^\circ\text{C}$, and $T_4 = -10^\circ\text{C}$. Energy transfer through the wall is steady. What is interface temperature T_{34} ?



Assuming the conduction rate of each section is P_1, P_2, P_3, P_4 . Since the heat flow is steady, then we have $P_1 = P_2 = P_3 = P_4$.

Assuming the unit area of the all is A , according to thermal conduction formula:

$$k_1 \frac{A}{L_1} (T_1 - T_{12}) = k_2 \frac{A}{L_2} (T_{12} - T_{23}) = k_3 \frac{A}{L_3} (T_{23} - T_{34}) = k_4 \frac{A}{L_4} (T_{34} - T_4).$$

We can further simplified it:

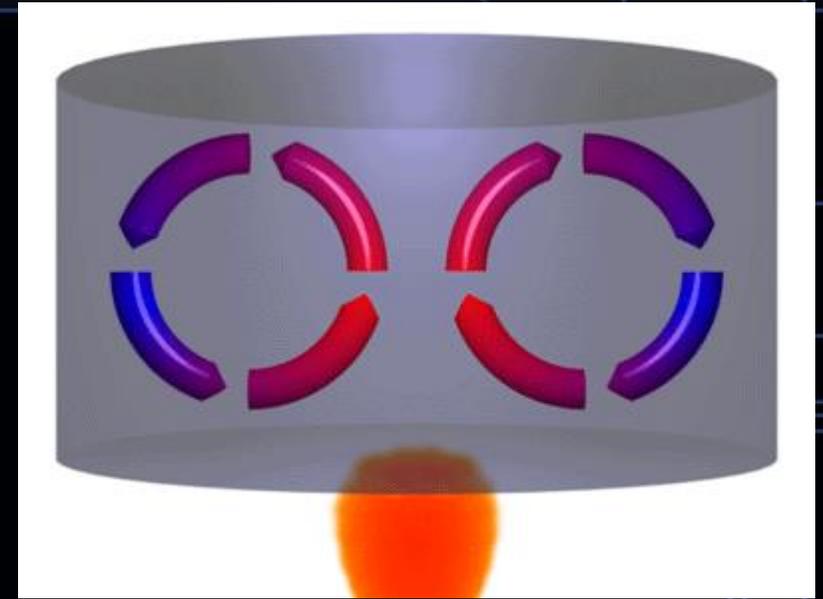
$$\frac{k_1}{L_1} (T_1 - T_{12}) = \frac{k_4}{L_4} (T_{34} - T_4)$$

$$\text{Thus } \frac{0.060(\text{W/m}\cdot\text{K})}{0.015(\text{m})} (30 - 25) = \frac{0.12(\text{W/m}\cdot\text{K})}{0.035(\text{m})} (T_{34} + 10)$$

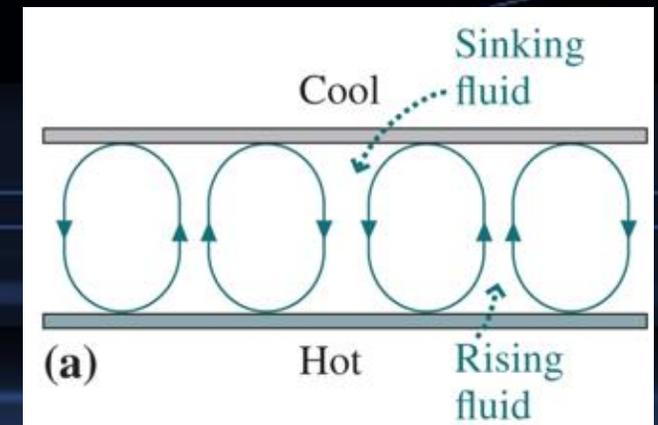
One can find $T_{34} = -4.16^\circ\text{C}$ (Answer)

Convection

- Convection: The energy transfer occurs with a fluid motion. When a fluid, such as air or water, comes in contact with an object whose temperature is higher than that of the fluid, heat can be transferred due to fluid flow



<https://commons.wikimedia.org/wiki/File:Convection.gif>



Radiation

- The third method by which an object and its environment can exchange energy as heat is via electromagnetic waves. Energy transferred in this way is often called **thermal radiation**.
- The rate P at which an object emits energy via electromagnetic radiation depends on the object's surface area A and the temperature T of that area in kelvins and is given by

$$P = e\sigma AT^4$$

Avogadro's Number

- All the materials are made out of atoms. Atoms form molecules, which is the smallest unit of the materials with the same chemical properties.

- Link between macroscopic mass and atomic mass: mole

One mole is the number of atoms in a 12 g sample of carbon-12.

- The number of molecules in a mole is:

$$N_A = 6.02 \times 10^{23} \text{ (Avogadro's number)}$$

Ideal Gas Law

- For a gas that the density is low enough, the state of the gas can be described by:

$$pV = nRT \quad (\text{ideal gas law})$$

p : the absolute pressure.

V : volume of the gas.

n : the number of moles of gas present.

T : the temperature in kelvins.

R : gas constant $R = 8.31 \text{ J/mol} \cdot \text{K}$

Ideal Gas Law

- A different form of idea gas law:

$$pV = NkT \quad (\text{ideal gas law})$$

where:

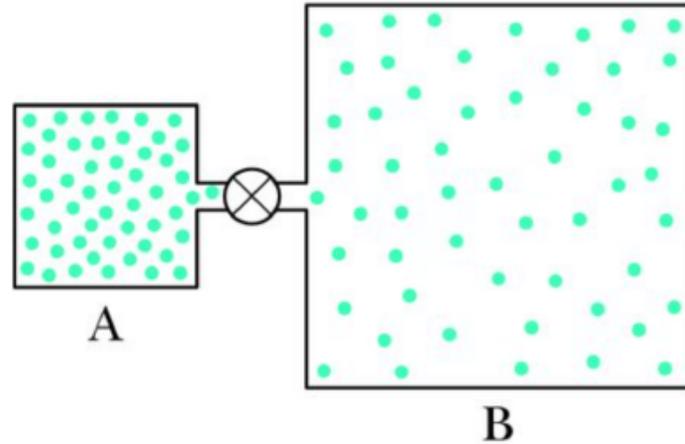
N: the number of **gas molecules**.

k: Boltzmann constant

$$k = \frac{R}{N_A} = \frac{8.31 \text{ J/mol}\cdot\text{K}}{6.02 \times 10^{23} \text{ mol}^{-1}} = 1.38 \times 10^{-23} \text{ J/K}$$

3.Ideal Gas

Container A in the following figure holds an ideal gas at a pressure of 5.0×10^5 Pa and a temperature of 300 K. It is connected by a thin tube (and a closed valve) to container B, with four times the volume of A. Container B holds the same ideal gas at a pressure of 1.0×10^5 Pa and a temperature of 400 K. The valve is opened to allow the pressures to equalize, but the temperature of each container is maintained. What then is the pressure?



Solution

Before the valve open, the ideal gases in the two containers follows:

$$p_A V_A = n_A R T_A \text{ and } p_B V_B = n_B R T_B$$

From the given condition, we have $p_A = 5.0 \times 10^5$ Pa, $p_B = 1.0 \times 10^5$ Pa = $\frac{1}{5} p_A$, $T_A = 300$ K, $T_B = 400$ K = $\frac{4}{3} T_A$, and $V_B = 4V_A$.

$$\text{Thus, we have } n_A + n_B = \frac{p_A V_A}{R T_A} + \frac{p_B V_B}{R T_B} = \frac{p_A V_A}{R T_A} + \frac{3}{5} \frac{p_A V_A}{R T_A} = \frac{8}{5} \frac{p_A V_A}{R T_A}.$$

After the valve is opened, the container will reach new pressure p' for both side while the volume, temperature, and the total number of molecules are the same.

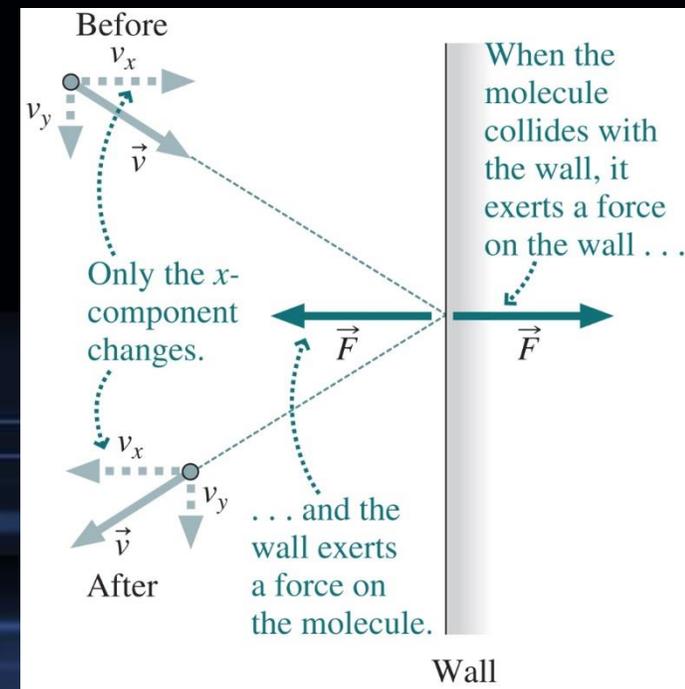
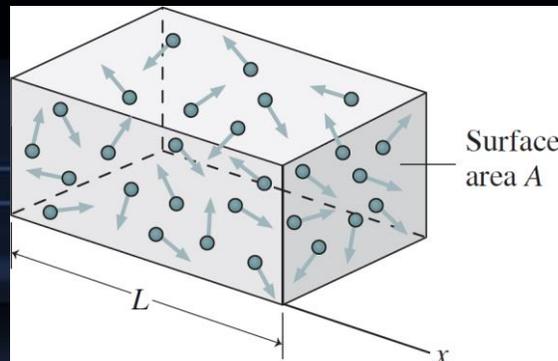
$$\text{Therefore, } \frac{p' V_A}{R T_A} + \frac{p' V_B}{R T_B} = p' \left(\frac{V_A}{R T_A} + \frac{4 V_A}{R \frac{4}{3} T_A} \right) = p' \left(4 \frac{V_A}{R T_A} \right) = n_A + n_B = \frac{8}{5} \frac{p_A V_A}{R T_A}.$$

$$\text{Then } p' = \frac{2}{5} p_A = 2.0 \times 10^5 \text{ Pa (Answer)}$$

Pressure due to change of momentum

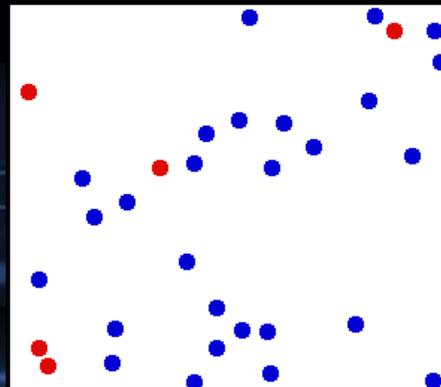
- A typical gas molecule, of mass m and velocity \vec{v} , that is about to collide with the shaded wall.
- The only change in the particle's momentum is along the x axis, and that change is

$$\Delta p_x = (-mv_x) - (mv_x) = -2mv_x$$



Meaning of temperature for idea gas molecules

- At a given temperature T , all ideal gas molecules—no matter what their mass—**have the same average translational kinetic energy**—namely, $\frac{3}{2}kT$. When we measure the temperature of a gas, we are also measuring **the average translational kinetic energy** of its molecules' random motion.



Thermal speed and temperature

- We introduce thermal speed or root-mean-square speed of the molecules and symbolized by v_{th} or v_{rms} . And we have $v_{th} = \sqrt{\overline{v^2}}$.
- We can find

$$v_{th} = \sqrt{\frac{3kT}{m}}$$

- The microscopic quantity v_{th} is linked to macroscopic quantity T

Example

A gas mixture consists of molecules of Helium (He, atomic mass=4), Neon (Ne, atomic mass=20), and Argon (Ar, atomic mass=40). The gas is at a status of equilibrium. Rank the three types according to (a) average kinetic energy (b) Thermal speed

Example

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(a) The kinetic energy is the same ($K_{\text{He}} = K_{\text{Ne}} = K_{\text{Ar}}$) because it is only related to temperature:

$$\frac{1}{2} m \overline{v^2} = \frac{3}{2} kT$$

(b) The $V_{\text{th,He}} > V_{\text{th,Ne}} > V_{\text{th,Ar}}$ because

$$V_{\text{th}} = \sqrt{\frac{3kT}{m}}$$

Maxwell's speed distribution law

- In 1852, Scottish physicist James Clerk Maxwell first solved the problem of finding the speed distribution of gas molecules. His result, known as Maxwell's speed distribution law, is

$$P(v) = 4\pi \left(\frac{M}{2\pi RT} \right)^{3/2} v^2 e^{-Mv^2/2RT}$$

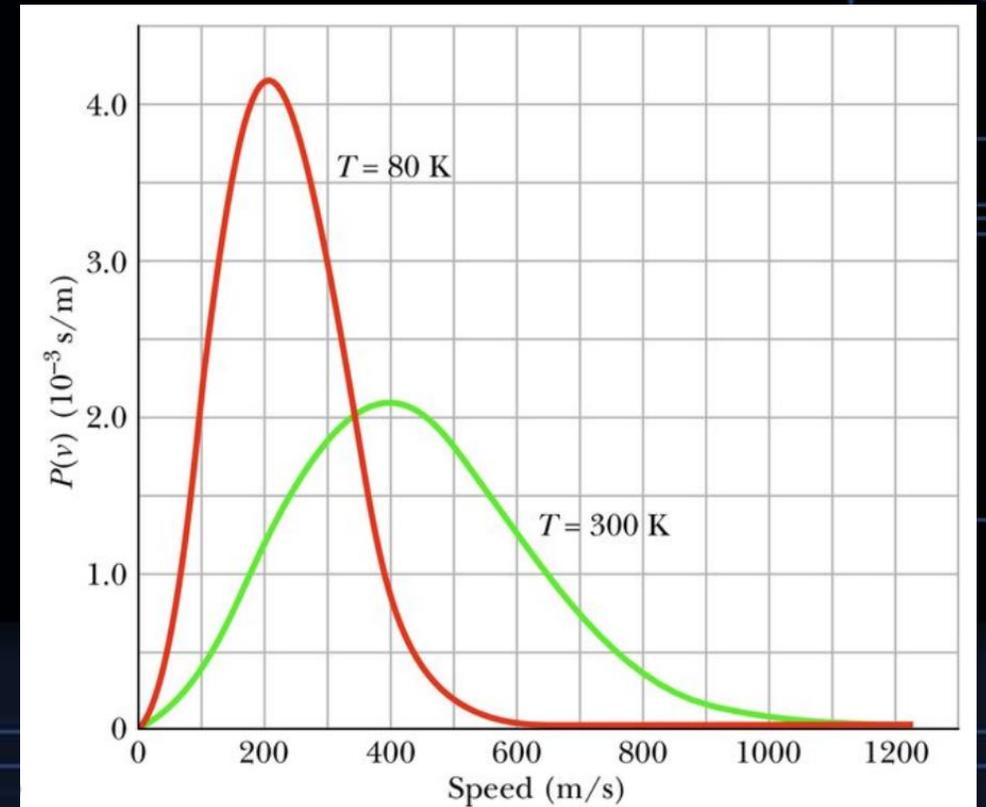
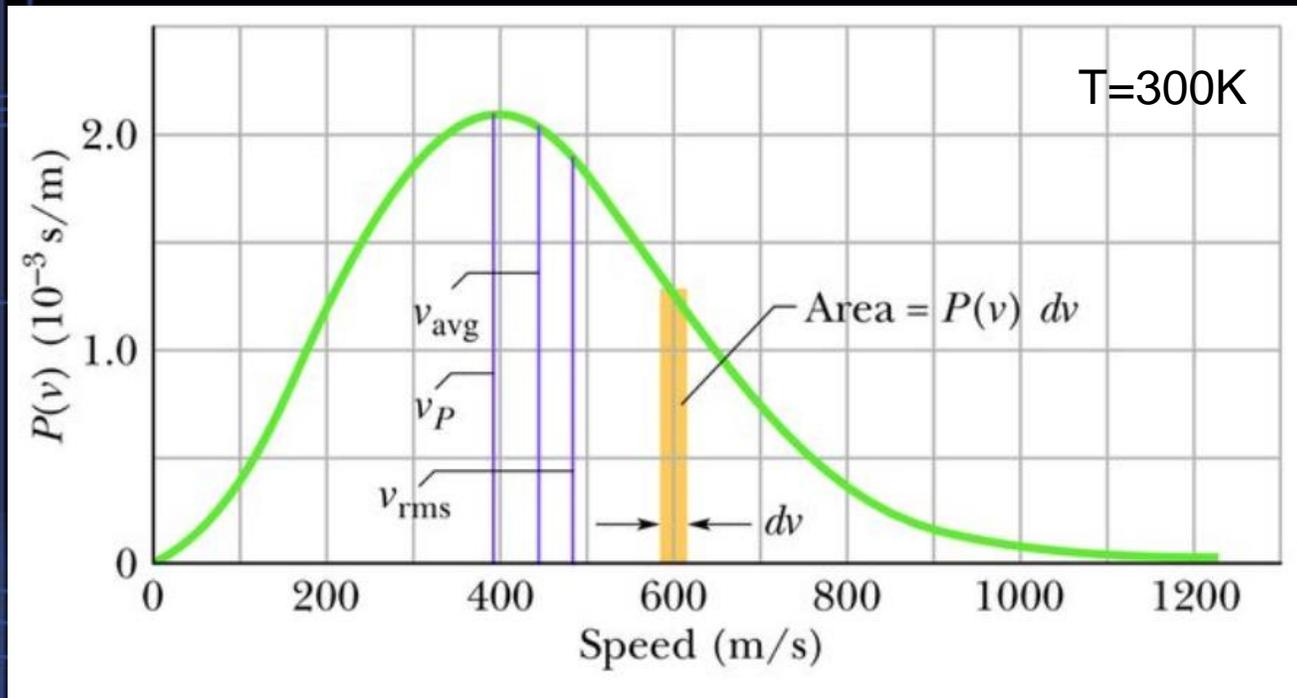
M:the molar mass of the gas,

R:the gas constant,

T: the gas temperature

v :the molecular speed.

Maxwell's speed distribution law



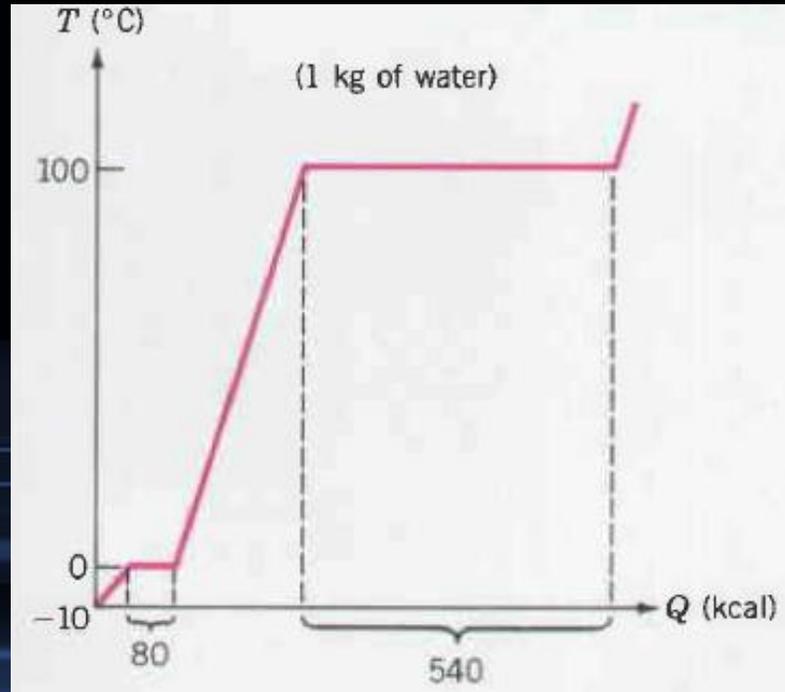
Phase Changes and Heats of Phase Transformation (Latent heat)

- Most substances occur in three phases—solid, liquid, gas.
- It takes energy, heat of transformation, L to effect phase changes from solid to liquid and liquid to gas. Example: melting and vaporization
- The amount of energy per unit mass that must be transferred as heat when a sample completely undergoes a phase change is called the heat of transformation L .

$$Q = Lm$$

Heats of Phase Transformation (Latent heat)

- The solid-liquid transition involves the heat of fusion, L_f .
- The liquid-gas transition involves the heat of vaporization, L_v .
- The direct transition from solid to gas involves the heat of sublimation, L_s .



Thermal expansion

Most materials expand when heated.

- Liquids are best characterized by the **coefficient of volume expansion**, β , which is the fractional change in volume per unit temperature change:

$$\beta = \frac{\Delta V/V}{\Delta T}$$

- Solids are best characterized by the **coefficient of linear expansion**, α , which is the fractional change in length per unit temperature change:

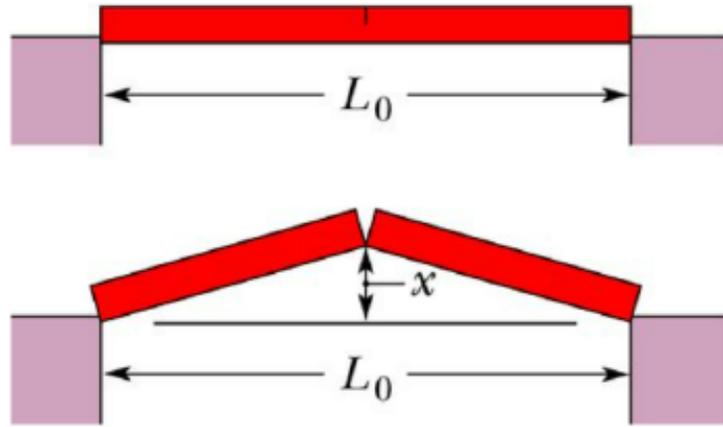
$$\alpha = \frac{\Delta L/L}{\Delta T}$$

- Typical values are

$$\beta \approx 10^{-3} \text{ K}^{-1}; \alpha \approx 10^{-5} \text{ K}^{-1}$$

4. Thermal expansion

As a result of a temperature rise of 32 C° , a bar with a crack at its center buckles upward as shown in the following figure. The fixed distance L_0 is 3.77 m and the coefficient of linear expansion of the bar is $25 \times 10^{-6}/\text{C}^\circ$. Find the rise x of the center.



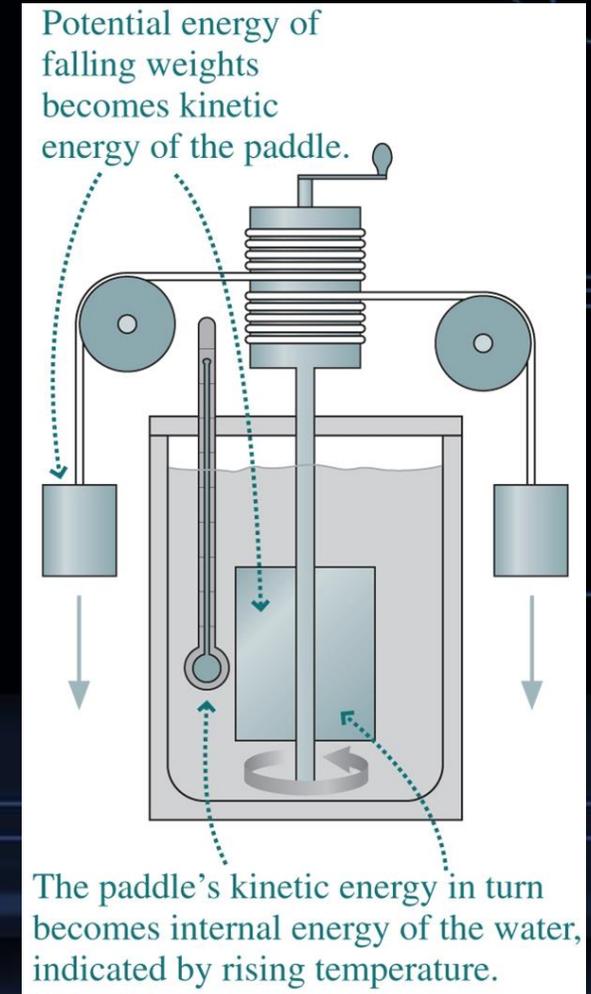
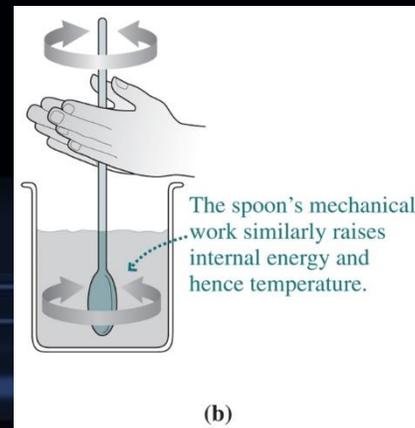
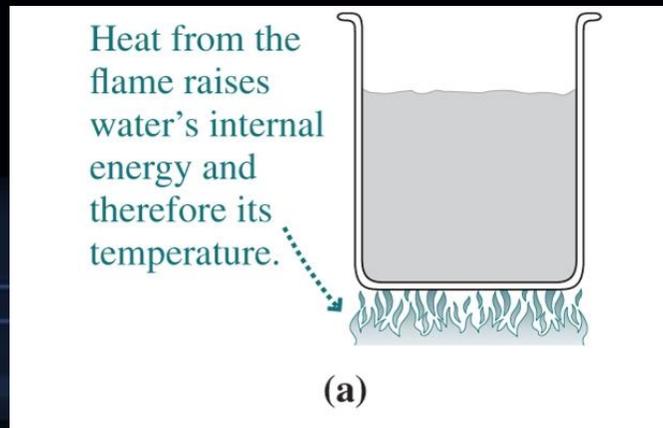
Solution:

After the temperature rise of 32 C° , the new total length is $L' = \alpha L_0 \Delta T$

$$\text{Thus, } x = \sqrt{\left(\frac{L'}{2}\right)^2 - \left(\frac{L_0}{2}\right)^2} = \frac{L_0 \sqrt{(1 + \alpha \Delta T)^2 - 1}}{2} = 7.5\text{ cm (Answer)}$$

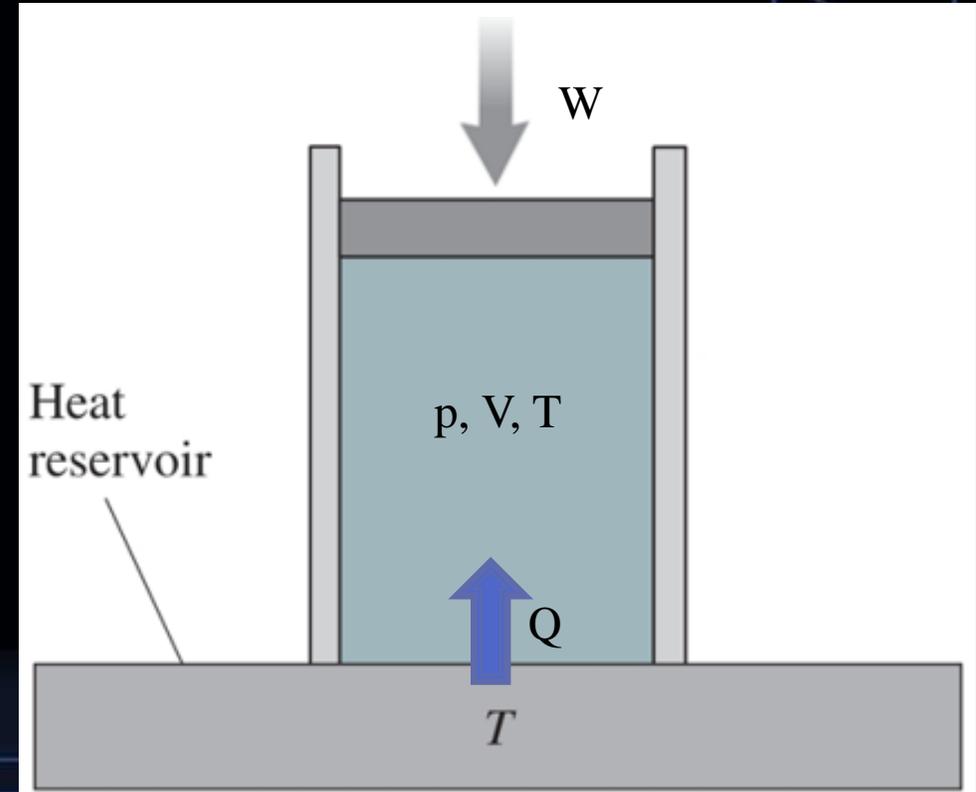
Heat and Energy

- From previous discussion, we know that heat is a form of energy. A conversion: $1 \text{ cal} = 4.1868 \text{ J}$



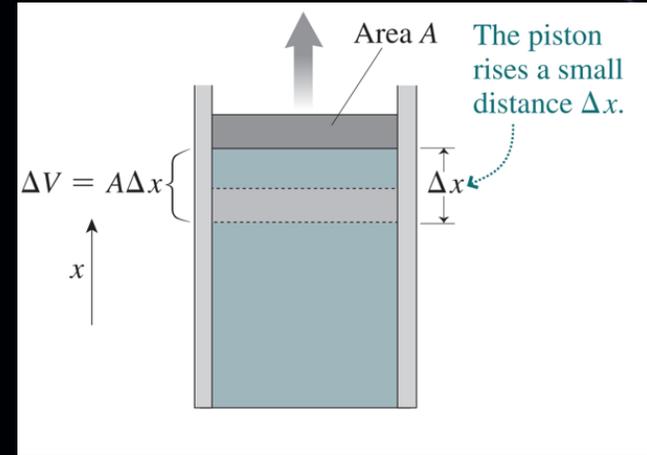
A gas system transferring heat to work

- The system (the gas) starts from an initial state i : described by a pressure p_i , a volume V_i , and a temperature T_i .
- The final state f : described by a pressure p_f , a volume V_f , and a temperature T_f .
- The procedure from its initial state to its final state is called a **thermodynamic process**.



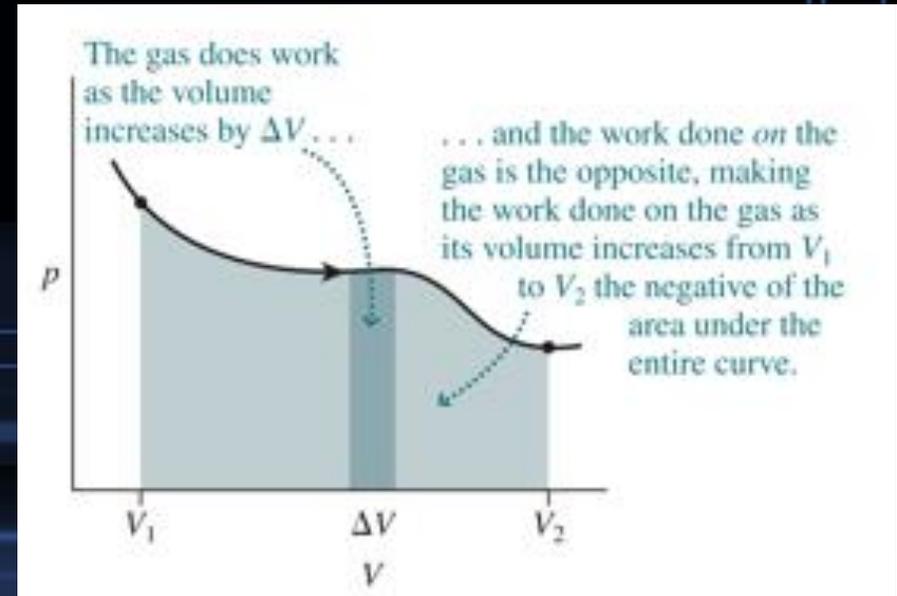
Work done on the gas

- The work done **on** the gas to the piston at a small displacement is:
 $dW = -Fdx = -pAdx = -pdV$



- The work done **on** gas during change from state with volume V_1 to state with volume V_2 :

$$W = -\int_{V_1}^{V_2} p dV$$



The first law of thermodynamics

- When a system changes from a given initial state to a given final state, both the work W and the heat Q depend on the nature of the process. Experimentally, however, we find a surprising thing: **The quantity $Q + W$ is the same for all processes.**
- The quantity $Q + W$ must represent a change in some intrinsic property of the system. We call this property the internal energy E_{int} and **the first law of thermodynamics** is:

$$\Delta E_{\text{int}} = Q + W$$

Meaning of the first law of thermodynamics

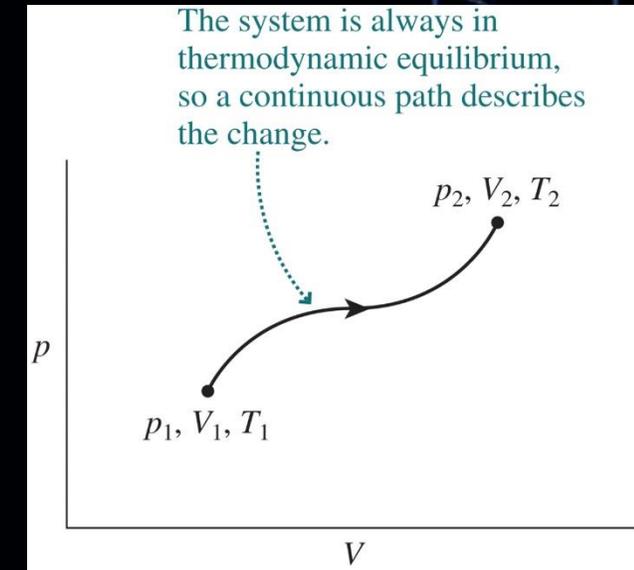
- If the thermodynamic system undergoes only a differential change:

$$dE_{int} = dQ + dW$$

- The internal energy dE_{int} of a system tends to increase if energy is added as heat Q and tends to decrease if energy is lost as work W done by the system. Therefore, the first law of thermodynamics is conservation of energy.

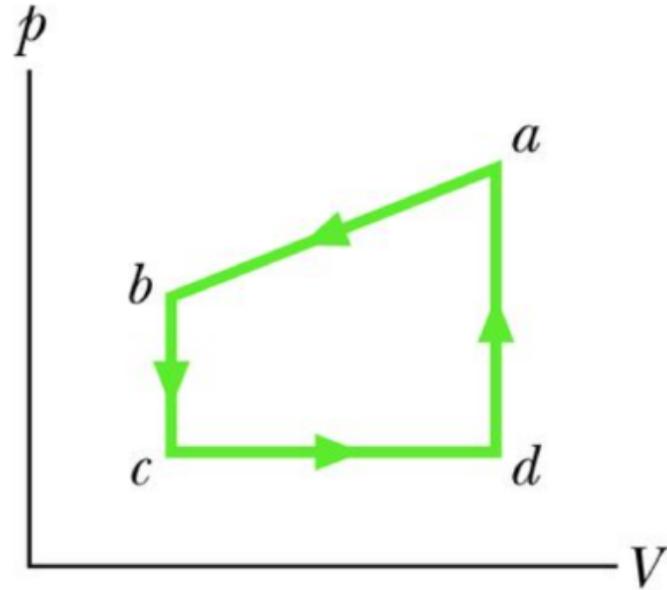
Quasi-static process

- To describe the status of ideal during thermodynamic processes, the system changes but is always in **thermodynamic equilibrium** and follows a continuous path in its pressure- versus-volume (pV) diagram.
- Any process carried out slowly enough to be **quasi-static** is **reversible**.
- In contrast, an **irreversible process** is the system goes temporarily out of equilibrium, without well-defined values for temperature, pressure, and other quantities.



1. The First Law of Thermodynamics

The bottom figure represents a closed cycle for a gas (the figure is not drawn to scale). The change in the internal energy of the gas as it moves from a to c along the path abc is -200 J . As it moves from c to d, 180 J must be transferred to it as heat. An additional transfer of 80 J to it as heat is needed as it moves from d to a. How much work is done on the gas as it moves from c to d?



Solution:

Since process abcda is a close cycle, the change of internal energy is zero. Given the internal energy change of process abc is -200 J , we can conclude that the internal energy change of process cda is 200 J . In process c to d, assume the work done on the gas is W_{cd} and the heat absorbed by gas is 180 J . In process d to a, the work done is zero (no volume change) and the heat absorbed by gas is 80 J .

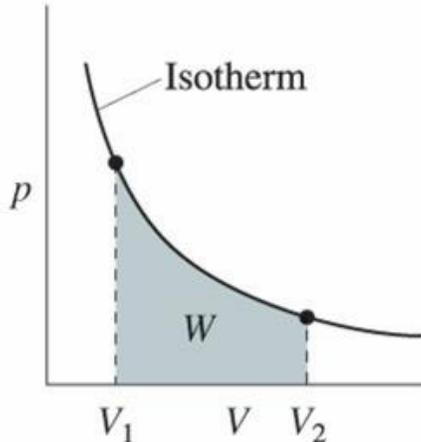
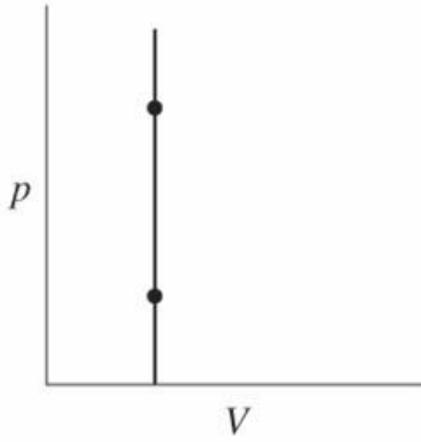
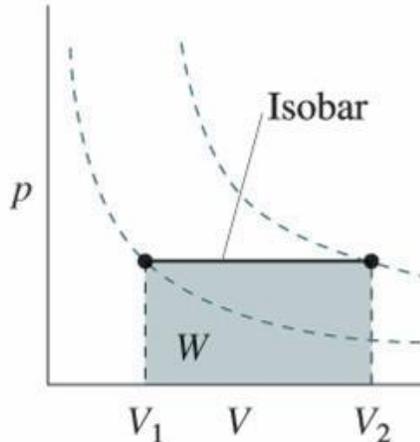
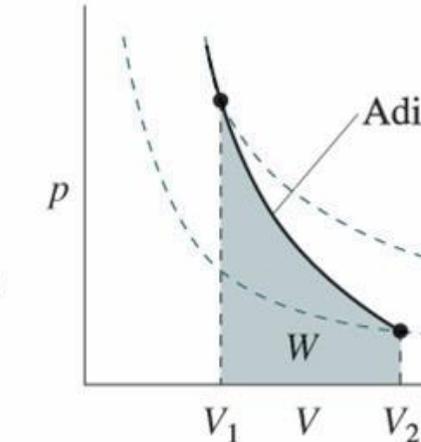
According to the first law of thermodynamics: $\Delta E = Q - W$, we can write down:

$$\Delta E_{cda} = 200\text{ J} = Q_{cd} + Q_{da} - W_{cd} - W_{da} = 180\text{ J} + 80\text{ J} - W_{cd} - 0\text{ J}$$

$$\text{Thus } W_{cd} = 260\text{ J} - 200\text{ J} = 60\text{ J} \text{ (Answer)}$$

Summary of ideal gas processes

Table 18.1 Ideal-Gas Processes

	ISOTHERMAL	CONSTANT-VOLUME	ISOBARIC	ADIABATIC
pV diagram				
Defining characteristic	$T = \text{constant}$	$V = \text{constant}$	$p = \text{constant}$	$Q = 0$
First law	$Q = -W$	$Q = \Delta E_{\text{int}}$	$Q = \Delta E_{\text{int}} - W$	$\Delta E_{\text{int}} = W$
Work done on gas	$W = -nRT \ln\left(\frac{V_2}{V_1}\right)$	$W = 0$	$W = -p(V_2 - V_1)$	$W = \frac{p_2V_2 - p_1V_1}{\gamma - 1}$
Other relationships	$pV = \text{constant}$	$Q = nC_V\Delta T$	$Q = nC_p\Delta T$ $C_p = C_V + R$	$pV^\gamma = \text{constant}$ $TV^{\gamma-1} = \text{constant}$

Example: Bubble in isothermal process

- A scuba diver is 25m down, where the pressure is 3.5atm. The air she exhales forms bubbles 8.0mm in radius. How much work does each bubble do as it rises to the surface, assuming the bubbles remain at the uniform 300K of water.

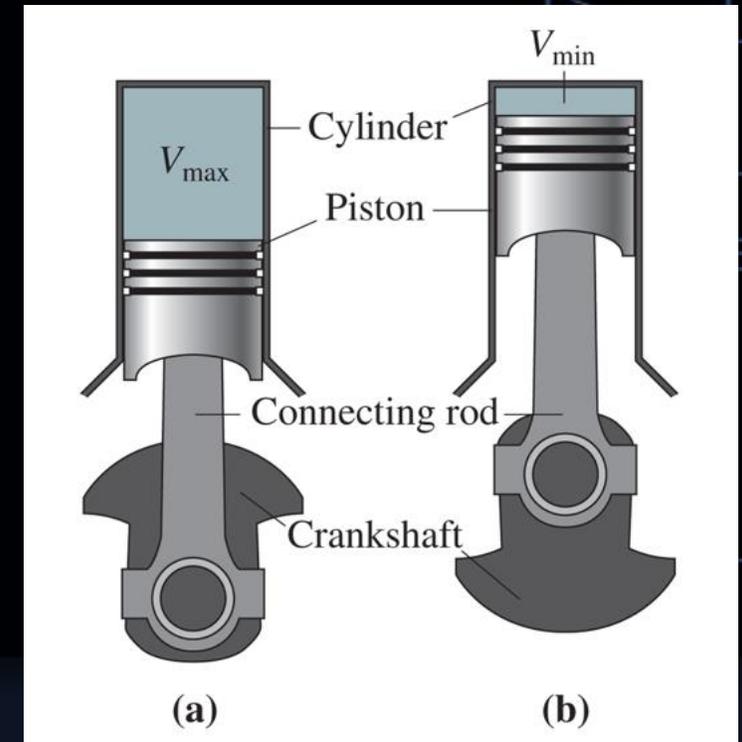


Example: Bubble in isothermal process

- The process is isothermal process (temperature is constant). For ideal gas, the work done in isothermal process is: $-W = nRT \ln\left(\frac{V_2}{V_1}\right)$
- From ideal gas law: $pV = nRT$, we can know that n , R , T are all constant for the bubble in isothermal process. We have $pV = p\left(\frac{4}{3}\pi r^3\right) = nRT = \text{constant}$. This means the volume will expand by a factor of 3.5 since the pressure is reduced from 3.5 atm to 1 atm.
- Therefore, $-W = nRT \ln\left(\frac{V_2}{V_1}\right) = p \frac{4}{3}\pi r^3 \ln(3.5) = 0.95 \text{ J}$

Example: an Adiabatic Process: Diesel Power

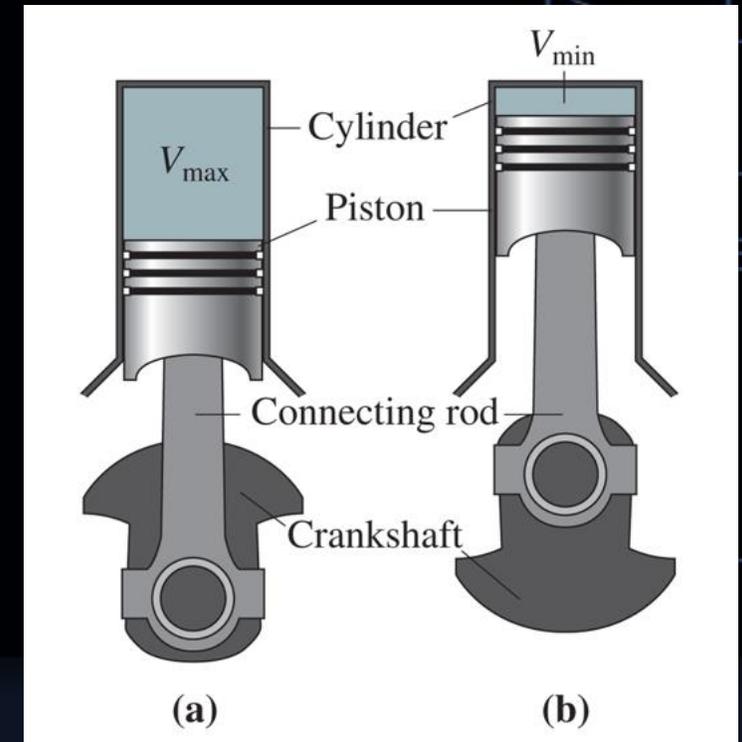
Fuel ignites in a diesel engine because of the temperature rise that results from compression as the piston moves toward the top of the cylinder; there's no spark plug as in a gasoline engine. Compression is fast enough that the process is essentially adiabatic. If the ignition temperature is 500°C , what compression ratio $V_{\text{max}}/V_{\text{min}}$ is needed? Air's specific-heat ratio γ is 1.4 and before compression the air is at 20°C .



Example: an Adiabatic Process: Diesel Power

Since the process is adiabatic, the gas follows $T_{min}V_{min}^{\gamma-1} = T_{max}V_{max}^{\gamma-1}$. Therefore, we can find out that:

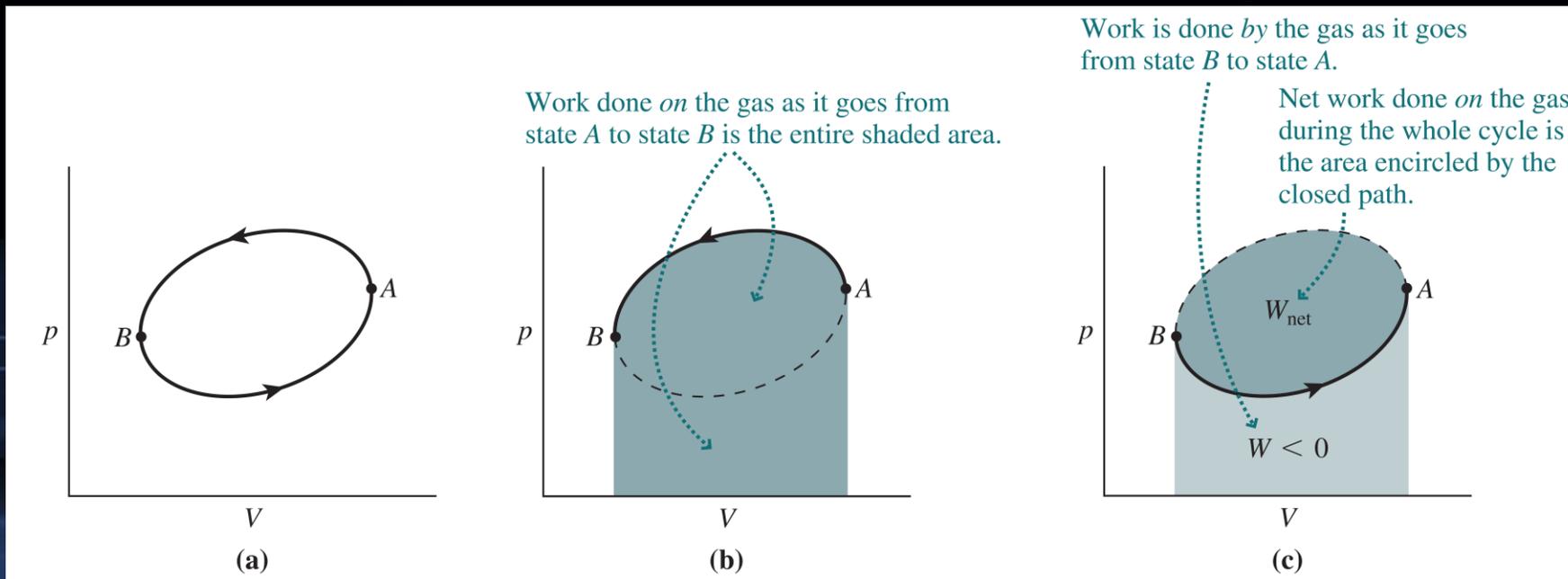
$$\frac{V_{max}}{V_{min}} = \left(\frac{T_{min}}{T_{max}}\right)^{\frac{1}{\gamma-1}} = \left(\frac{773K}{293K}\right)^{\frac{1}{0.4}} = 11$$



Cyclic Processes

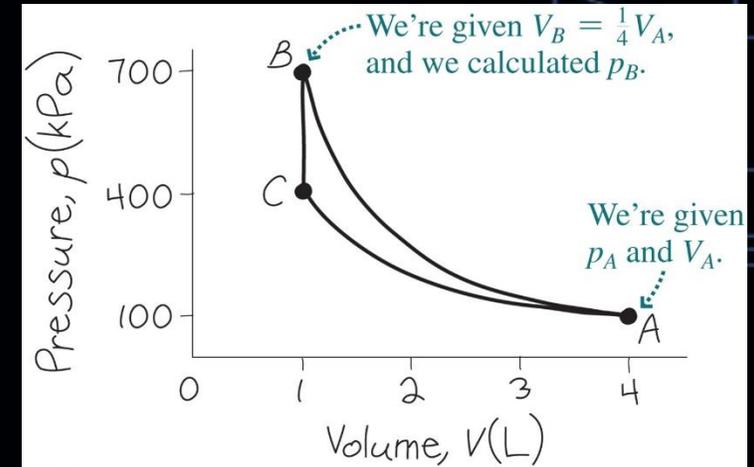
Cyclic processes combine the basic processes of other thermodynamic processes to take a system around a complete cycle and back to its starting state.

- Cyclic processes are important in technological systems like engines.
- The net work done on a gas in a cyclic process can be found from the area enclosed by the process curve in the pV diagram.



Example: Finding the work of a cyclic process

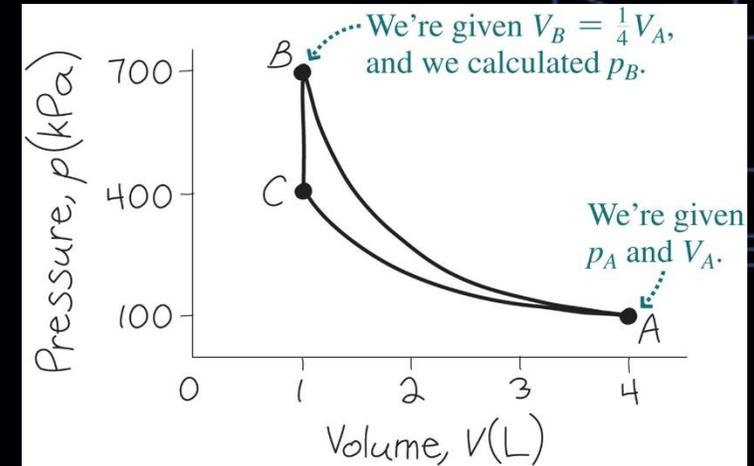
An ideal gas with $\gamma = 1.4$ occupies 4.0L at 300K and 100 kPa pressure. It's compressed adiabatically to one-fourth of its original volume, then cooled at constant volume back to 300K, and finally allowed to expand isothermally to its original volume. How much work is done on the gas?



Example: Finding the work of a cyclic process

- The cyclic process can be expressed as the path A-B-C-A in the right plot. The work done on the gas of this process is the area in the close loop. We need to find our work done in every process.
- It is adiabatic compression from A to B. We have $p_A V_A^\gamma = p_B V_B^\gamma$. So, we have

$$p_B = p_A (V_A/V_B)^\gamma = 696.4 \text{ kPa}$$
$$W_{AB} = \frac{p_B V_B - p_A V_A}{\gamma - 1} = 741 \text{ J}$$



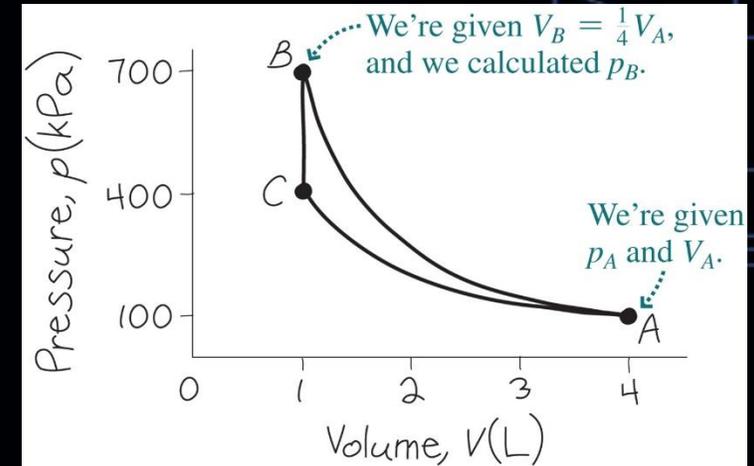
Example: Finding the work of a cyclic process

- There is no work done from B to C
- It is isothermal expansion compression from C to A. We have:

$$\begin{aligned}W_{CA} &= -nRT \ln \left(\frac{V_A}{V_C} \right) = -p_A V_A \ln \left(\frac{V_A}{V_C} \right) \\ &= -(400J)(\ln 4) = -555J\end{aligned}$$

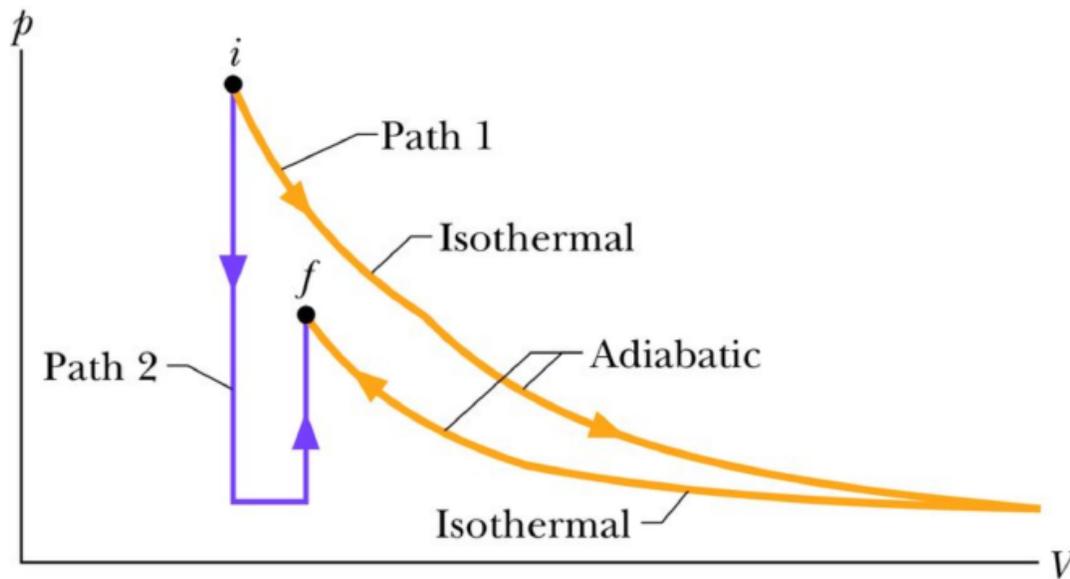
- Thus the total work done on the gas is

$$W_{ABCA} = 741J + 0J - 555J = 186J$$



5. The Adiabatic and Isothermal Expansion of an Ideal Gas

The following figure, shows two paths that may be taken by a gas from an initial point i to a final point f . Path 1 consists of an isothermal expansion (work is 50 J in magnitude), an adiabatic expansion (work is 40 J in magnitude), an isothermal compression (work is 30 J in magnitude), and then an adiabatic compression (work is 25 J in magnitude). What is the change in the internal energy of the gas if the gas goes from point i to point f along path 2?



Solution

Since internal energy only depends temperature, the change in the internal energy of the gas goes from point i to point f along path 2 will be the same along path one. For path one, there is no change in internal energy in all the isothermal process and the work is equal to heat according to the first law of thermal dynamics. There is heat exchange in all the adiabatic process and the work is equal to change of internal energy. Thus, only the two adiabatic processes have change in internal energy and therefore $\Delta E_{i \rightarrow f} = (-40J) + (+25J) = -15J$ (Answer)

Specific heat of monatomic ideal gases

we are now able to derive an expression for the molar specific heat of monatomic ideal gas. There will be two cases:

- C_V (specific heat of constant volume):

The volume of the gas remains constant as energy is transferred to or from it as heat.

- C_p (specific heat of constant pressure):

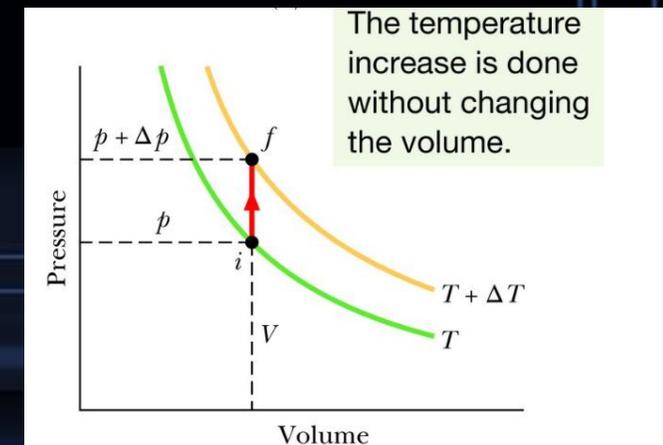
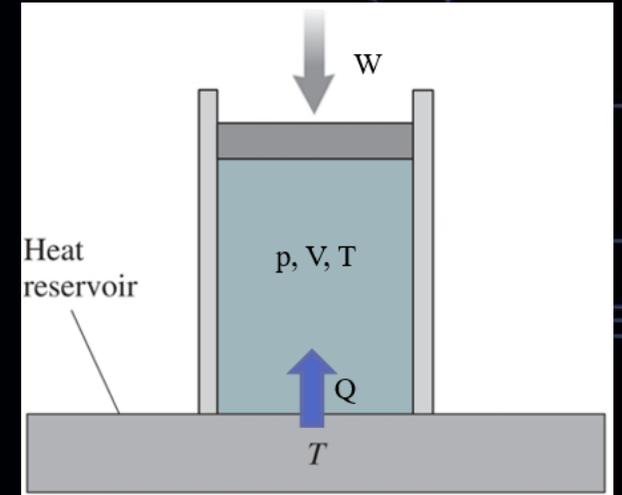
The pressure of the gas remains constant as energy is transferred to or from it as heat.

C_V of Monatomic Ideal Gases

- We can conclude that the molar specific heat at constant volume of monatomic ideal gas is:

$$C_V = \frac{3}{2}R = 12.5 \text{ J/mol} \cdot \text{K} \quad (\text{monatomic gas})$$

Molecule	Example	C_V (J/mol · K)	
Monatomic	Ideal	$\frac{3}{2}R = 12.5$	
	Real	He	12.5
		Ar	12.6



Cp of Monatomic Ideal Gases

With constant pressure, the gas does work:

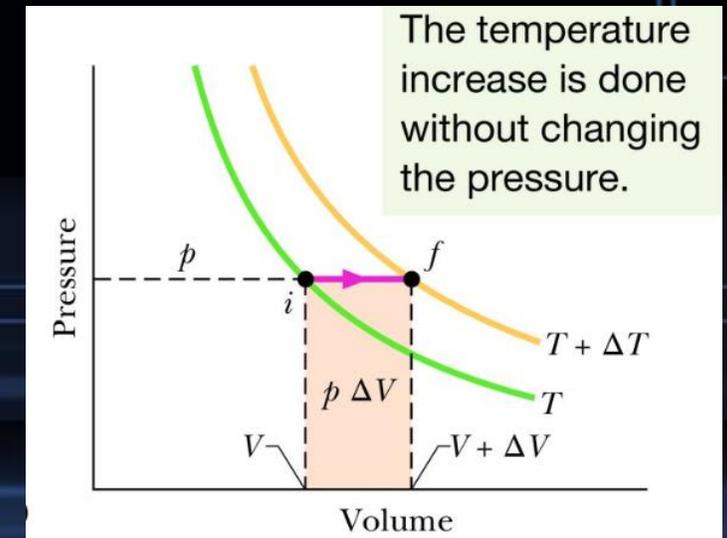
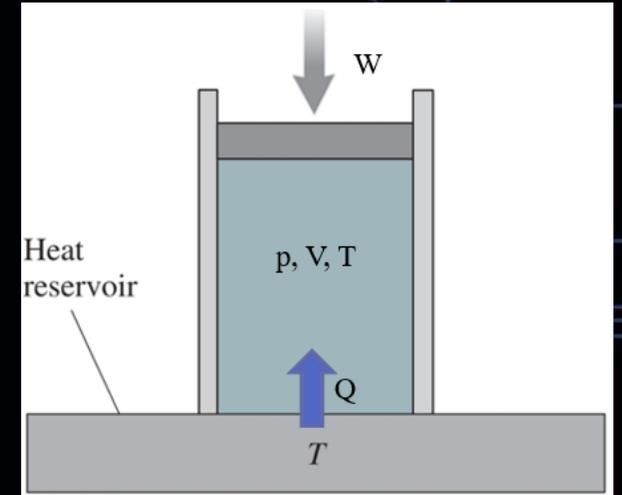
$$W = p \Delta V = nR \Delta T$$

And change of internal energy is:

$$\Delta E_{\text{int}} = nC_V \Delta T \quad (\text{ideal gas, any process})$$

Thus, $nC_p \Delta T = nC_V \Delta T + nR \Delta T$ and

$$C_p = C_v + R = \frac{5}{2} R$$



Degree of freedom vs specific heat

		Degrees of Freedom			Predicted Molar Specific Heats	
Molecule	Example	Translational	Rotational	Total (f)	C_V (Eq. 19-51)	$C_p = C_V + R$
Monatomic	He	3	0	3	$\frac{3}{2}R$	$\frac{5}{2}R$
Diatomic	O ₂	3	2	5	$\frac{5}{2}R$	$\frac{7}{2}R$
Polyatomic	CH ₄	3	3	6	$3R$	$4R$

Example: Specific Heat of gas mixture

- A gas mixture consists of 2.0mol of oxygen (O_2) and 1.0 mole of argon (Ar). Find the volume specific heat of the mixture.

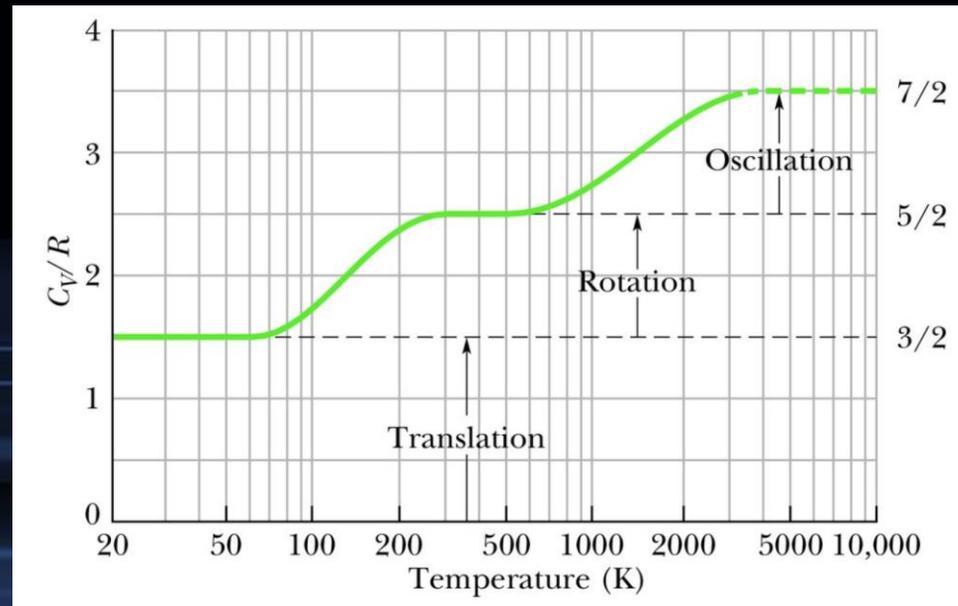
Example: Specific Heat of gas mixture

- A gas mixture consists of 2.0mol of oxygen (O₂) and 1.0 mole of argon (Ar). Find the volume specific heat of the mixture.
- For O₂, the degree of freedom of the molecule is 5:
 $C_{v_{O_2}} = \frac{5}{2}R$. Argon is monatomic molecule gas: $C_{v_{Ar}} = \frac{3}{2}R$.
- For the mixture we can find:

$$C_v = \frac{n_{O_2}C_{v_{O_2}} + n_{Ar}C_{v_{Ar}}}{n_{O_2} + n_{Ar}} = 2.2R$$

Oscillational degree of freedom

- The C_v of oxygen actually depends on temperature. Shown in the plot is the C_v of H_2 gas. At higher temperature there will be another R added in for C_v , which associated with two atom vibrating respect to COM.



Definition of Entropy

- Entropy is a state property just like pressure and volume that describing the status of a system at a certain state.
- Since we know the change in entropy can tell the direction of a process, we start the definition with changing in entropy:

Change in entropy $S_f - S_i$ of a system during a process that takes the system from an initial state i to a final state f as

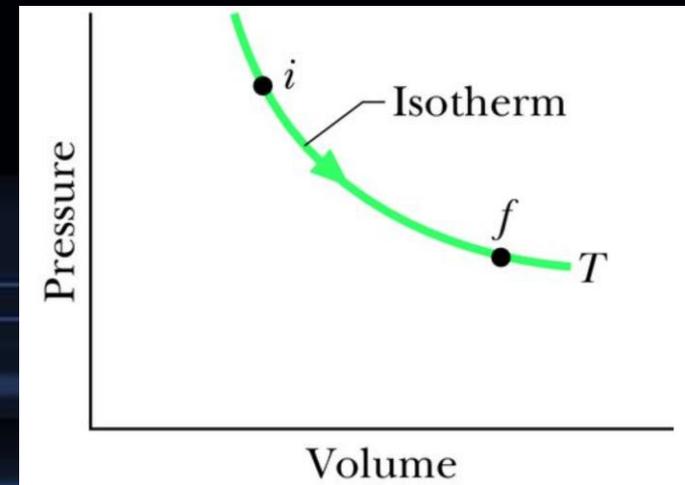
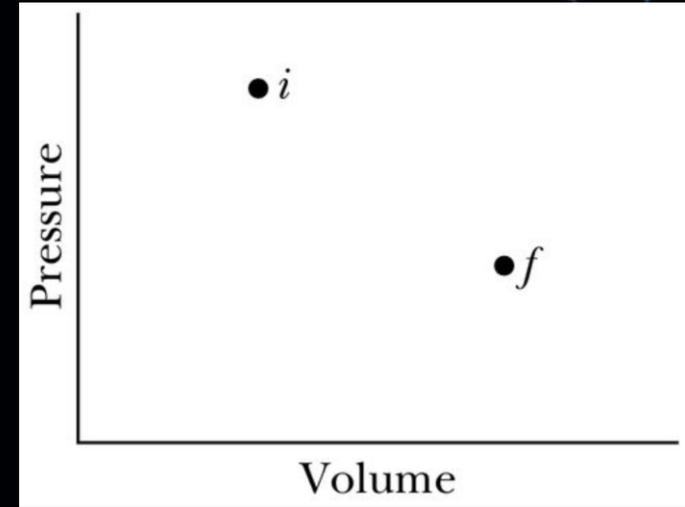
$$\Delta S = S_f - S_i = \int_i^f \frac{dQ}{T}$$

Finding entropy in a process

- In this case, we use an isotherm process to find out change of entropy from state i to f :

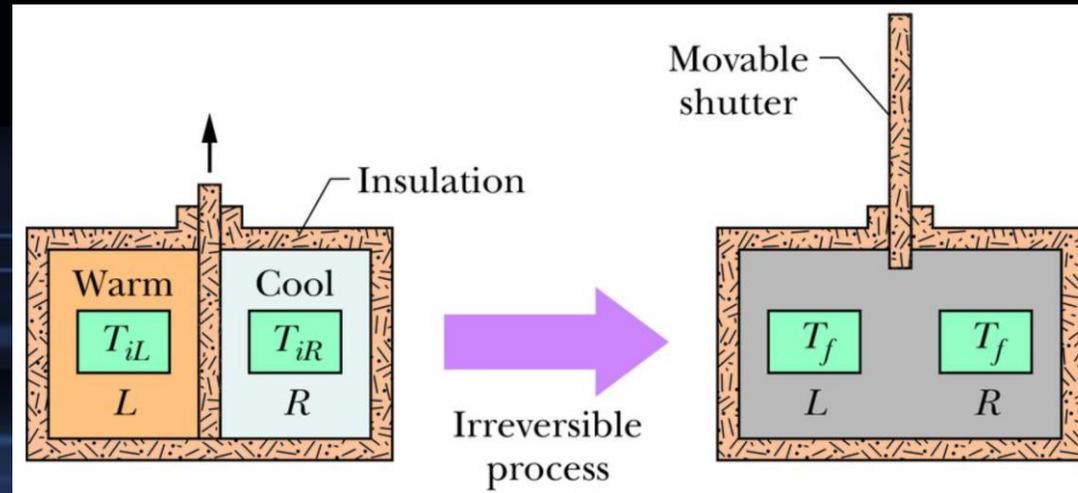
$$\Delta S = S_f - S_i = \frac{Q}{T} \quad (\text{change in entropy, isothermal process})$$

- To keep the temperature T of the gas constant during the isothermal expansion, heat Q must have been energy transferred from the reservoir to the gas. Thus, Q is positive and the entropy of the gas increases during the isothermal process and during the free expansion.



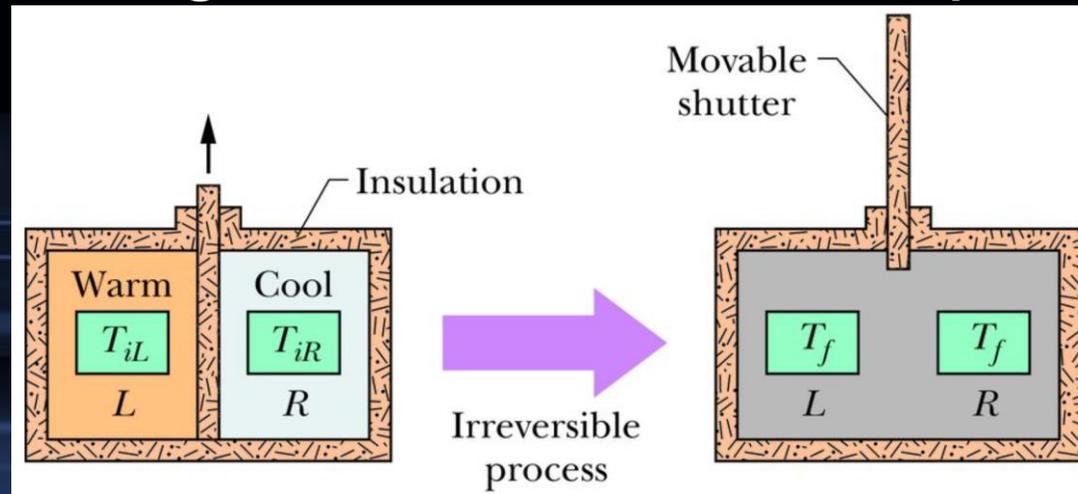
Example: Entropy change of two blocks coming to thermal equilibrium

Two identical copper blocks of mass $m = 1.5 \text{ kg}$: block L at temperature $T_{iL} = 60^\circ\text{C}$ and block R at temperature $T_{iR} = 20^\circ\text{C}$. The blocks are in a thermally insulated box and are separated by an insulating shutter. When we lift the shutter, the blocks eventually come to the equilibrium temperature $T_f = 40^\circ\text{C}$. What is the net entropy change of the two-block system during this irreversible process? The specific heat of copper is $386 \text{ J/kg}\cdot\text{K}$.



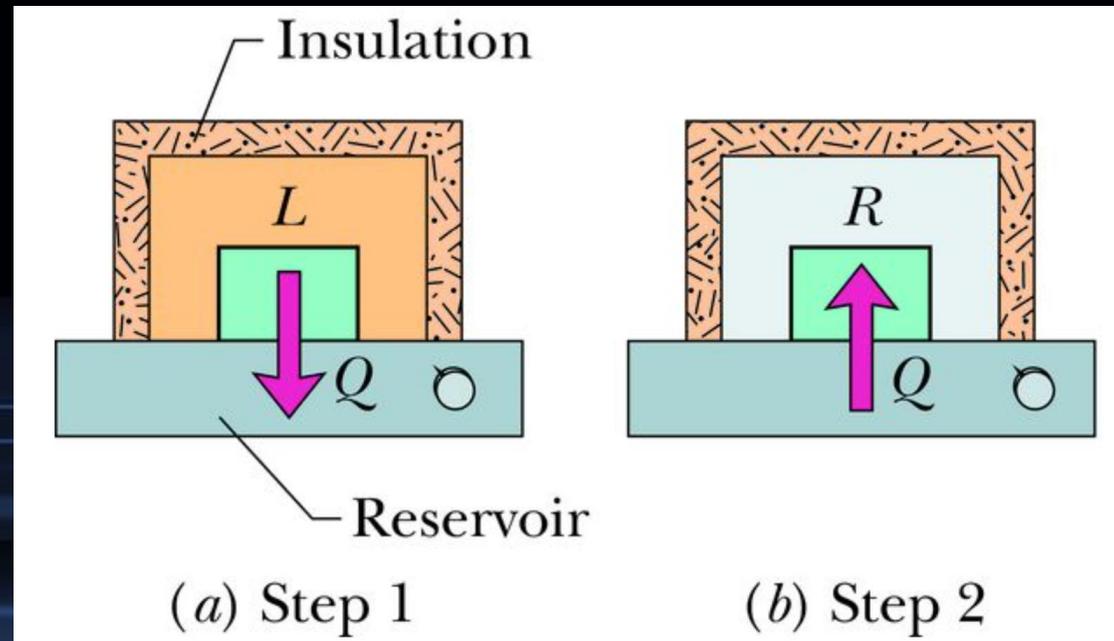
Entropy change of two blocks coming to thermal equilibrium

- Key: To calculate the entropy change, we must find a reversible process that takes the system from the initial state to the final state. We can calculate the net entropy change ΔS_{rev} of the reversible process using, and then the entropy change for the irreversible process is equal to ΔS_{rev} .



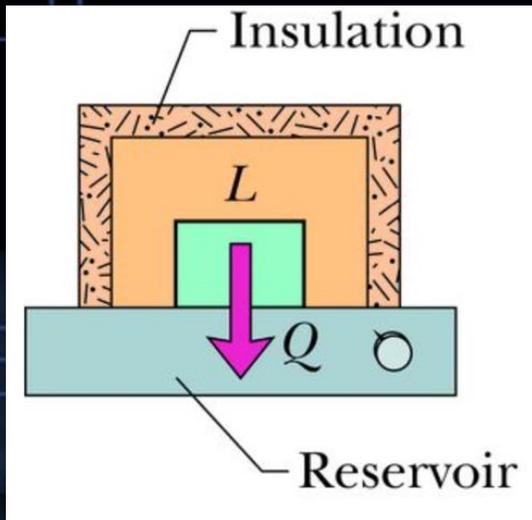
Entropy change of two blocks coming to thermal equilibrium

For the reversible process, we need a thermal reservoir whose temperature can be changed slowly (say, by turning a knob).



Entropy change of two blocks coming to thermal equilibrium

The entropy change ΔS_L of block L during the full temperature change from initial temperature T_{iL} ($= 60^\circ\text{C} = 333\text{ K}$) to final temperature T_f ($= 40^\circ\text{C} = 313\text{ K}$) is

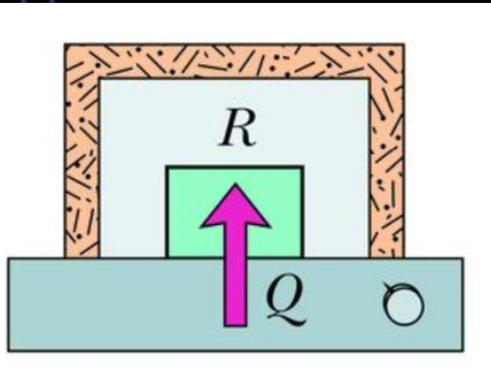


$$\begin{aligned}\Delta S_L &= \int_i^f \frac{dQ}{T} = \int_{T_{iL}}^{T_f} \frac{mc dT}{T} = mc \int_{T_{iL}}^{T_f} \frac{dT}{T} \\ &= mc \ln \frac{T_f}{T_{iL}}.\end{aligned}$$

$$\begin{aligned}\Delta S_L &= (1.5\text{ kg})(386\text{ J/kg}\cdot\text{K}) \ln \frac{313\text{ K}}{333\text{ K}} \\ &= -35.86\text{ J/K}.\end{aligned}$$

Entropy change of two blocks coming to thermal equilibrium

With the same reasoning used to find ΔS_L , you can show that the entropy change ΔS_R of block R during this process is



$$\begin{aligned}\Delta S_R &= (1.5 \text{ kg}) (386 \text{ J/kg} \cdot \text{K}) \ln \frac{313 \text{ K}}{293 \text{ K}} \\ &= +38.23 \text{ J/K}.\end{aligned}$$

Entropy change of two blocks coming to thermal equilibrium

- The net entropy change ΔS_{rev} of the two-block system undergoing this two-step reversible process is then

$$\begin{aligned}\Delta S_{\text{rev}} &= \Delta S_L + \Delta S_R \\ &= -35.86 \text{ J/K.} + 38.23 \text{ J/K} = 2.4 \text{ J/K.}\end{aligned}$$

Thus, the net entropy change ΔS_{irrev} for the two-block system undergoing the actual irreversible process is

$$\Delta S_{\text{irrev}} = \Delta S_{\text{rev}} = 2.4 \text{ J/K. (Answer)}$$

This result is positive, in accordance with the entropy postulate.

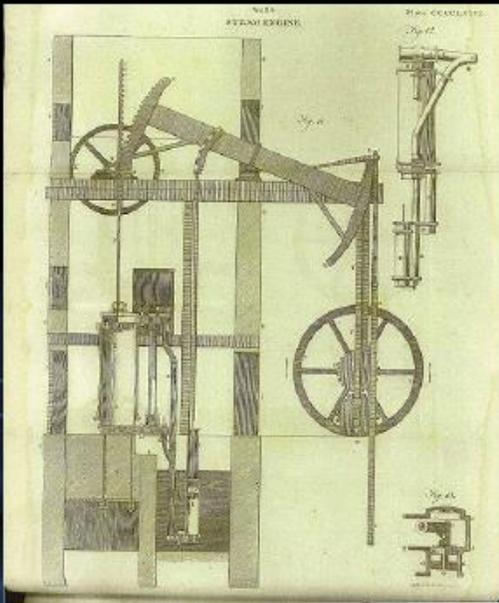
The 2nd law of thermodynamics

If a process occurs in a closed system, the entropy of the system increases for irreversible processes and remains constant for reversible processes. It never decreases.

$$\Delta S \geq 0 \quad (\text{second law of thermodynamics})$$

Heat Engine

- A heat engine, or more simply, an engine, is a device that extracts energy from its environment in the form of heat and does useful work.
- The invention and improvement of steam engine start the Industrial Revolution.



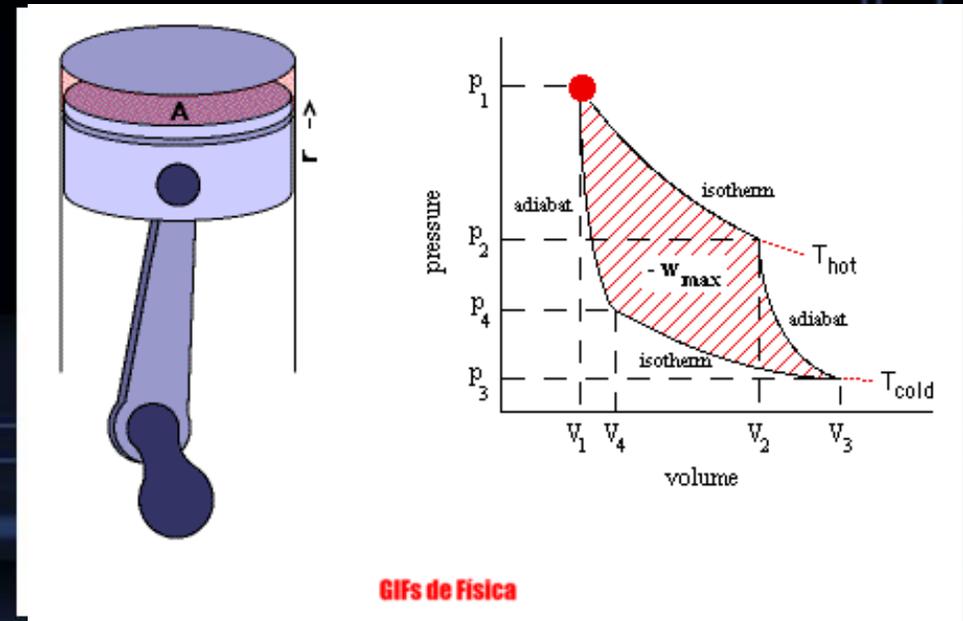
<https://en.wikipedia.org/wiki/File:WattsSteamEngine.jpeg>



https://en.wikipedia.org/wiki/Engine#/media/File:Mercedes_V6_DTM_Rennmotor_1996.jpg

Cycle in Heat Engine

- At the heart of every engine is a working **substance**. In a steam engine, the working substance is water, in both its vapor and its liquid form. In an automobile engine the working substance is a gasoline–air mixture.
- The working substance must operate in a **cycle**; that is, the working substance must pass through a closed series of thermodynamic processes (strokes).



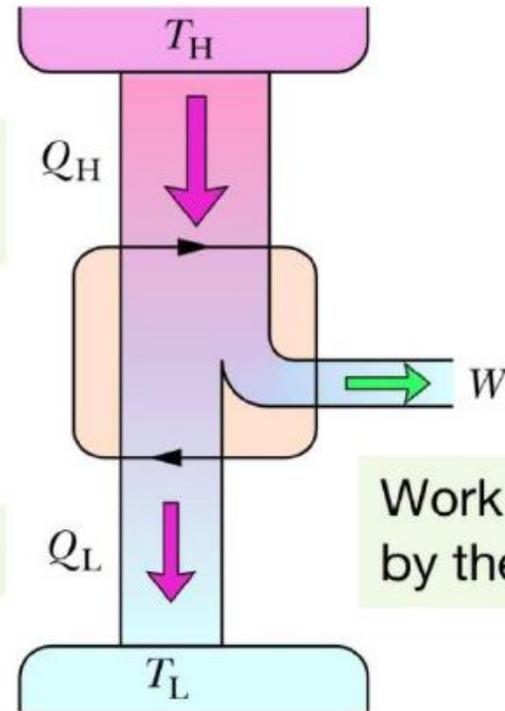
Ideal Engine and Carnot Engine

- Let's use ideal gas as working substance (obeys the simple law $pV = nRT$).
- In an ideal engine, all processes are **reversible** and **no wasteful energy** transfers occur due to friction and turbulence.
- French scientist and engineer N. L. Sadi Carnot who first proposed the engine's concept in 1824. He was able to analyze the performance of this engine before the first law of thermodynamics and the concept of entropy had been discovered.

Scheme of Carnot Engine

Schematic of
a Carnot engine

Heat is
absorbed.

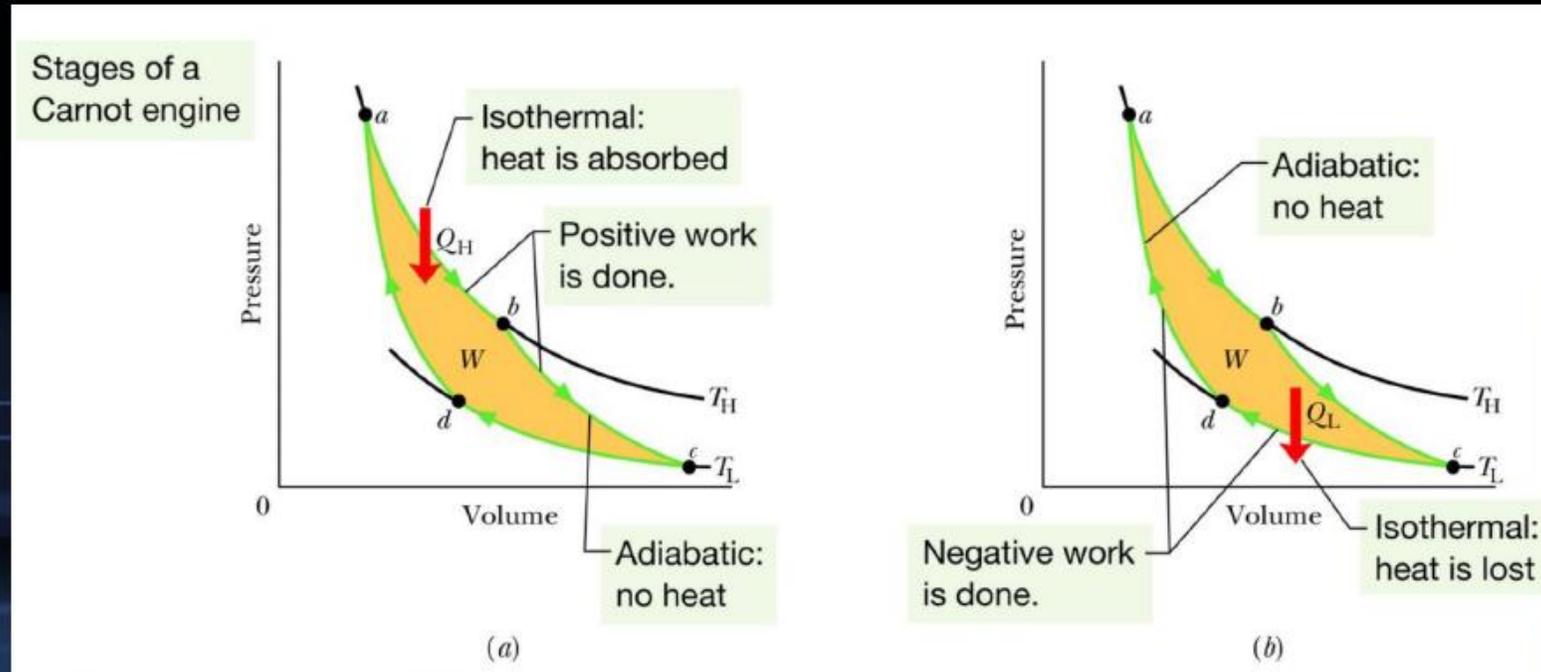


Heat is lost.

Work is done
by the engine.

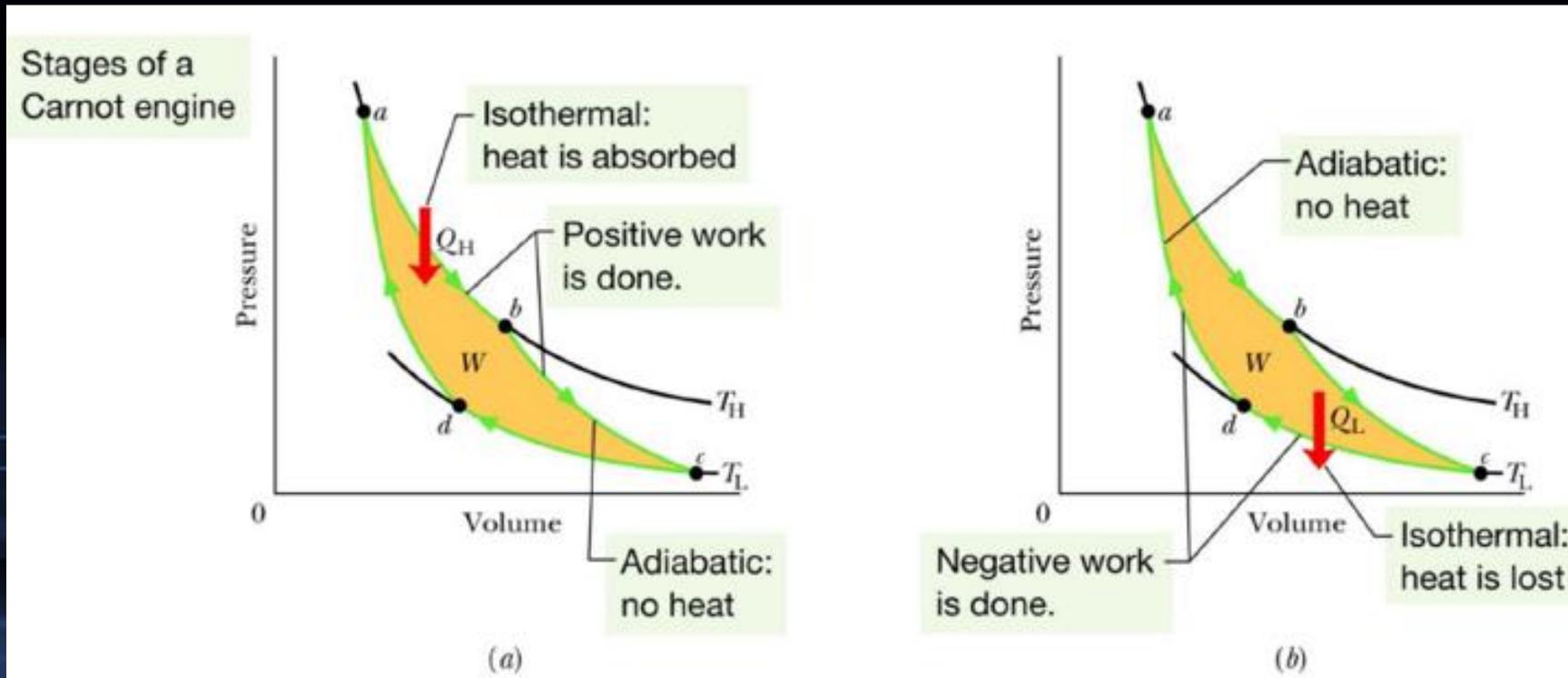
Carnot Cycle

Carnot cycle is how a working substance cycling in a Carnot engine. It contain four steps: Isothermal expansion(ab), adiabatic expansion(bc), isothermal compression(cd), and adiabatic compression(da).



Analysis of Carnot Cycle

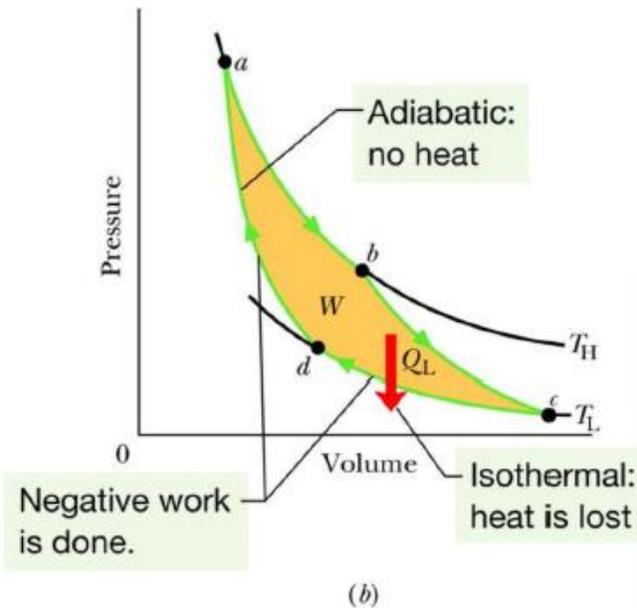
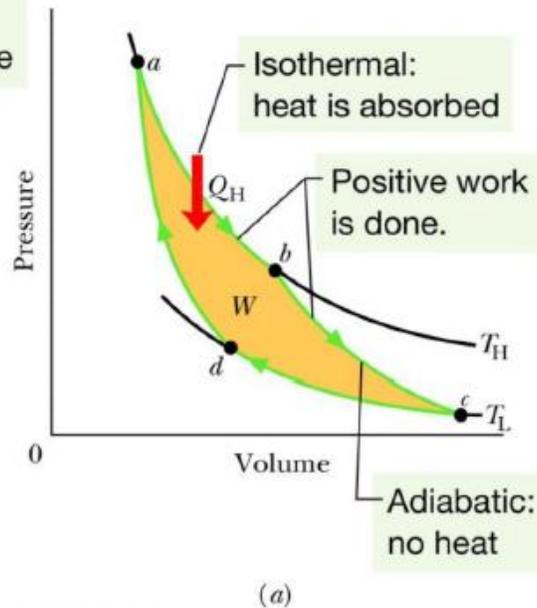
- ab: No change in E_{int} . Absorb heat Q_H . Do positive work to outside.
- bc: Decrease in E_{int} . No heat exchange. Do positive work to outside.
- cd: No change in E_{int} . Release heat Q_L . Do negative work to outside.
- da: Increase in E_{int} . No heat exchange. Do negative work to outside.



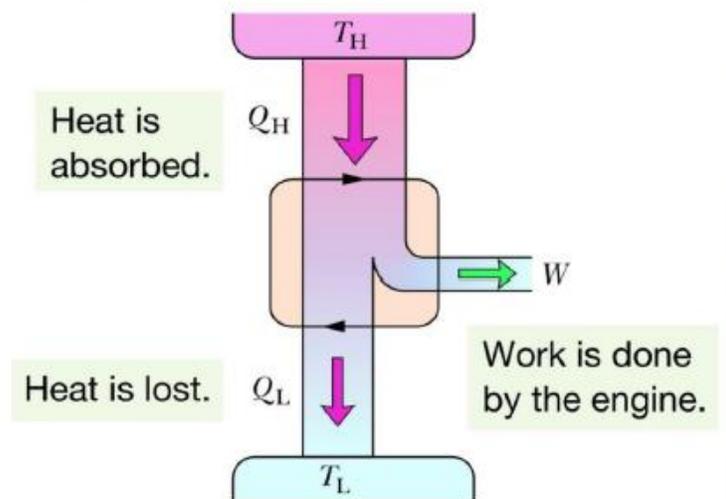
Summing up of Carnot Cycle

- With above analysis, we can see that after one cycle:
Work done W : the colored area in the p - V plot
 $\Delta E_{int} = 0$, since it goes back to a state.
Total heat absorbed: $Q_H - Q_L$.
With 1st law of thermodynamics: $W = Q_H - Q_L$

Stages of a Carnot engine



Schematic of a Carnot engine



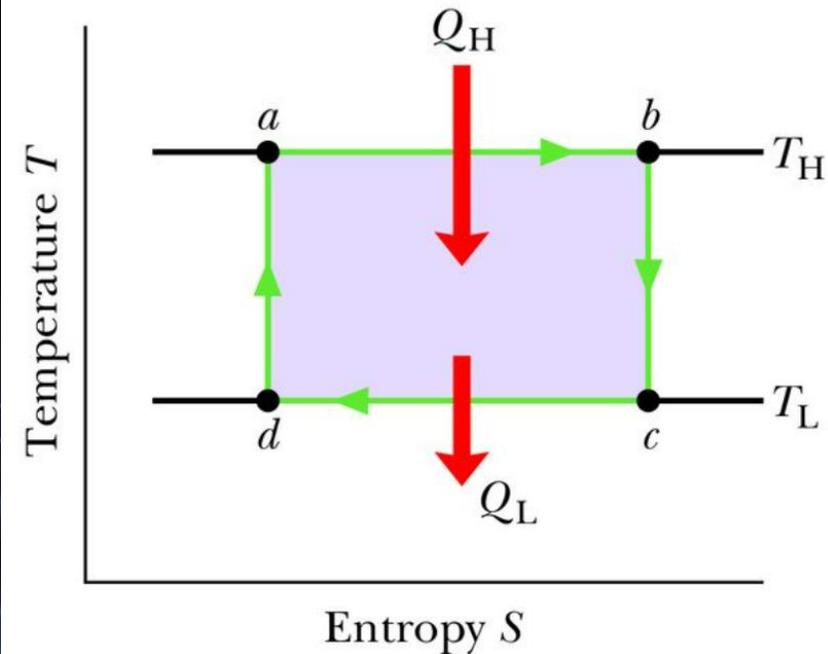
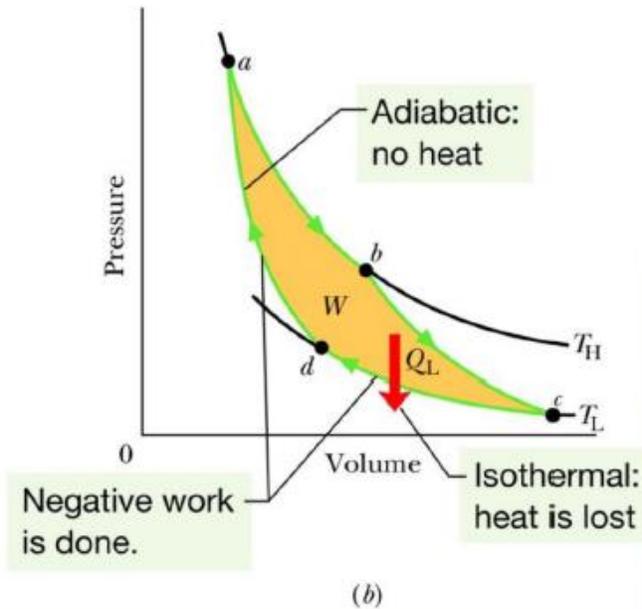
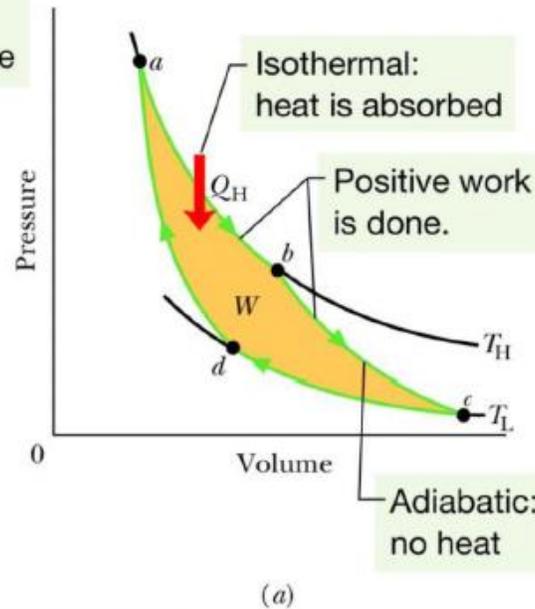
Entropy change in a Carnot Cycle

Total entropy change: $\Delta S_{cycle} = \frac{Q_H}{T_H} - \frac{Q_L}{T_L}$

Since entropy is a state function and we go back to a ($\Delta S_{cycle} = 0$):

$$\frac{Q_H}{T_H} = \frac{Q_L}{T_L}$$

Stages of a Carnot engine



Efficiency of a heat engine

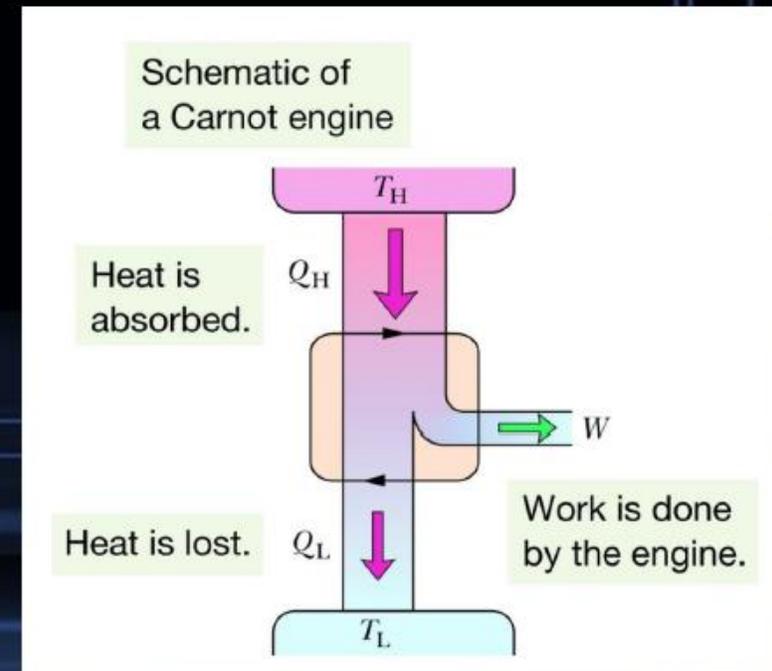
- Let's consider what is efficiency of a heat engine:

We measure its success in doing so by its thermal efficiency ϵ

$$\epsilon = \frac{\text{energy we get}}{\text{energy we pay for}} = \frac{|W|}{|Q_H|} \quad (\text{efficiency, any engine})$$

In a cycle:

$$\epsilon = \frac{|Q_H| - |Q_L|}{|Q_H|} = 1 - \frac{|Q_L|}{|Q_H|}$$



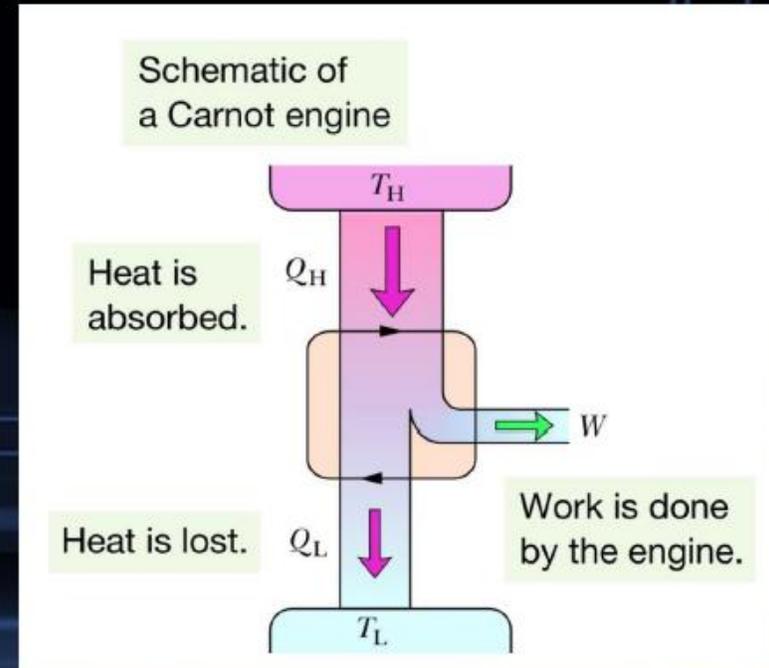
Efficiency of a heat engine

In a Carnot cycle:

$$\varepsilon_C = 1 - \frac{T_L}{T_H} \quad (\text{efficiency, Carnot engine})$$

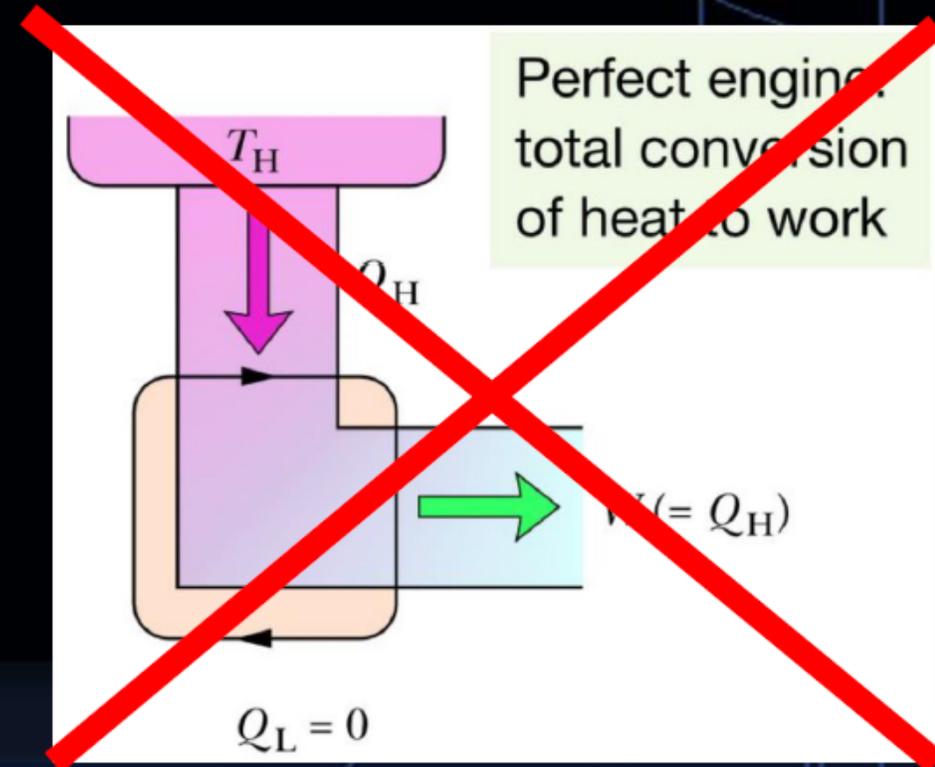
where the temperatures T_L and T_H are in kelvins. Therefore, the efficiency is decided by the thermal reservoirs' temperatures.

Because $T_L < T_H$, the Carnot engine necessarily has a thermal efficiency less than unity—that is, less than 100%.



No perfect engine

- The inventor's dream is to produce the perfect engine, as the figure, in which is reduced to zero and is converted completely into work.
- A perfect engine is only a dream: we can achieve 100% engine efficiency (that is, $\varepsilon = 1$) only if $T_L = 0$ or $T_H \rightarrow \infty$, which is an impossible requirements.
- No series of processes is possible whose sole result is the transfer of energy as heat from a thermal reservoir and the complete conversion of this energy to work.



The 2nd law of thermodynamics (Kelvin-Planck Statement)

It is impossible to construct a heat engine operating in a cycle that extracts heat from a reservoir and delivers an equal amount of work.

Example: Impossibly efficient engine

- An inventor claims to have constructed an engine that has an efficiency of 75% when operated between the boiling and freezing points of water. Is this possible?

不耗能源引擎 環保新革命

華視 CTS 更新日期: 2007/11/20 14:39



現在能源節節高漲，不過，屏東科技大學的學生就發明一種引擎，不必吃油，也可以用水電解氫氣加酒精，不但不耗能源，排放出來的也是水，而不是二氧化碳，被認為是劃時代的進步，還獲得德國紐堡發明展的金牌獎。

德國倫堡發明展現場，這台揚名國際的引擎靠著四個透明瓶子的能源，其中有水，滾滾揮發的是甲醇，讓機器動起來。甚至是水電解出的氫，也可以讓接上引擎的焊槍，發出能量。台灣駐德人員充當翻譯，這項發明立刻引起各國評審注意，入神傾聽議論紛紛。發明者就是他，屏科大的謝孟翰，深度鏡片後專注的眼睛，平時就窩在電腦前試數據，不然就是在機房試機器。這台引擎吃的是水電解的氫氣，或是甲醇和酒精這類的揮發性燃料，牧草或麻

Example: Impossibly efficient engine

- An inventor claims to have constructed an engine that has an efficiency of 75% when operated between the boiling and freezing points of water. Is this possible?

We find that the efficiency of a Carnot engine operating between the boiling and freezing points of water is

$$\epsilon = 1 - \frac{T_L}{T_H} = 1 - \frac{(0 + 273) \text{ K}}{(100 + 273) \text{ K}} = 0.268 \approx 27\%$$

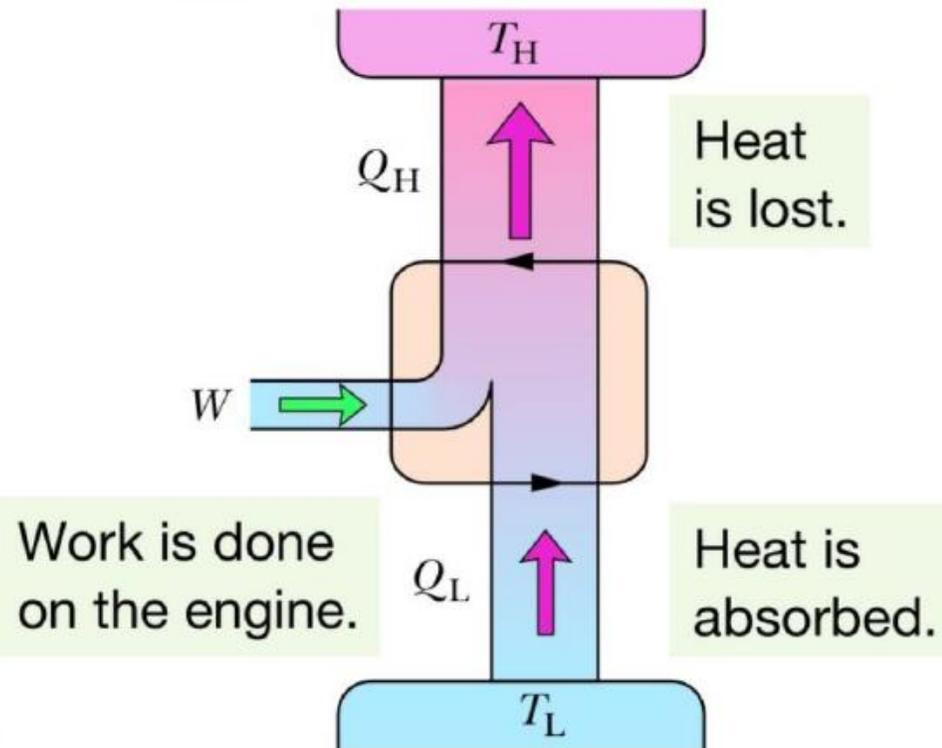
Thus, for the given temperatures, the claimed efficiency of 75% for a real engine (with its irreversible processes and wasteful energy transfers) is impossible.

Refrigerators

- A refrigerator is a device that uses work in order to transfer energy from a low-temperature reservoir to a high-temperature reservoir as the device continuously repeats a set series of thermodynamic processes.
- In an ideal refrigerator, all processes are reversible and no wasteful energy transfers occur as a result of, say, friction and turbulence.
- If we do backward loop of Carnot cycle, we can obtain a Carnot refrigerator!

Scheme of Carnot Refrigerators

Schematic of
a refrigerator



Work is done
on the engine.

Heat
is lost.

Heat is
absorbed.

Efficiency of a refrigerator

- As our analysis of engine, the efficiency of a refrigerator is:

$$K = \frac{\text{what we want}}{\text{what we pay for}} = \frac{|Q_L|}{|W|} \quad (\text{coefficient of performance, any refrigerator})$$

With 1st law of thermodynamics:

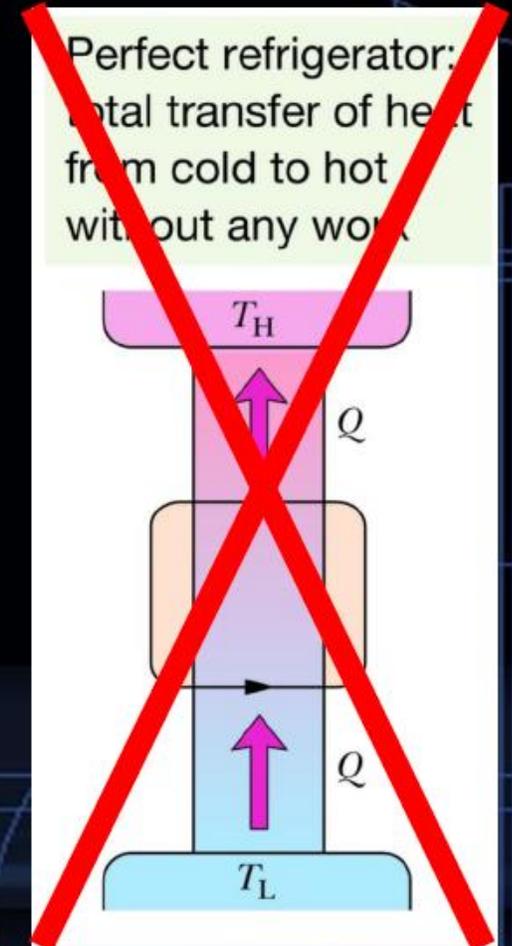
$$K = \frac{|Q_L|}{|Q_H| - |Q_L|}$$

- For a Carnot refrigerator:

$$K_C = \frac{T_L}{T_H - T_L} \quad (\text{coefficient of performance, Carnot refrigerator})$$

No perfect refrigerator

- A perfect refrigerator that transfers energy as heat Q from a cold reservoir to a warm reservoir without the need for work.
- The entropy changes due to two reservoirs in one cycle are:
$$\Delta S = -\frac{|Q|}{T_L} + \frac{|Q|}{T_H}$$
- With the 2nd law of thermodynamics, it is required that $T_L \leq T_H$. Thus efficiency can only be 1 when $T_L = T_H$. This is not a refrigerator. Thus no perfect refrigerator.



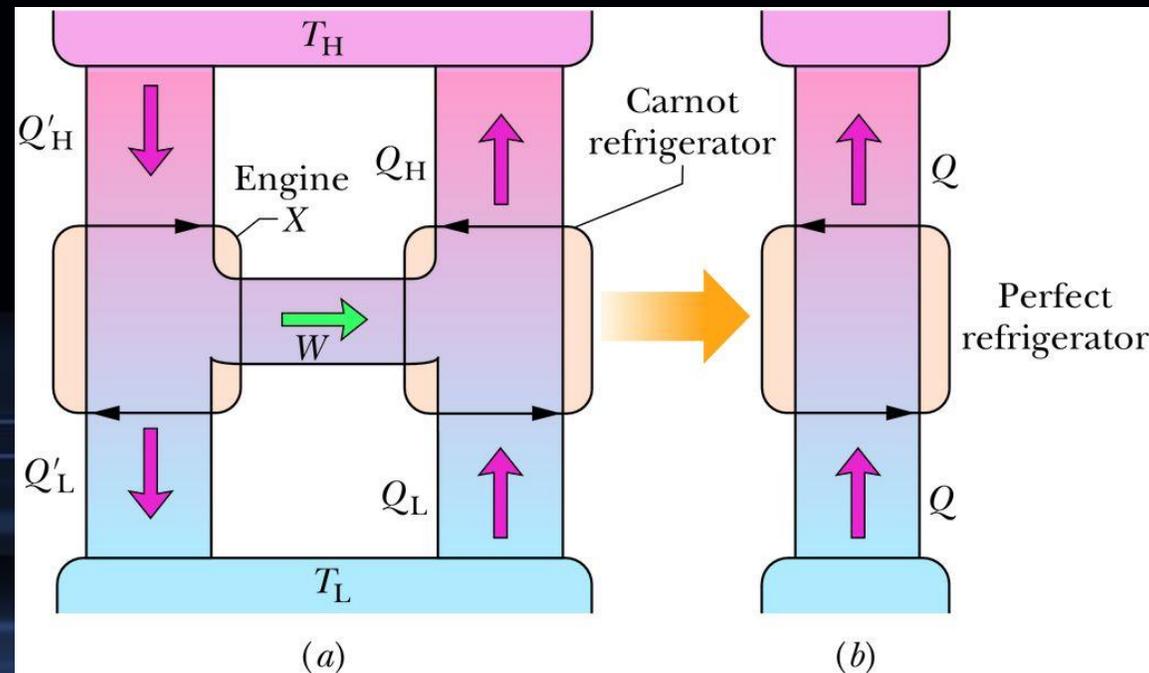
The 2nd law of thermodynamics (Clausius Statement)

It is impossible to construct a refrigerator operating in a cycle whose sole effect is to transfer heat from a cooler object to a hotter one

Carnot's theorem

All Carnot engines operating between temperature T_H and T_L have the same efficiency: $\epsilon_C = 1 - \frac{T_L}{T_H}$ (efficiency, Carnot engine).

And no other heat engine operating between the same two temperatures can have a greater efficiency.

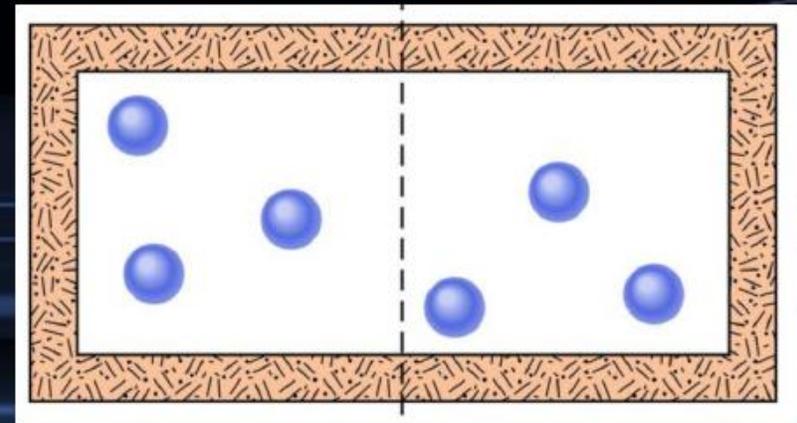
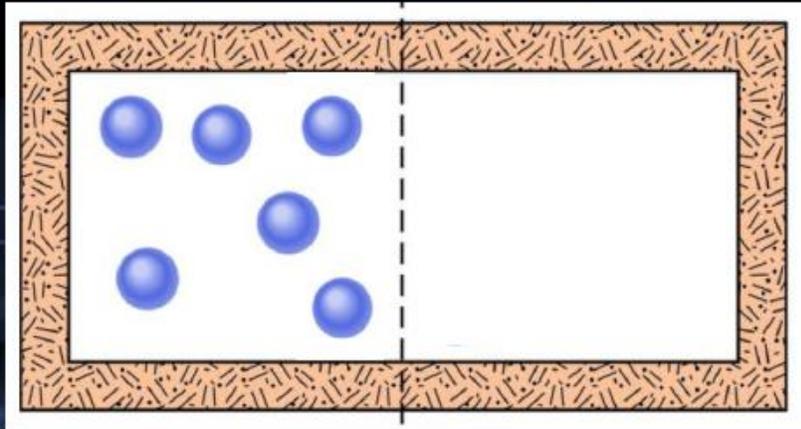


Multiplicity of molecules

- Define multiplicity of configuration as following:

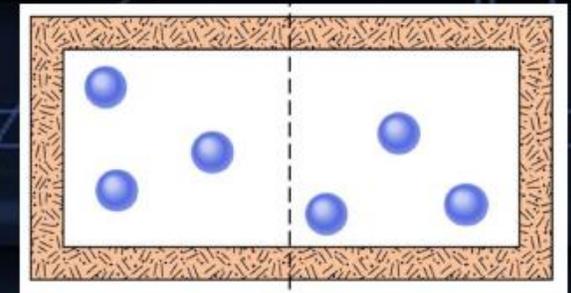
$$W = \frac{N!}{n_1! n_2!} \quad (\text{multiplicity of configuration})$$

N: total number of molecules. n1: molecules in right box, n2: molecules in left box



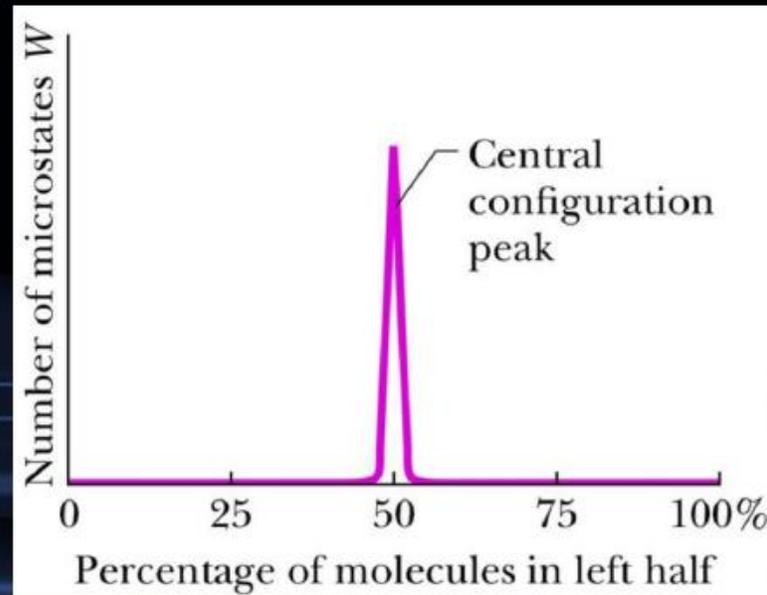
Multiplicity of molecules

Configuration			Multiplicity W (number of microstates)	Calculation of W
Label	n_1	n_2		
I	6	0	1	$6!/(6! 0!) = 1$
II	5	1	6	$6!/(5! 1!) = 6$
III	4	2	15	$6!/(4! 2!) = 15$
IV	3	3	20	$6!/(3! 3!) = 20$
V	2	4	15	$6!/(2! 4!) = 15$
VI	1	5	6	$6!/(1! 5!) = 6$
VII	0	6	1	$6!/(0! 6!) = 1$
			Total = 64	



The central configuration peak

- When $N=10^{22}$, nearly all the microstates correspond to an approximately equal sharing of the molecules between the two halves of the box; those microstates form the central configuration peak on the plot.



Probability and Entropy

- In 1877, Austrian physicist Ludwig Boltzmann (the Boltzmann of Boltzmann's constant k) derived a relationship between the entropy S of a configuration of a gas and the multiplicity W of that configuration.

$$S = k \ln W \quad (\text{Boltzmann's entropy equation})$$

