

# Course announcement

- The 2<sup>nd</sup> midterm score has been posted on eLearn today. I will bring the answer sheet to class for review from Friday until 12/30. You can also review it during Monday's office hours. If you have any questions about the score, please contact me.
- The 5<sup>th</sup> homework set will be posted today (12/27). And it will be due on (1/3), 5pm.

15	12/23(Fri.)	<b>The First Law of Thermal Dynamics: 1<sup>st</sup> law of thermal dynamics</b>
16	12/27(Tue.)	<b>The First Law of Thermal Dynamics: Thermodynamic processes (Homework5)</b>
16	12/30(Fri.)	<b>Entropy and the Second Law of Thermal Dynamics: entropy</b>
17	1/3(Tue.)	<b>Entropy and the Second Law of Thermal Dynamics: engines and refrigerator</b>

# GENERAL PHYSICS B1

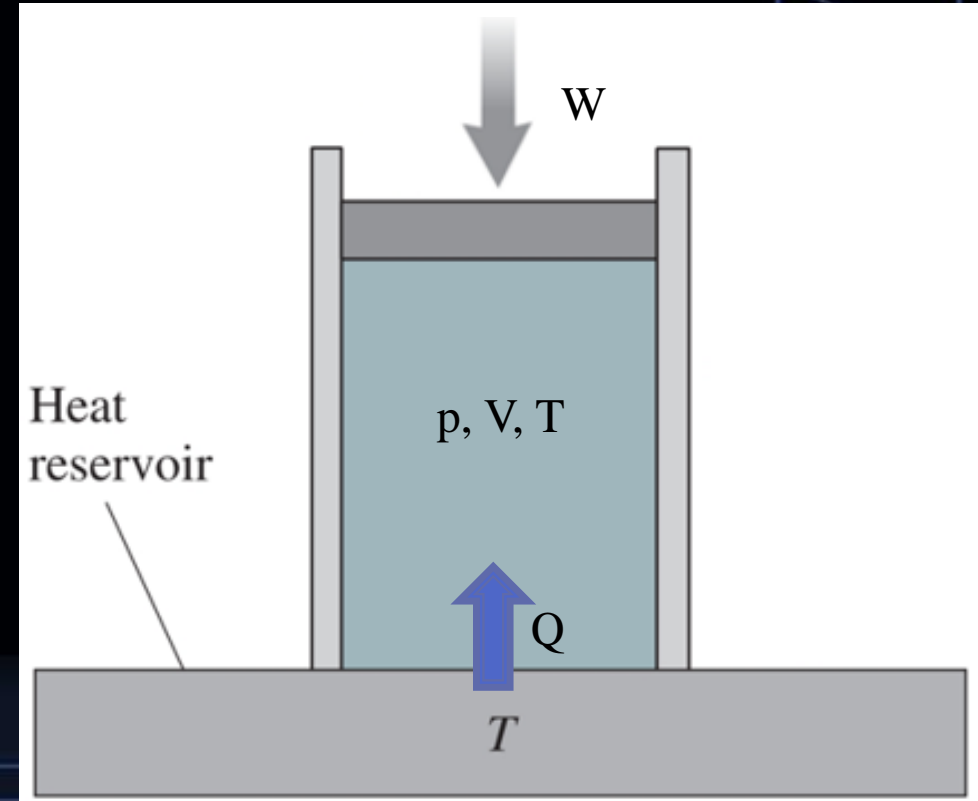
## HEAT, WORK, AND THE FIRST LAW OF THERMODYNAMICS

2022/12/20

Thermodynamic process  
Specific Heats of an ideal gas

# A gas system transferring heat to work

- The system (the gas) starts from an initial state  $i$ : described by a pressure  $p_i$ , a volume  $V_i$ , and a temperature  $T_i$ .
- The final state  $f$ : described by a pressure  $p_f$ , a volume  $V_f$ , and a temperature  $T_f$ .
- The procedure from its initial state to its final state is called a **thermodynamic process**.



# Summary of ideal gas processes

Table 18.1 Ideal-Gas Processes

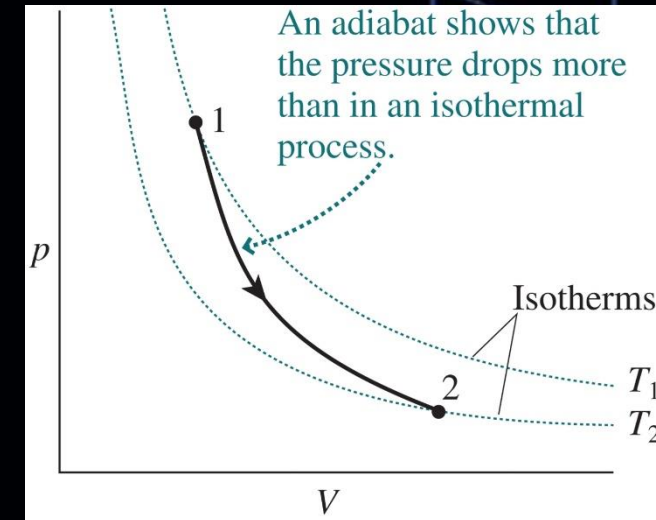
	ISOTHERMAL	CONSTANT-VOLUME	ISOBARIC	ADIABATIC
$pV$ diagram				
Defining characteristic	$T = \text{constant}$	$V = \text{constant}$	$p = \text{constant}$	$Q = 0$
First law	$Q = -W$	$Q = \Delta E_{\text{int}}$	$Q = \Delta E_{\text{int}} - W$	$\Delta E_{\text{int}} = W$
Work done on gas	$W = -nRT \ln\left(\frac{V_2}{V_1}\right)$	$W = 0$	$W = -p(V_2 - V_1)$	$W = \frac{p_2V_2 - p_1V_1}{\gamma - 1}$
Other relationships	$pV = \text{constant}$	$Q = nC_V\Delta T$	$Q = nC_p\Delta T$ $C_p = C_V + R$	$pV^\gamma = \text{constant}$ $TV^{\gamma-1} = \text{constant}$

# Work in an Adiabatic Process

- In an adiabatic process, we have  $pV^\gamma = p_1V_1^\gamma$ . Thus the work done on the gas can be found:

$$W = - \int_{V_1}^{V_2} p dV = - \int_{V_1}^{V_2} p_1 V_1^\gamma \frac{dV}{V^\gamma}$$
$$= -p_1 V_1^\gamma \int_{V_1}^{V_2} V^{-\gamma} dV$$

$$= p_1 V_1^\gamma \frac{V_2^{1-\gamma} - V_1^{1-\gamma}}{\gamma - 1} = \frac{p_2 V_2 - p_1 V_1}{\gamma - 1}$$





# Example: Bubble in isothermal process

- A scuba diver is 25m down, where the pressure is 3.5atm. The air she exhales forms bubbles 8.0mm in radius. How much work does each bubble do as it rises to the surface, assuming the bubbles remain at the uniform 300K of water.



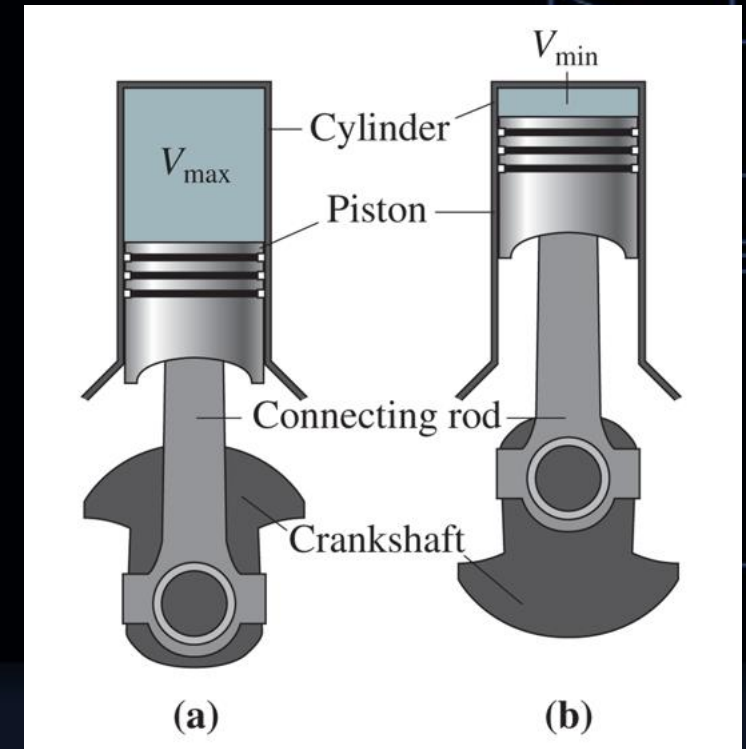
# Example: Bubble in isothermal process

- The process is isothermal process (temperature is constant). For ideal gas, the work done in isothermal process is:  $-W = nRT \ln\left(\frac{V_2}{V_1}\right)$
- From ideal gas law:  $pV = nRT$ , we can know that  $n$ ,  $R$ ,  $T$  are all constant for the bubble in isothermal process. We have  $pV = p\left(\frac{4}{3}\pi r^3\right) = nRT = \text{constant}$ . This means the volume will expand by a factor of 3.5 since the pressure is reduced from 3.5 atm to 1 atm.
- Therefore,  $-W = nRT \ln\left(\frac{V_2}{V_1}\right) = p \frac{4}{3}\pi r^3 \ln(3.5) = 0.95 \text{ J}$



# Example: an Adiabatic Process: Diesel Power

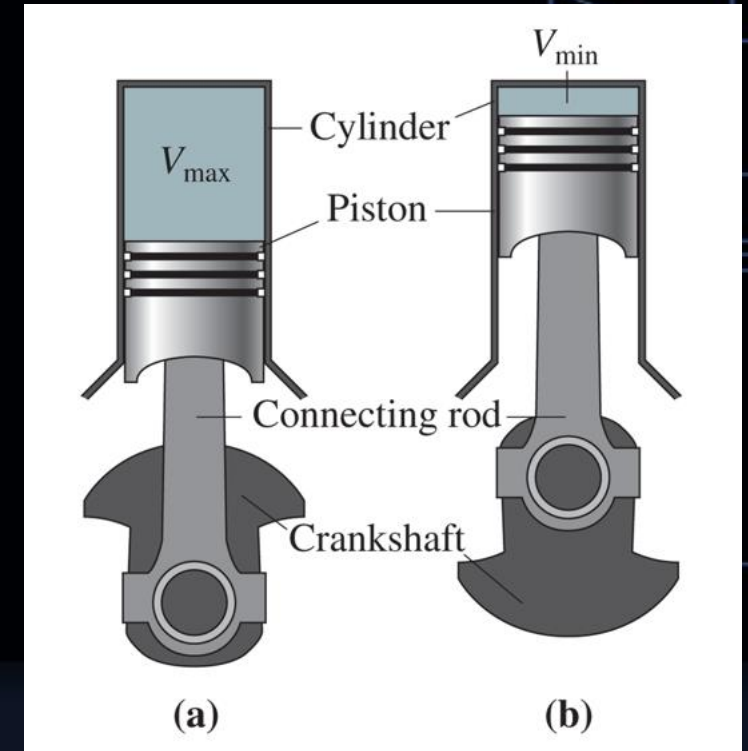
Fuel ignites in a diesel engine because of the temperature rise that results from compression as the piston moves toward the top of the cylinder; there's no spark plug as in a gasoline engine. Compression is fast enough that the process is essentially adiabatic. If the ignition temperature is  $500^{\circ}\text{C}$ , what compression ratio  $V_{\text{max}}/V_{\text{min}}$  is needed? Air's specific-heat ratio  $\gamma$  is 1.4 and before compression the air is at  $20^{\circ}\text{C}$ .



# Example: an Adiabatic Process: Diesel Power

Since the process is adiabatic, the gas follows  $T_{min}V_{min}^{\gamma-1} = T_{max}V_{max}^{\gamma-1}$ . Therefore, we can find out that:

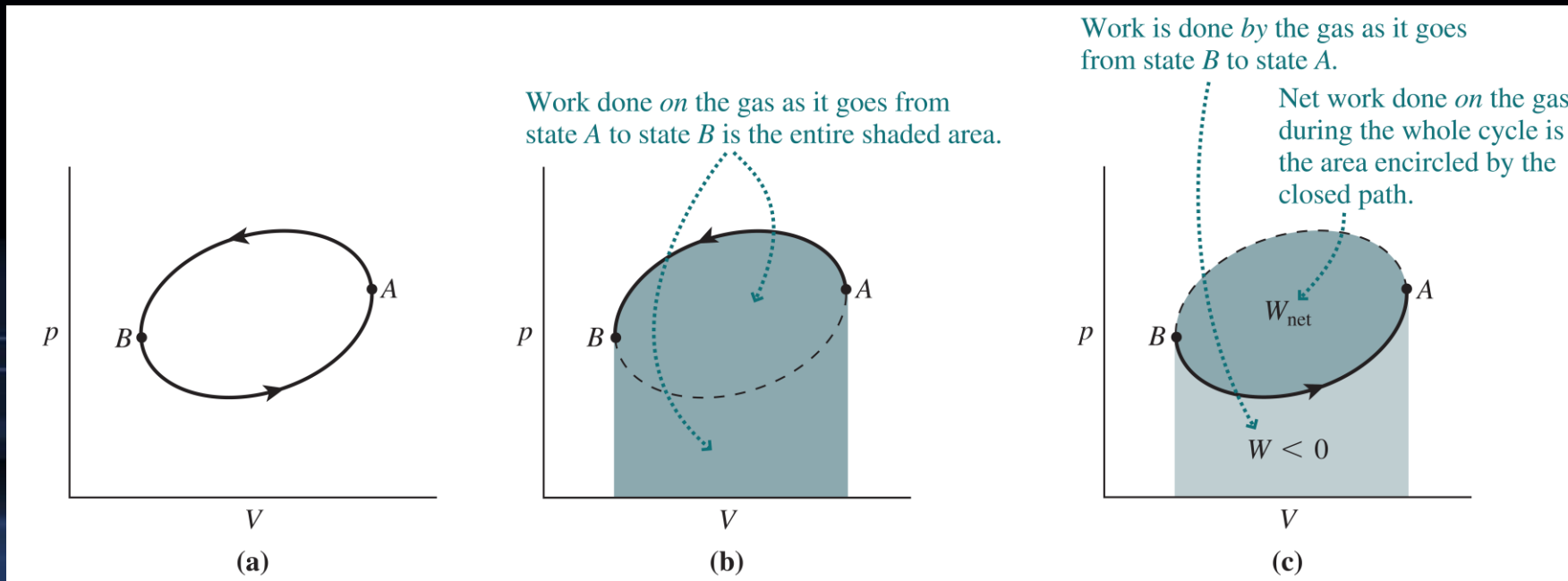
$$\frac{V_{max}}{V_{min}} = \left(\frac{T_{min}}{T_{max}}\right)^{\frac{1}{\gamma-1}} = \left(\frac{773K}{293K}\right)^{\frac{1}{0.4}} = 11$$



# Cyclic Processes

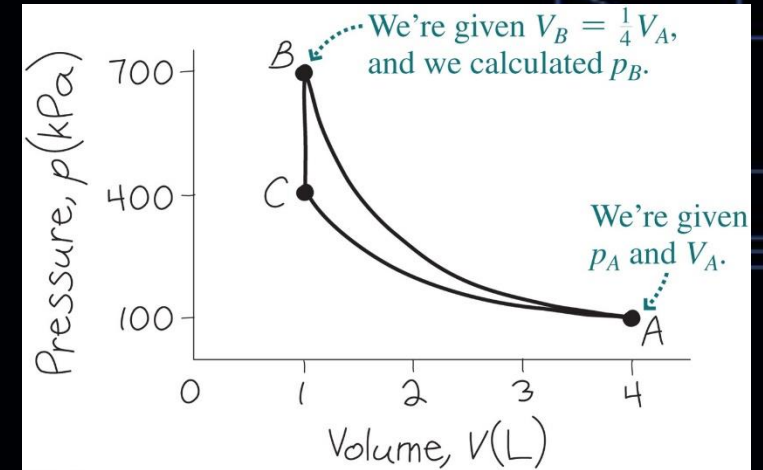
Cyclic processes combine the basic processes of other thermodynamic processes to take a system around a complete cycle and back to its starting state.

- Cyclic processes are important in technological systems like engines.
- The net work done on a gas in a cyclic process can be found from the area enclosed by the process curve in the  $pV$  diagram.



# Example: Finding the work of a cyclic process

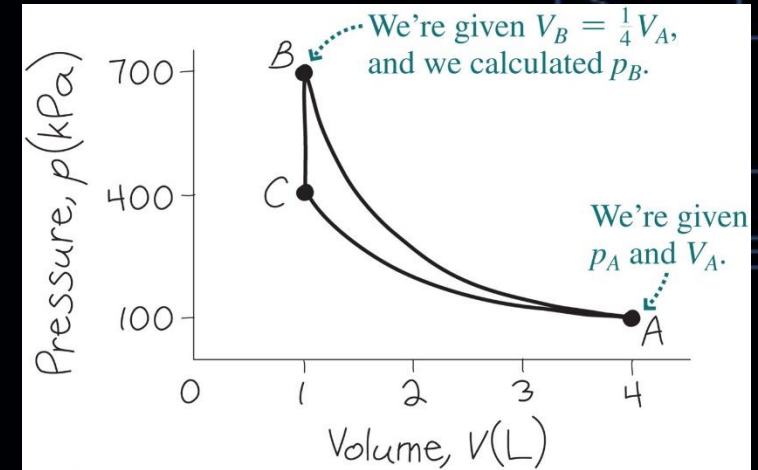
An ideal gas with  $\gamma = 1.4$  occupies 4.0L at 300K and 100 kPa pressure. It's compressed adiabatically to one-fourth of its original volume, then cooled at constant volume back to 300K, and finally allowed to expand isothermally to its original volume. How much work is done on the gas?



# Example: Finding the work of a cyclic process

- The cyclic process can be expressed as the path A-B-C-A in the right plot. The work done on the gas of this process is the area in the close loop. We need to find our work done in every process.
- It is adiabatic compression from A to B. We have  $p_A V_A^\gamma = p_B V_B^\gamma$ . So, we have

$$p_B = p_A (V_A/V_B)^\gamma = 696.4 \text{ kPa}$$
$$W_{AB} = \frac{p_B V_B - p_A V_A}{\gamma - 1} = 741 \text{ J}$$





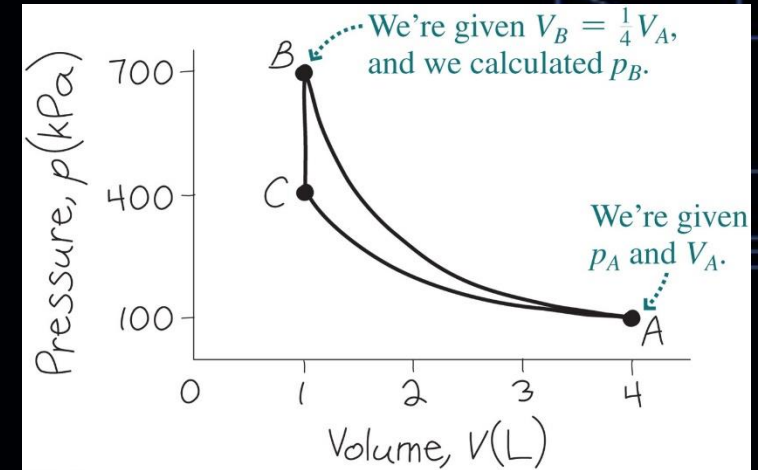
# Example: Finding the work of a cyclic process

- There is no work done from B to C
- It is isothermal expansion compression from C to A. We have:

$$\begin{aligned}W_{CA} &= -nRT \ln \left( \frac{V_A}{V_C} \right) = -p_A V_A \ln \left( \frac{V_A}{V_C} \right) \\ &= -(400J)(\ln 4) = -555J\end{aligned}$$

- Thus the total work done on the gas is

$$W_{ABCA} = 741J + 0J - 555J = 186J$$





# Specific heat of monatomic ideal gases

we are now able to derive an expression for the molar specific heat of monatomic ideal gas. There will be two cases:

- $C_V$  (specific heat of constant volume):

The volume of the gas remains constant as energy is transferred to or from it as heat.

- $C_p$  (specific heat of constant pressure):

The pressure of the gas remains constant as energy is transferred to or from it as heat.

# $C_V$ of Monatomic Ideal Gases

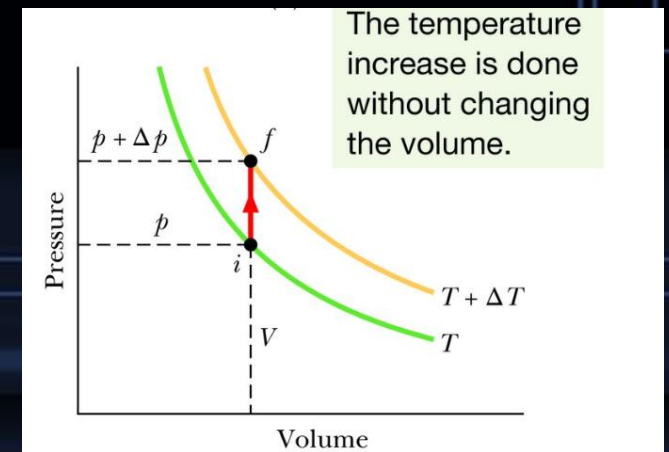
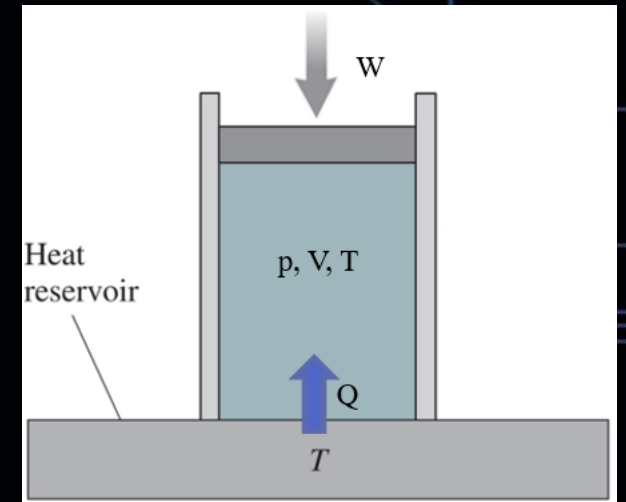
- During a constant volume process (red arrow), the heat absorbed:

$$Q = nC_V \Delta T \quad (\text{constant volume})$$

- Since there is no work done during this process, the heat absorbed is the change of internal energy (1<sup>st</sup> law of thermal dynamics). Thus:

$$C_V = \frac{\Delta E_{\text{int}}}{n \Delta T}$$

$$\Delta E_{\text{int}} = \frac{3}{2} nR \Delta T$$

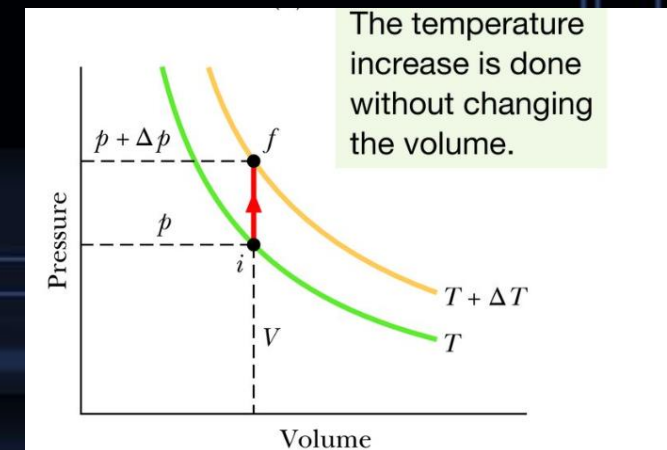
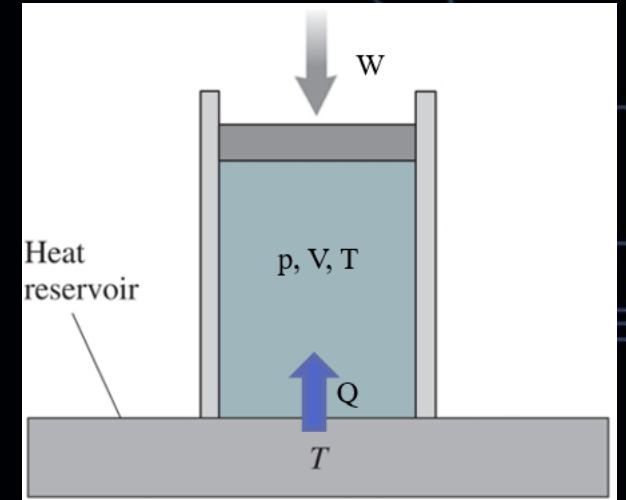


# $C_V$ of Monatomic Ideal Gases

- We can conclude that the molar specific heat at constant volume of monatomic ideal gas is:

$$C_V = \frac{3}{2}R = 12.5 \text{ J/mol} \cdot \text{K} \quad (\text{monatomic gas})$$

Molecule	Example	$C_V$ (J/mol · K)	
Monatomic	Ideal	$\frac{3}{2}R = 12.5$	
	Real	He	12.5
		Ar	12.6



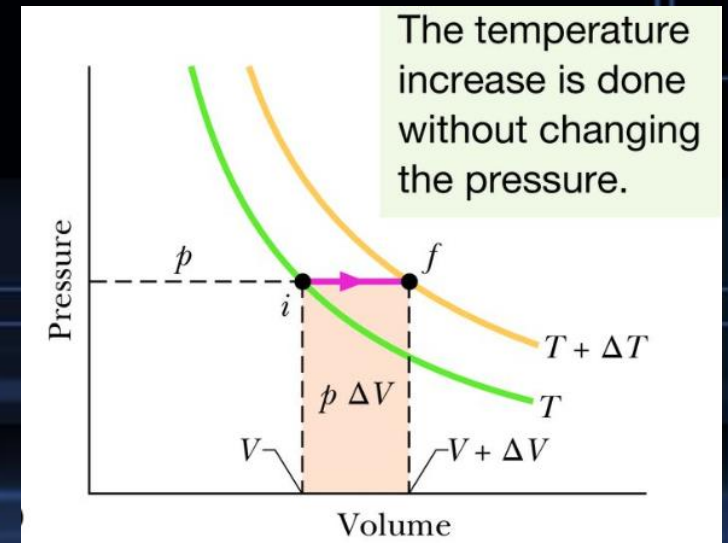
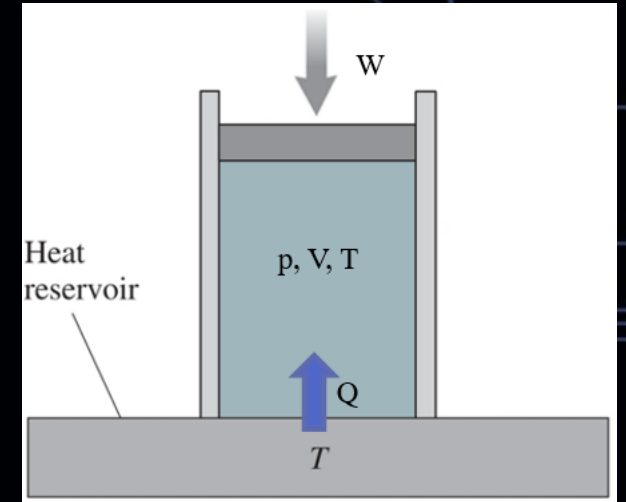
# Cp of Monatomic Ideal Gases

We now assume that the temperature of our ideal gas is increased by the same small amount  $\Delta T$  as previously but now the necessary energy (heat  $Q$ ) is added with the gas under constant pressure.

Thus  $Q = nC_p \Delta T$  (constant pressure)

With 1<sup>st</sup> law of thermal dynamics, we have

$$\Delta E_{\text{int}} = Q - W$$



# Cp of Monatomic Ideal Gases

With constant pressure, the gas does work:

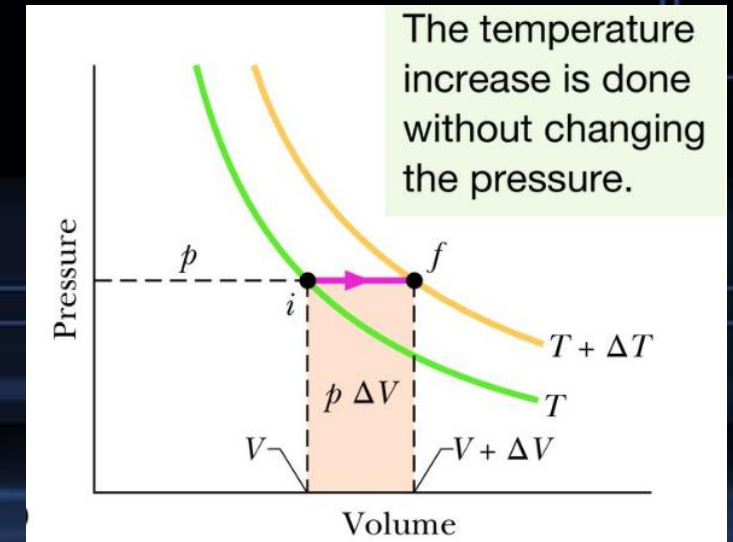
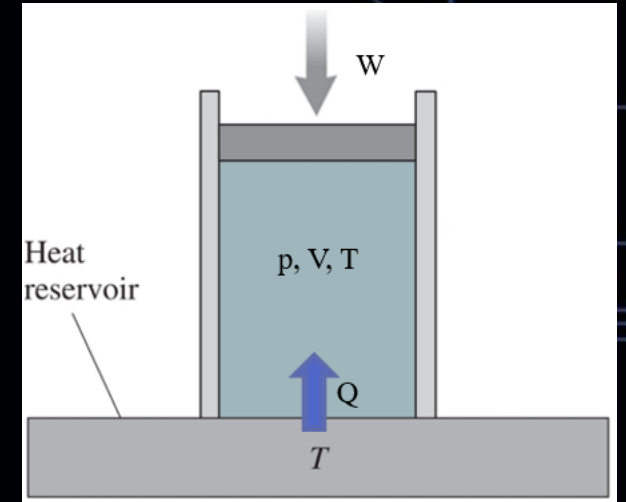
$$W = p \Delta V = nR \Delta T$$

And change of internal energy is:

$$\Delta E_{\text{int}} = nC_V \Delta T \quad (\text{ideal gas, any process})$$

Thus,  $nC_p \Delta T = nC_V \Delta T + nR \Delta T$  and

$$C_p = C_v + R = \frac{5}{2} R$$



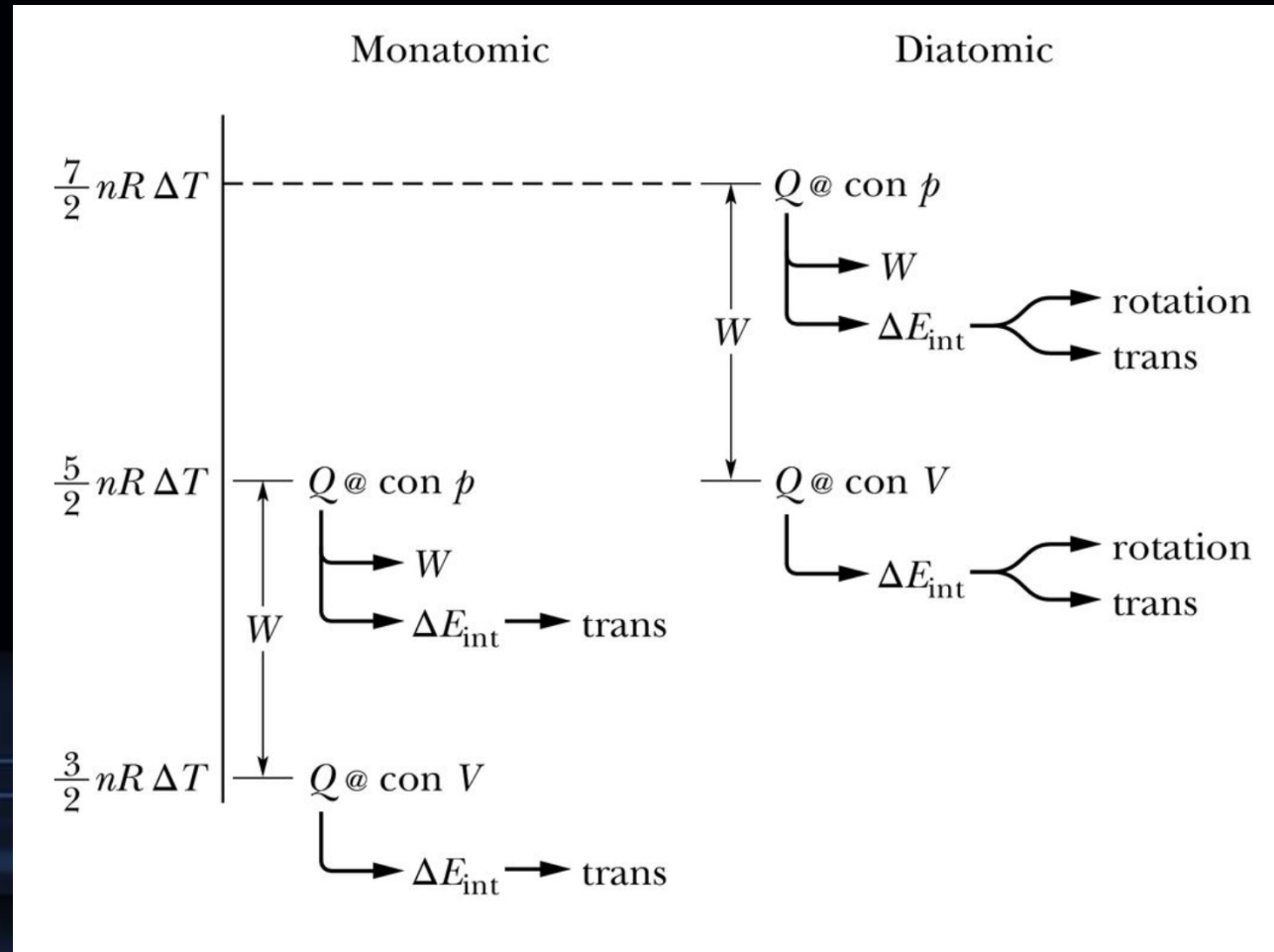
# $C_V$ of non-monatomic gas

Molecule	Example	$C_V$ (J/mol · K)	
Monatomic	Ideal	$\frac{3}{2}R = 12.5$	
	Real	He	12.5
		Ar	12.6

Diatomic	Ideal	$\frac{5}{2}R = 20.8$	
	Real	N <sub>2</sub>	20.7
		O <sub>2</sub>	20.8
Polyatomic	Ideal	$3R = 24.9$	
	Real	NH <sub>4</sub>	29.0
		CO <sub>2</sub>	29.7



# Monatomic vs Diatomic gas

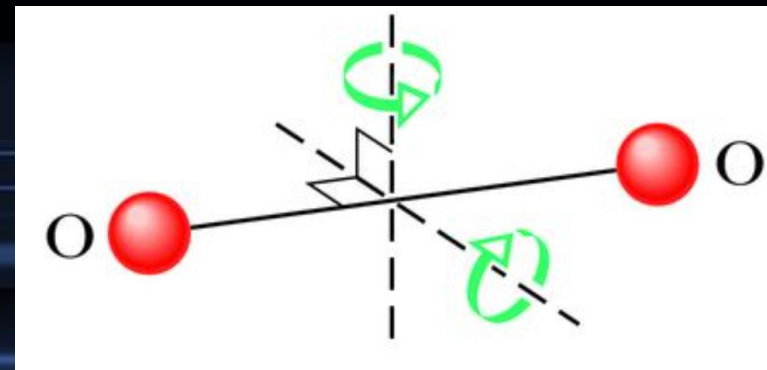
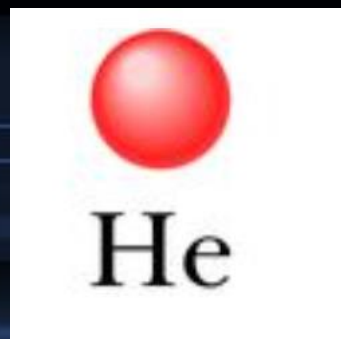


# Degree of freedom and molar specific heat

The type of motion of gas molecules can have:

- Helium(monatomic): translational motion in x, y, z direction: (3)
- Oxygen(diatomic): translational motion in x, y, z and rotational along x, and z direction: (5)

$E_{\text{int}} = (f/2)nRT$ , where f is the number of degrees of freedom



# Equipartition of energy

- To keep account of the various ways in which energy can be stored in a gas, James Clerk Maxwell introduced the theorem of the equipartition of energy:

Every kind of molecule has a certain number  $f$  of degrees of freedom, which are independent ways in which the molecule can store energy. Each such degree of freedom has associated with it—on average—an energy of  $\frac{1}{2}kT$  per molecule (or  $\frac{1}{2}RT$  per mole).

# Degree of freedom vs specific heat

		Degrees of Freedom			Predicted Molar Specific Heats	
Molecule	Example	Translational	Rotational	Total ( $f$ )	$C_V$ (Eq. 19-51)	$C_p = C_V + R$
Monatomic	He	3	0	3	$\frac{3}{2}R$	$\frac{5}{2}R$
Diatomic	O <sub>2</sub>	3	2	5	$\frac{5}{2}R$	$\frac{7}{2}R$
Polyatomic	CH <sub>4</sub>	3	3	6	$3R$	$4R$

## Example: Specific Heat of gas mixture

- A gas mixture consists of 2.0mol of oxygen ( $O_2$ ) and 1.0 mole of argon (Ar). Find the volume specific heat of the mixture.

## Example: Specific Heat of gas mixture

- A gas mixture consists of 2.0mol of oxygen (O<sub>2</sub>) and 1.0 mole of argon (Ar). Find the volume specific heat of the mixture.
- For O<sub>2</sub>, the degree of freedom of the molecule is 5:  
 $C_{v_{O_2}} = \frac{5}{2}R$ . Argon is monatomic molecule gas:  $C_{v_{Ar}} = \frac{3}{2}R$ .
- For the mixture we can find:

$$C_v = \frac{n_{O_2}C_{v_{O_2}} + n_{Ar}C_{v_{Ar}}}{n_{O_2} + n_{Ar}} = 2.2R$$



# Oscillational degree of freedom

- The  $C_v$  of oxygen actually depends on temperature. Shown in the plot is the  $C_v$  of  $H_2$  gas. At higher temperature there will be another  $R$  added in for  $C_v$ , which associated with two atom vibrating respect to COM.

