

Course announcement

- The 2nd midterm score has been updated on eLearn. You can also review it during Monday's office hours. If you have any questions about the score, please contact me.
- The 5th homework set has been posted. And it will be due today (1/3), 5pm.
- The review section 3 on 1/6 will be a pre-recorded section. It will be uploaded on eLearn.

16	12/30(Fri.)	Entropy and the Second Law of Thermal Dynamics: entropy
17	1/3(Tue.)	Entropy and the Second Law of Thermal Dynamics: engines and refrigerator
17	1/6(Fri.)	Review III
18	1/10(Tue.)	Final Exam

GENERAL PHYSICS B1 THE SECOND LAW OF THERMODYNAMICS

2022/12/30

Entropy, Engine and Refrigerator

Topic Today

Entropy and the second law of thermodynamics

- Heat Engine and Refrigerator
- Statistical Interpretation of Entropy

Definition of Entropy

- Entropy is a state property just like pressure and volume that describing the status of a system at a certain state.
- Since we know the change in entropy can tell the direction of a process, we start the definition with changing in entropy:

Change in entropy $S_f - S_i$ of a system during a process that takes the system from an initial state i to a final state f as

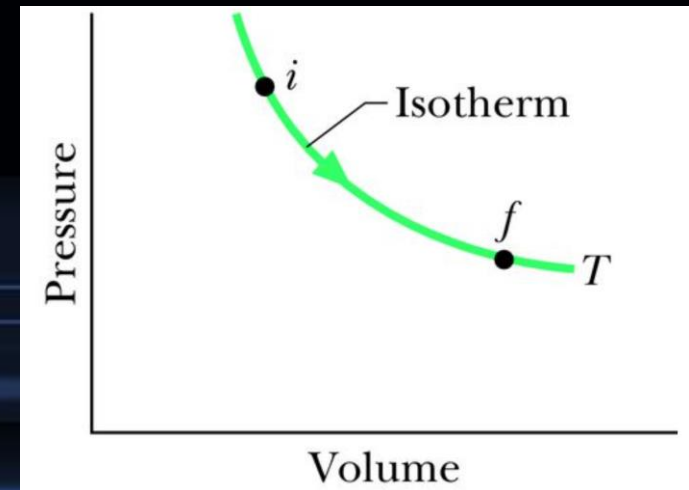
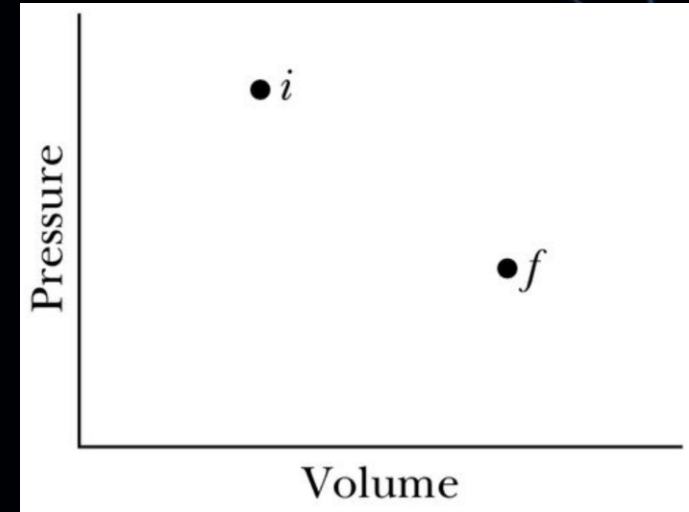
$$\Delta S = S_f - S_i = \int_i^f \frac{dQ}{T}$$

Finding entropy in a process

- In this case, we use an isotherm process to find out change of entropy from state i to f :

$$\Delta S = S_f - S_i = \frac{Q}{T} \quad (\text{change in entropy, isothermal process})$$

- To keep the temperature T of the gas constant during the isothermal expansion, heat Q must have been energy transferred from the reservoir to the gas. Thus, Q is positive and the entropy of the gas increases during the isothermal process and during the free expansion.



The 2nd law of thermodynamics

If a process occurs in a closed system, the entropy of the system increases for irreversible processes and remains constant for reversible processes. It never decreases.

$$\Delta S \geq 0 \quad (\text{second law of thermodynamics})$$

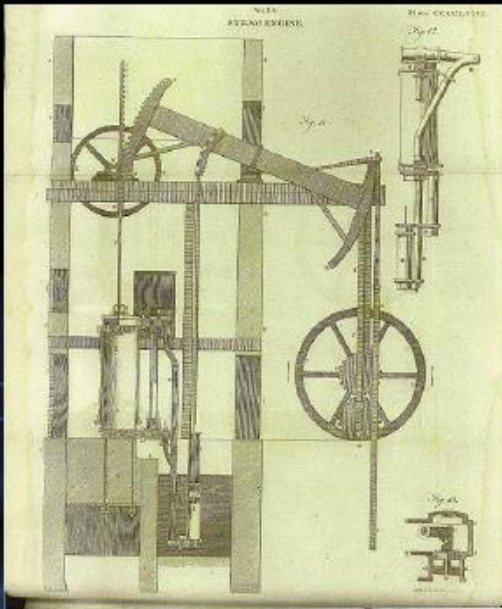
Topic Today

Entropy and the second law of thermodynamics

- **Heat Engine and Refrigerator**
- Statistical Interpretation of Entropy

Heat Engine

- A heat engine, or more simply, an engine, is a device that extracts energy from its environment in the form of heat and does useful work.
- The invention and improvement of steam engine start the Industrial Revolution.



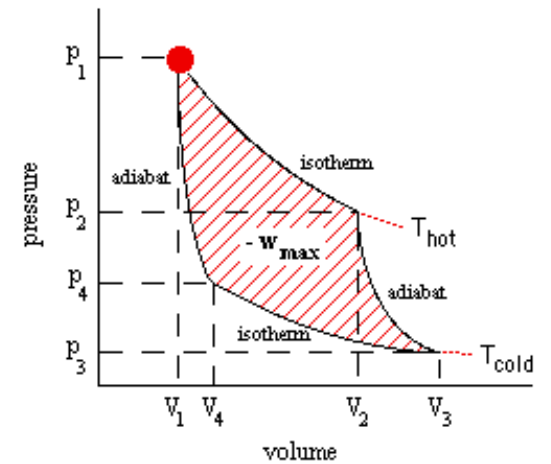
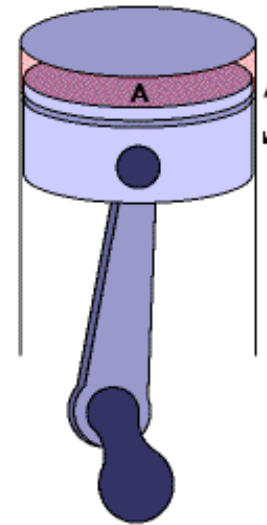
<https://en.wikipedia.org/wiki/File:WattsSteamEngine.jpeg>



https://en.wikipedia.org/wiki/Engine#/media/File:Mercedes_V6_DTM_Rennmotor_1996.jpg

Cycle in Heat Engine

- At the heart of every engine is a working **substance**. In a steam engine, the working substance is water, in both its vapor and its liquid form. In an automobile engine the working substance is a gasoline–air mixture.
- The working substance must operate in a **cycle**; that is, the working substance must pass through a closed series of thermodynamic processes (strokes).



GIFs de fisica

Question

- How much work can we get from each cycle?
- What is the efficiency of an engine? Can we maximize it?

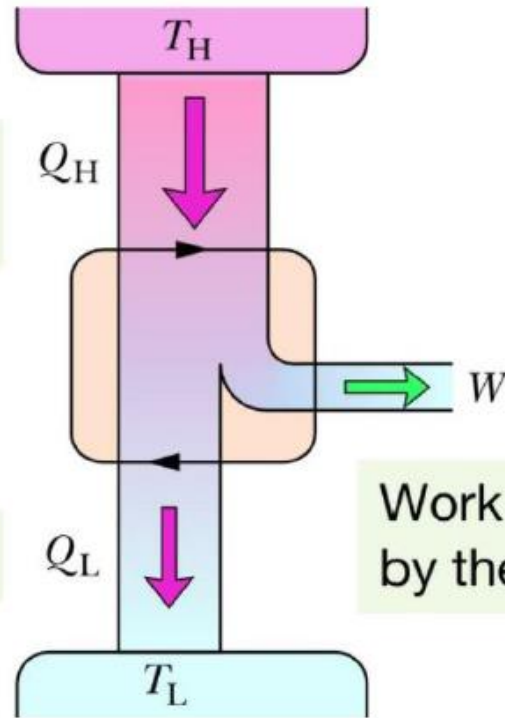
Ideal Engine and Carnot Engine

- Let's use ideal gas as working substance (obeys the simple law $pV = nRT$).
- In an ideal engine, all processes are **reversible** and **no wasteful energy** transfers occur due to friction and turbulence.
- French scientist and engineer N. L. Sadi Carnot who first proposed the engine's concept in 1824. He was able to analyze the performance of this engine before the first law of thermodynamics and the concept of entropy had been discovered.

Scheme of Carnot Engine

Schematic of
a Carnot engine

Heat is
absorbed.

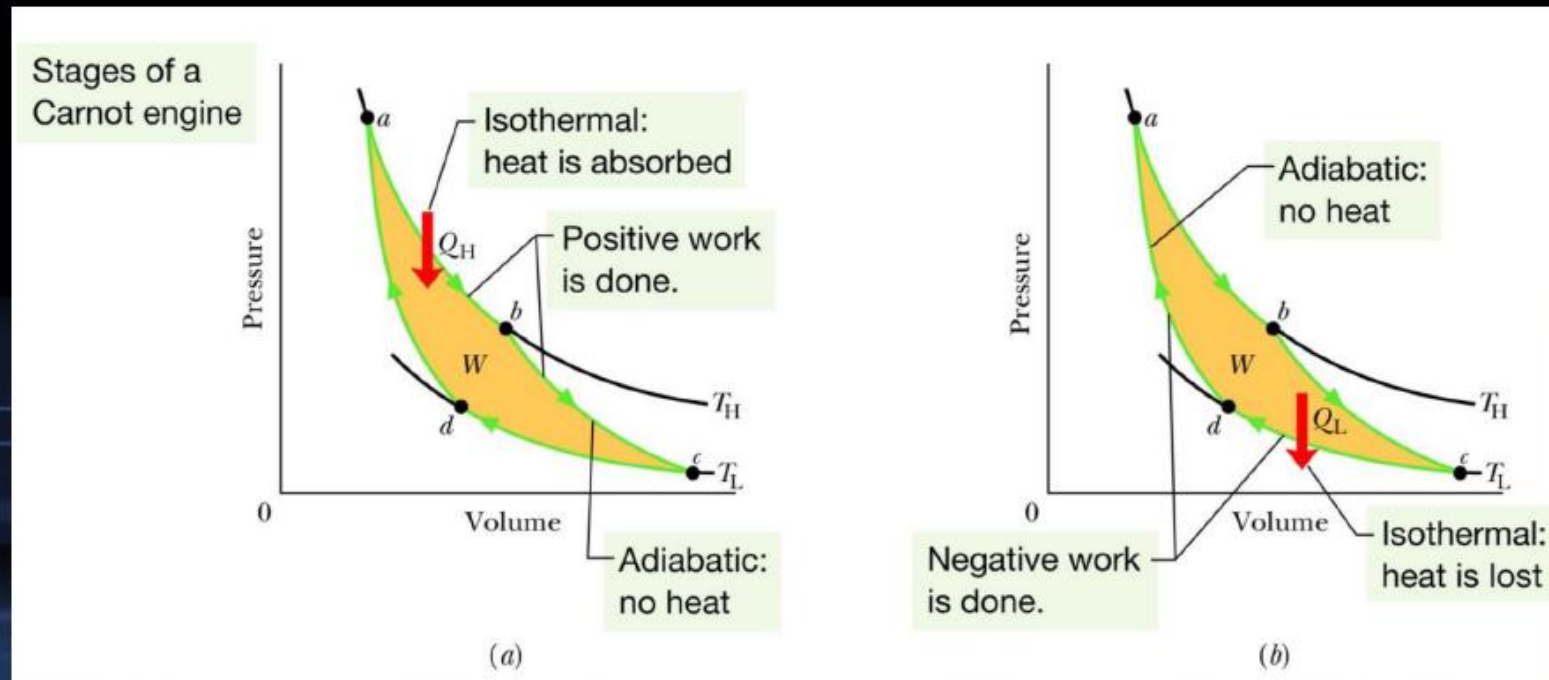


Heat is lost.

Work is done
by the engine.

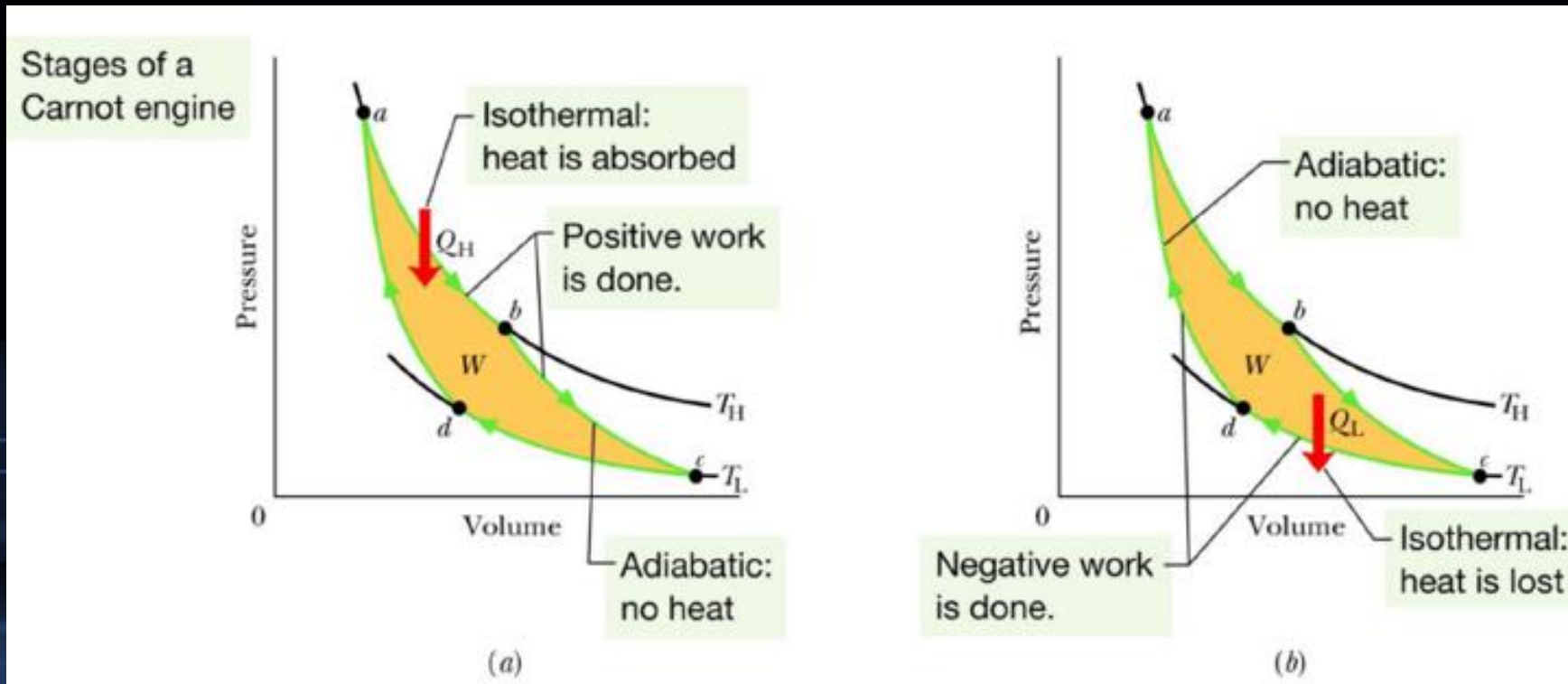
Carnot Cycle

Carnot cycle is how a working substance cycling in a Carnot engine. It contain four steps: Isothermal expansion(ab), adiabatic expansion(bc), isothermal compression(cd), and adiabatic compression(da).



Analysis of Carnot Cycle

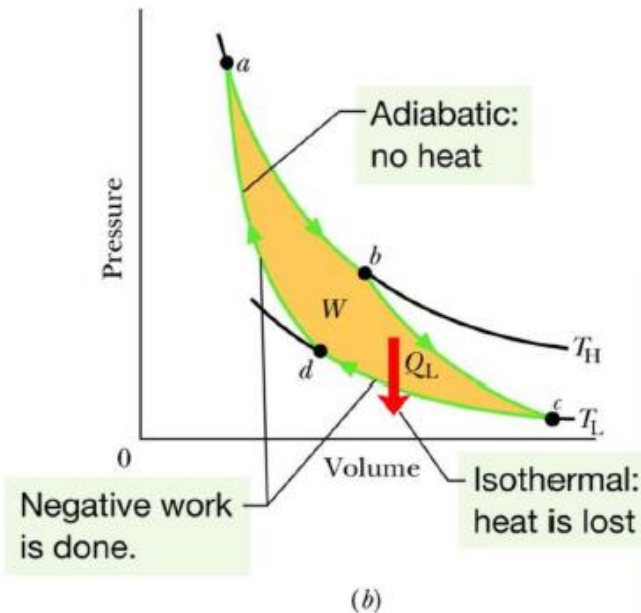
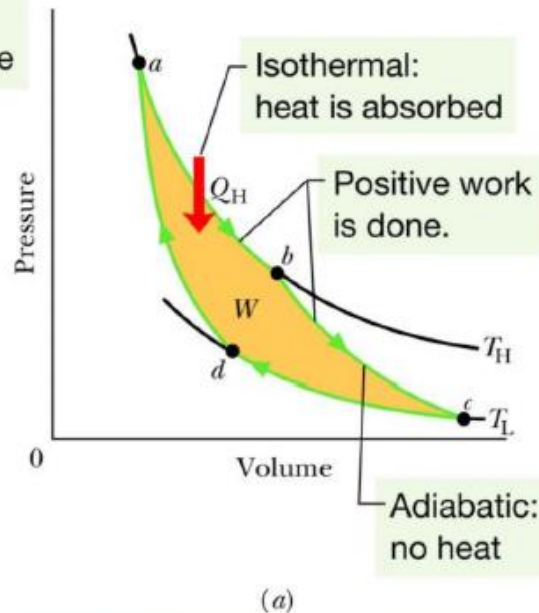
- ab: No change in E_{int} . Absorb heat Q_H . Do positive work to outside.
- bc: Decrease in E_{int} . No heat exchange. Do positive work to outside.
- cd: No change in E_{int} . Release heat Q_L . Do negative work to outside.
- da: Increase in E_{int} . No heat exchange. Do negative work to outside.



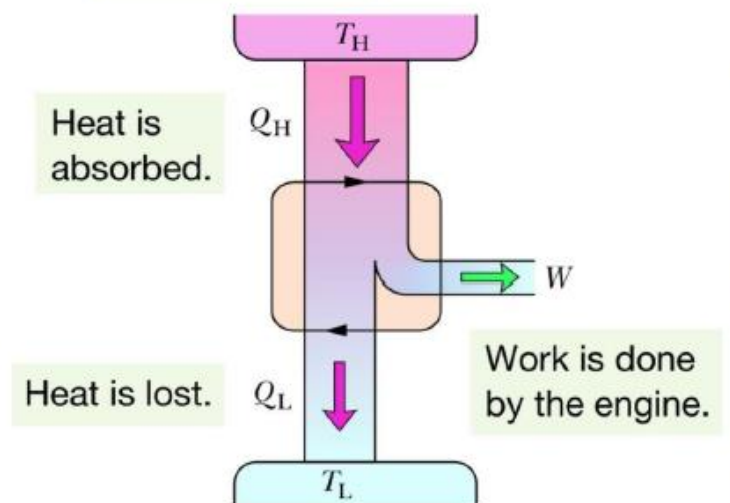
Summing up of Carnot Cycle

- With above analysis, we can see that after one cycle:
Work done W : the colored area in the p-V plot
 $\Delta E_{int} = 0$, since it goes back to a state.
Total heat absorbed: $Q_H - Q_L$.
With 1st law of thermodynamics: $W = Q_H - Q_L$

Stages of a Carnot engine



Schematic of a Carnot engine



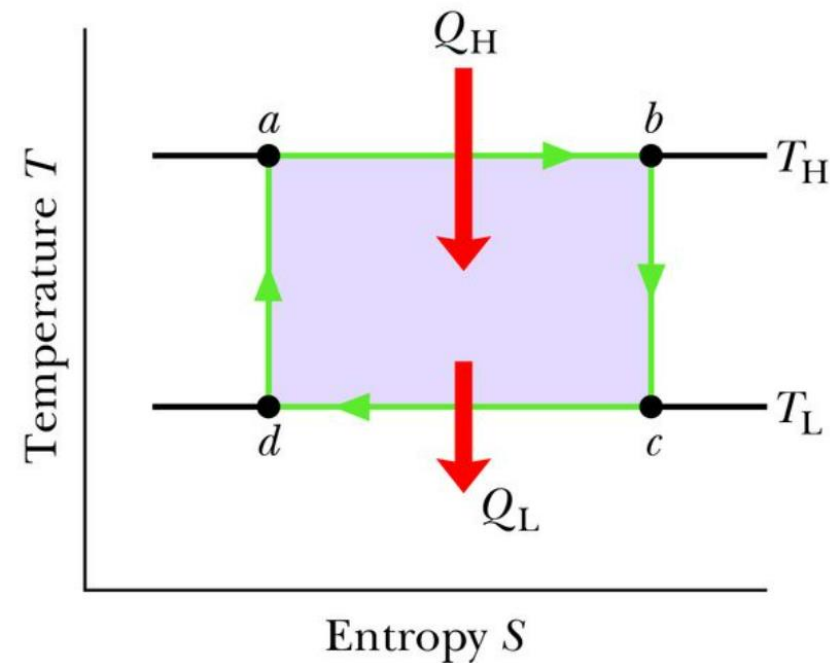
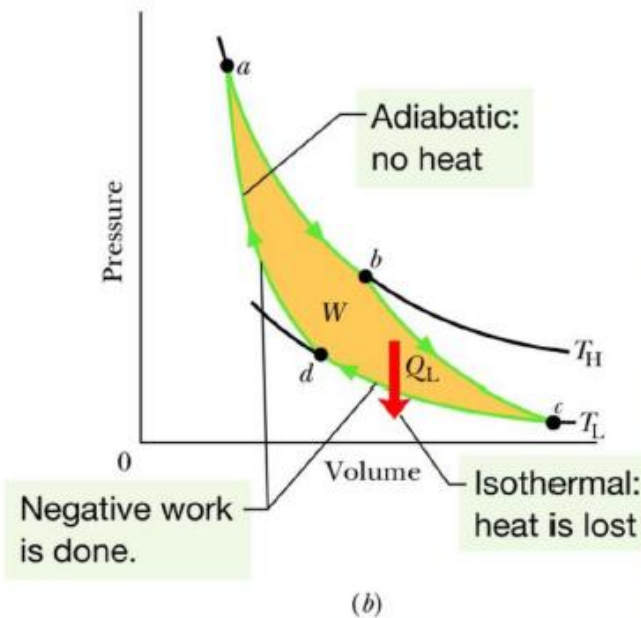
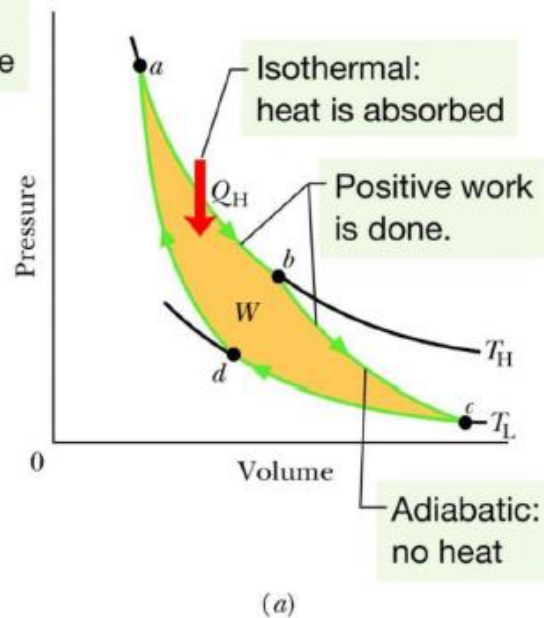
Entropy change in a Carnot Cycle

- The change of entropy:

$$ab: +\frac{Q_H}{T_H}, \quad bc: 0, \quad cd: -\frac{Q_L}{T_L}, \quad da: 0,$$

$$\text{Total entropy change: } \Delta S_{\text{cycle}} = \frac{Q_H}{T_H} - \frac{Q_L}{T_L}$$

Stages of a Carnot engine



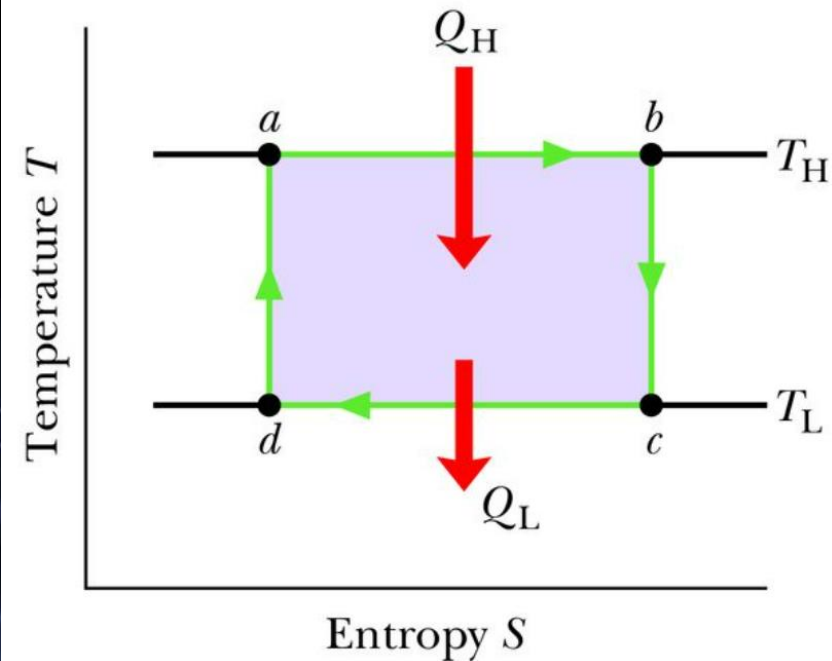
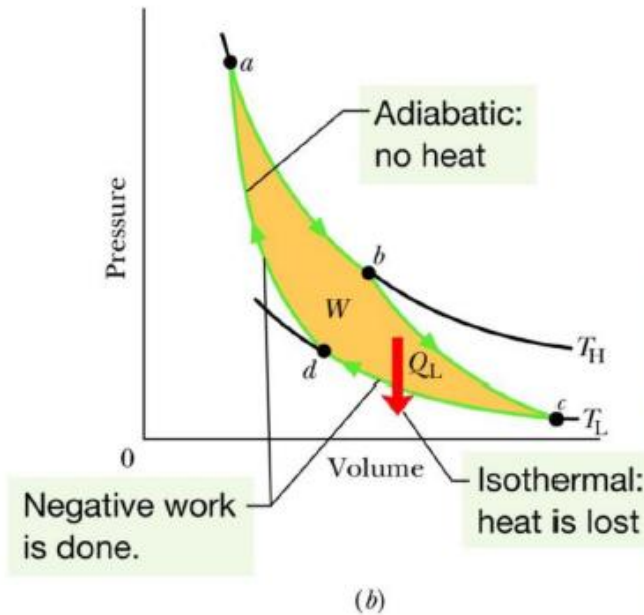
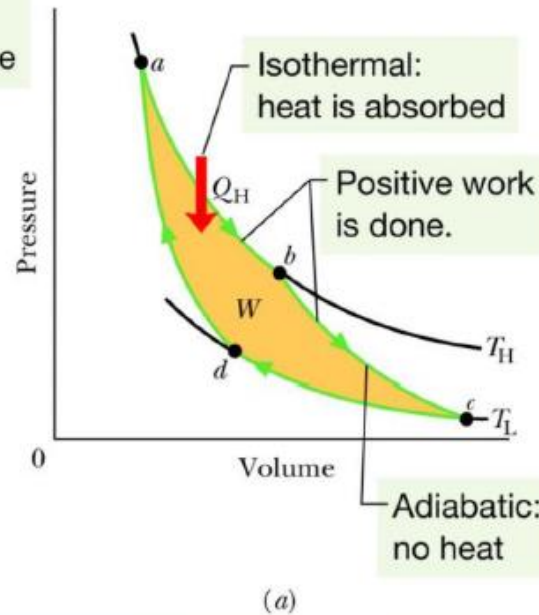
Entropy change in a Carnot Cycle

Total entropy change: $\Delta S_{cycle} = \frac{Q_H}{T_H} - \frac{Q_L}{T_L}$

Since entropy is a state function and we go back to a ($\Delta S_{cycle} = 0$):

$$\frac{Q_H}{T_H} = \frac{Q_L}{T_L}$$

Stages of a Carnot engine



Efficiency of a heat engine

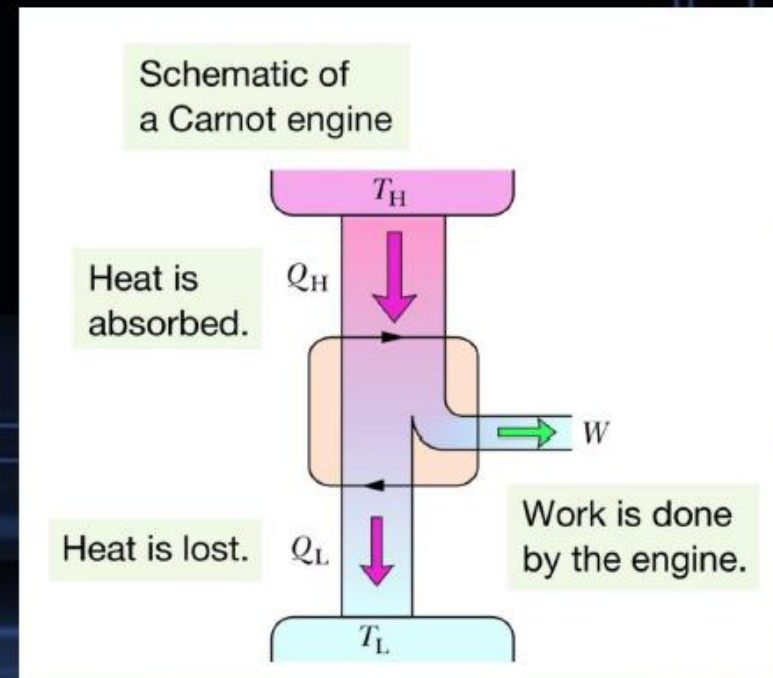
- Let's consider what is efficiency of a heat engine:

We measure its success in doing so by its thermal efficiency ϵ

$$\epsilon = \frac{\text{energy we get}}{\text{energy we pay for}} = \frac{|W|}{|Q_H|} \quad (\text{efficiency, any engine})$$

In a cycle:

$$\epsilon = \frac{|Q_H| - |Q_L|}{|Q_H|} = 1 - \frac{|Q_L|}{|Q_H|}$$



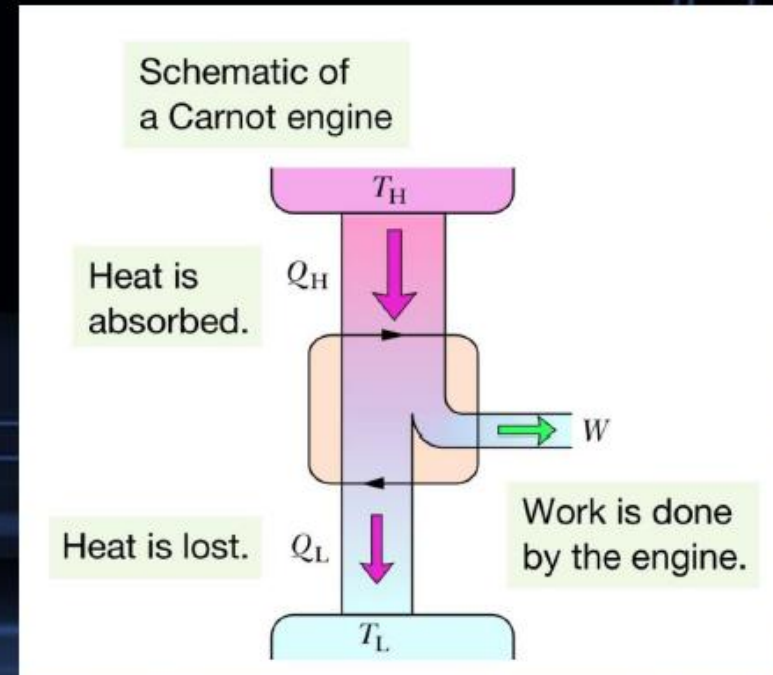
Efficiency of a heat engine

In a Carnot cycle:

$$\varepsilon_C = 1 - \frac{T_L}{T_H} \quad (\text{efficiency, Carnot engine})$$

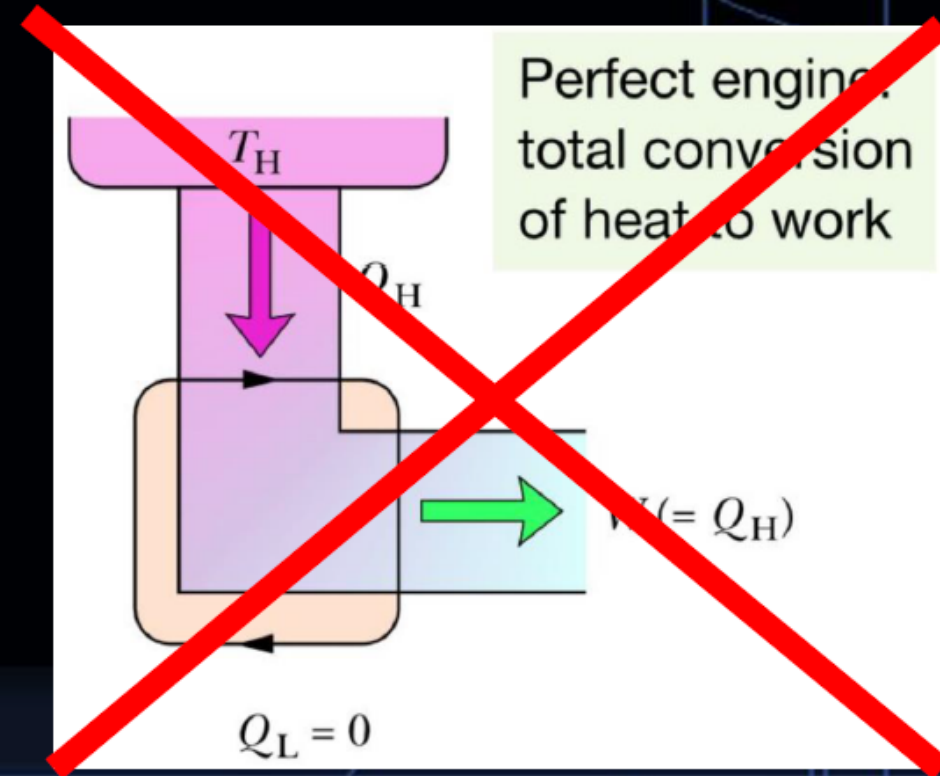
where the temperatures T_L and T_H are in kelvins. Therefore, the efficiency is decided by the thermal reservoirs' temperatures.

Because $T_L < T_H$, the Carnot engine necessarily has a thermal efficiency less than unity—that is, less than 100%.



No perfect engine

- The inventor's dream is to produce the perfect engine, as the figure, in which T_L is reduced to zero and T_H is converted completely into work.
- A perfect engine is only a dream: we can achieve 100% engine efficiency (that is, $\varepsilon = 1$) only if $T_L = 0$ or $T_H \rightarrow \infty$, which is an impossible requirements.
- No series of processes is possible whose sole result is the transfer of energy as heat from a thermal reservoir and the complete conversion of this energy to work.



The 2nd law of thermodynamics (Kelvin-Planck Statement)

It is impossible to construct a heat engine operating in a cycle that extracts heat from a reservoir and delivers an equal amount of work.

Example: Impossibly efficient engine

- An inventor claims to have constructed an engine that has an efficiency of 75% when operated between the boiling and freezing points of water. Is this possible?

不耗能源引擎 環保新革命

華視 CTS 更新日期: 2007/11/20 14:39



現在能源節節高漲，不過，屏東科技大學的學生就發明一種引擎，不必吃油，也可以用水電解氫氣加酒精，不但不耗能源，排放出來的也是水，而不是二氧化碳，被認為是劃時代的進步，還獲得德國紐堡發明展的金牌獎。

德國倫堡發明展現場，這台揚名國際的引擎靠著四個透明瓶子的能源，其中有水，滾滾揮發的是甲醇，讓機器動起來。甚至是水電解出的氫，也可以讓接上引擎的焊槍，發出能量。台灣駐德人員充當翻譯，這項發明立刻引起各國評審注意，入神傾聽議論紛紛。發明者就是他，屏科大的謝孟翰，深度鏡片後專注的眼睛，平時就窩在電腦前試數據，不然就是在機房試機器。這台引擎吃的是水電解的氫氣，或是甲醇和酒精這類的揮發性燃料，牧草或麻

Example: Impossibly efficient engine

- An inventor claims to have constructed an engine that has an efficiency of 75% when operated between the boiling and freezing points of water. Is this possible?

We find that the efficiency of a Carnot engine operating between the boiling and freezing points of water is

$$\epsilon = 1 - \frac{T_L}{T_H} = 1 - \frac{(0 + 273) \text{ K}}{(100 + 273) \text{ K}} = 0.268 \approx 27\%$$

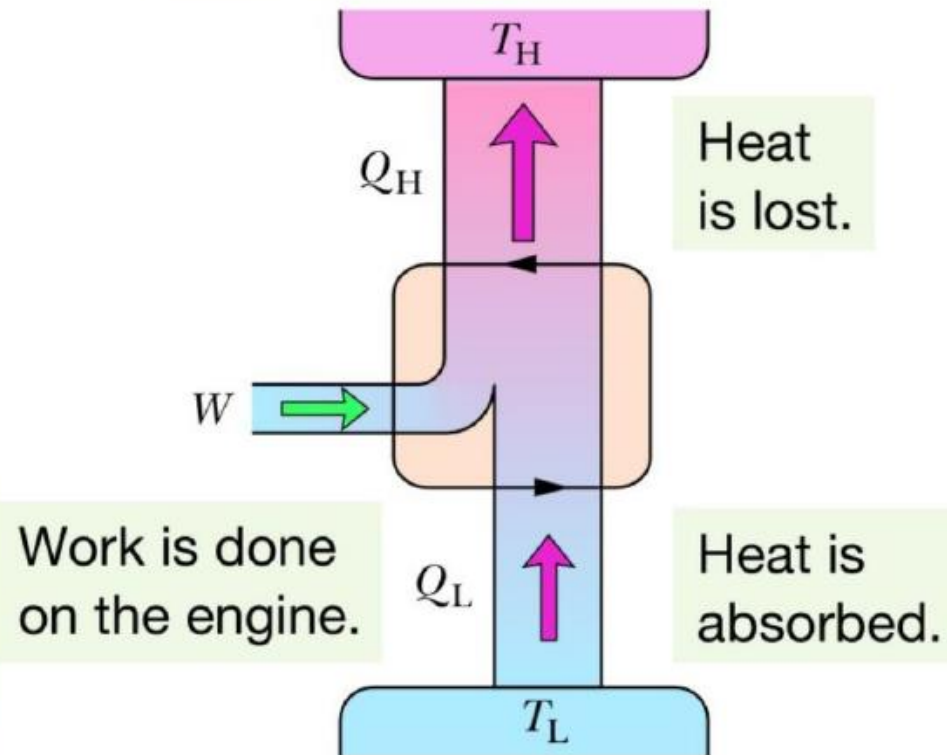
Thus, for the given temperatures, the claimed efficiency of 75% for a real engine (with its irreversible processes and wasteful energy transfers) is impossible.

Refrigerators

- A refrigerator is a device that uses work in order to transfer energy from a low-temperature reservoir to a high-temperature reservoir as the device continuously repeats a set series of thermodynamic processes.
- In an ideal refrigerator, all processes are reversible and no wasteful energy transfers occur as a result of, say, friction and turbulence.
- If we do backward loop of Carnot cycle, we can obtain a Carnot refrigerator!

Scheme of Carnot Refrigerators

Schematic of
a refrigerator



Efficiency of a refrigerator

- As our analysis of engine, the efficiency of a refrigerator is:

$$K = \frac{\text{what we want}}{\text{what we pay for}} = \frac{|Q_L|}{|W|} \quad (\text{coefficient of performance, any refrigerator})$$

With 1st law of thermodynamics:

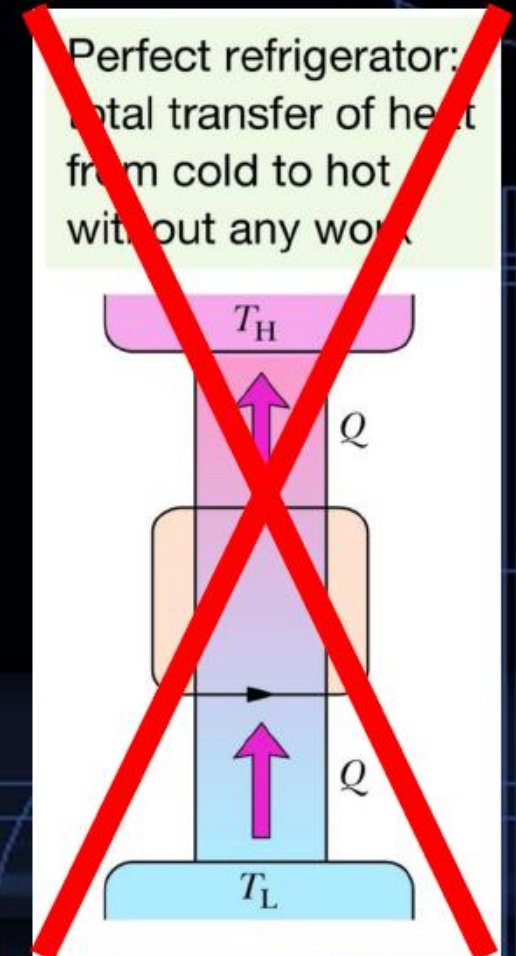
$$K = \frac{|Q_L|}{|Q_H| - |Q_L|}$$

- For a Carnot refrigerator:

$$K_C = \frac{T_L}{T_H - T_L} \quad (\text{coefficient of performance, Carnot refrigerator})$$

No perfect refrigerator

- A perfect refrigerator that transfers energy as heat Q from a cold reservoir to a warm reservoir without the need for work.
- The entropy changes due to two reservoirs in one cycle are:
$$\Delta S = -\frac{|Q|}{T_L} + \frac{|Q|}{T_H}$$
- With the 2nd law of thermodynamics, it is required that $T_L \leq T_H$. Thus efficiency can only be 1 when $T_L = T_H$. This is not a refrigerator. Thus no perfect refrigerator.



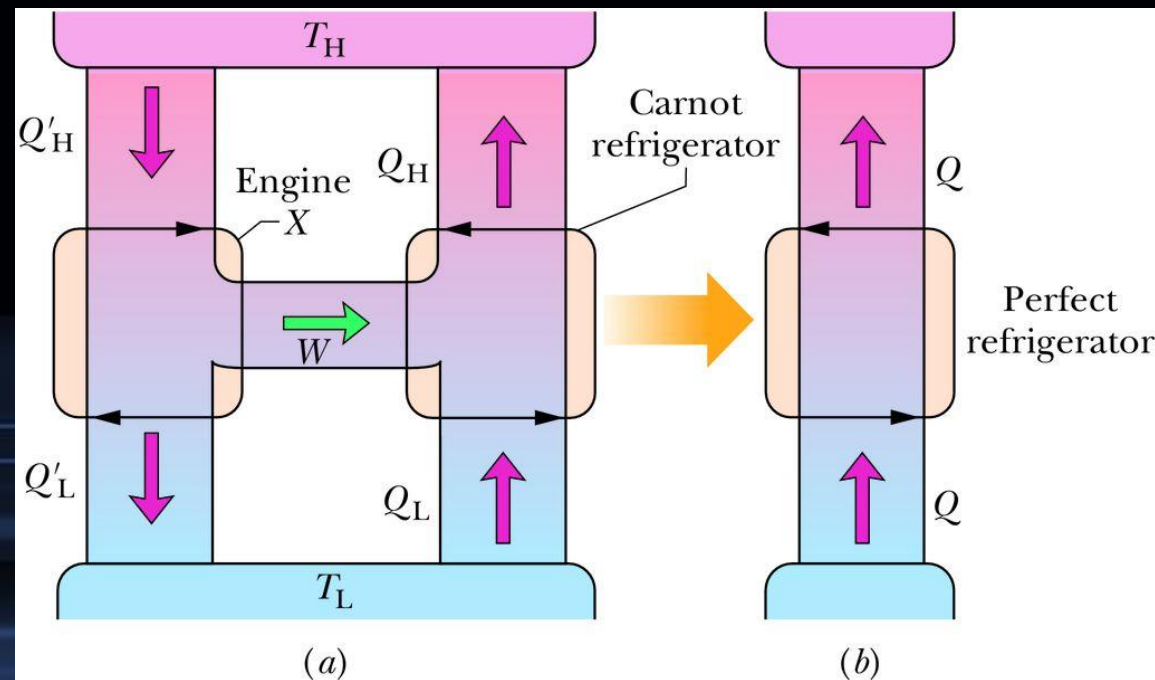
The 2nd law of thermodynamics (Clausius Statement)

It is impossible to construct a refrigerator operating in a cycle whose sole effect is to transfer heat from a cooler object to a hotter one

Carnot's theorem

All Carnot engines operating between temperature T_H and T_L have the same efficiency: $\epsilon_C = 1 - \frac{T_L}{T_H}$ (efficiency, Carnot engine).

And no other heat engine operating between the same two temperatures can have a greater efficiency.



Topic Today

Entropy and the second law of thermodynamics

- Heat Engine and Refrigerator
- **A Statistical View of Entropy**

Link between Entropy to microscopic quantities

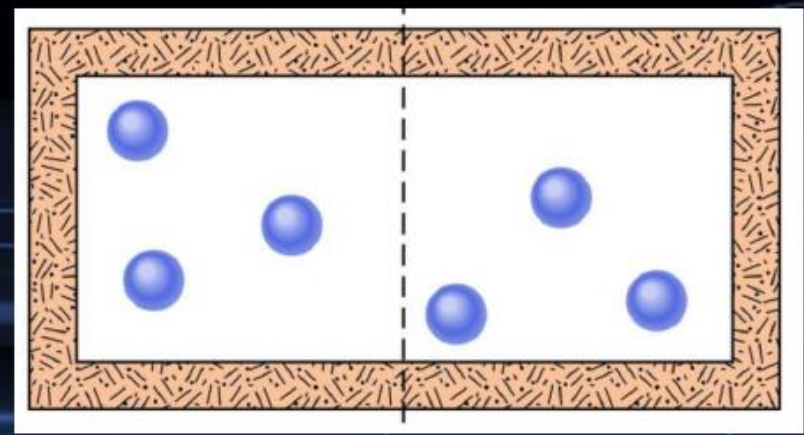
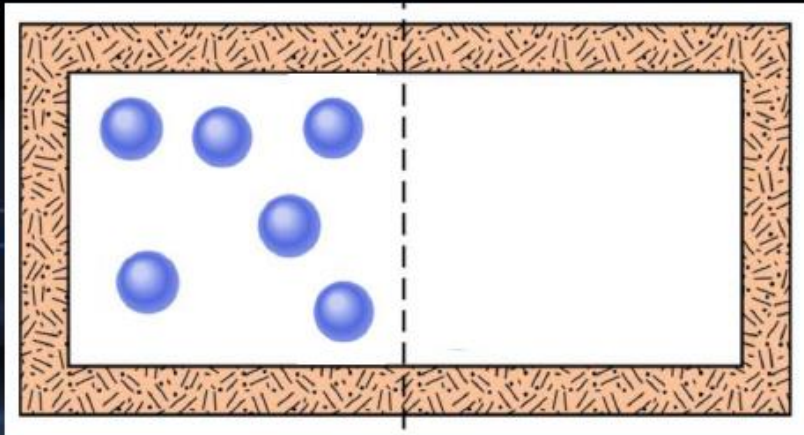
- We mentioned that temperature of idea gas can link to averaged internal energy of each molecule. This is a link between macroscopic physical quantities to microscopic physical quantities.

$$E_{\text{int}} = \frac{3}{2}nRT \quad (\text{monatomic ideal gas})$$

- Can entropy link to any microscopic physical quantities?

Multiplicity of molecules

- Consider the following two different configuration of molecules in a box. In nature, the configuration with molecules equally distributed in box is preferable.
- (Presumably the entropy is higher)

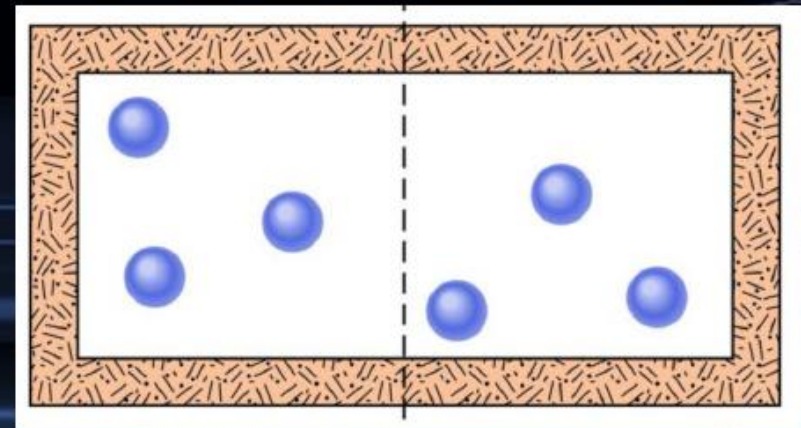
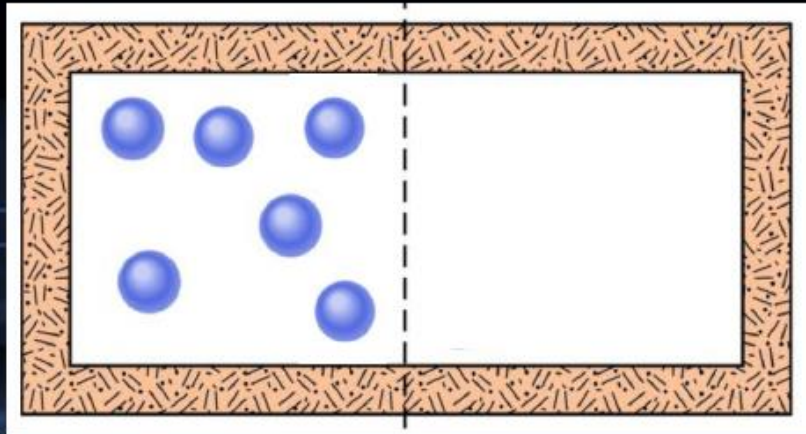


Multiplicity of molecules

- Define multiplicity of configuration as following:

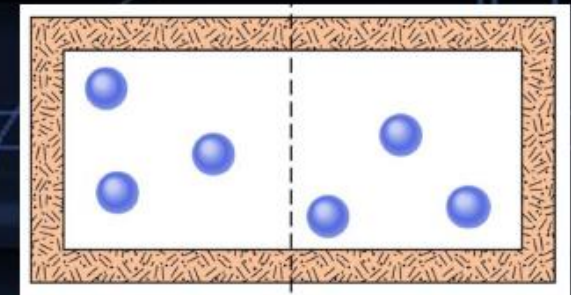
$$W = \frac{N!}{n_1! n_2!} \quad (\text{multiplicity of configuration})$$

N : total number of molecules. n_1 : molecules in right box, n_2 : molecules in left box



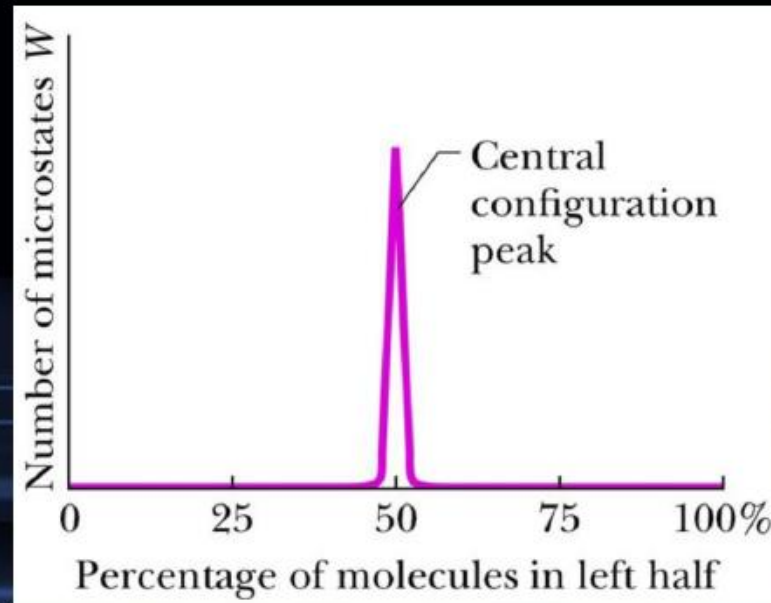
Multiplicity of molecules

Configuration			Multiplicity W (number of microstates)	Calculation of W
Label	n_1	n_2		
I	6	0	1	$6!/(6! 0!) = 1$
II	5	1	6	$6!/(5! 1!) = 6$
III	4	2	15	$6!/(4! 2!) = 15$
IV	3	3	20	$6!/(3! 3!) = 20$
V	2	4	15	$6!/(2! 4!) = 15$
VI	1	5	6	$6!/(1! 5!) = 6$
VII	0	6	1	$6!/(0! 6!) = 1$
			Total = 64	



The central configuration peak

- When $N=10^{22}$, nearly all the microstates correspond to an approximately equal sharing of the molecules between the two halves of the box; those microstates form the central configuration peak on the plot.



Probability and Entropy

- In 1877, Austrian physicist Ludwig Boltzmann (the Boltzmann of Boltzmann's constant k) derived a relationship between the entropy S of a configuration of a gas and the multiplicity W of that configuration.

$$S = k \ln W \quad (\text{Boltzmann's entropy equation})$$

