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|---|---|---|
| 14 | 12/16(Fri.) | **Thermal Behavior of Matter:** ideal gases, and kinetic theory of ideal gas |
| 15 | 12/20(Tue.) | **Thermal Behavior of Matter:** phase changes and thermal expansion |
| 15 | 12/23(Fri.) | **The First Law of Thermal Dynamics:** 1st law of thermal dynamics |
| 16 | 12/27(Tue.) | **The First Law of Thermal Dynamics:** Thermodynamic processes (Homework5) |

# GENERAL PHYSICS B1
# THE THERMAL BEHAVIOR OF MATTER

2022/12/16

Kinetic Theory of the Ideal Gas

# Topic Today

Kinetic Theory of Gases:

- Ideal gases Law
- Pressure
- Temperature
- Thermal speed of ideal gas
- Distribution of molecular speed

# Avogadro's Number

- All the materials are made out of atoms. Atoms form molecules, which is the smallest unit of the materials with the same chemical properties.
- Link between macroscopic mass and atomic mass: mole

One mole is the number of atoms in a 12 g sample of carbon-12.

- The number of molecules in a mole is:

$$N_A = 6.02 \times 10^{23} (Avogadro's\ number)$$

# Avogadro's Number

- The number of moles n contained in a sample of any substance is equal to the ratio of the number of molecules N in the sample to the number of molecules $N_A$ in 1 mol:

$$n = \frac{N}{N_{\mathrm{A}}}$$

- The number of moles n in a sample from the mass $M_{sam}$ of the sample and either the molar mass M (the mass of 1 mol) or the molecular mass m (the mass of one molecule)

$$n = \frac{M_{\mathrm{sam}}}{M} = \frac{M_{\mathrm{sam}}}{mN_{\mathrm{A}}}$$

# Ideal Gas Law

- For a gas that the density is low enough, the state of the gas can be described by:

$$pV = nRT \quad \text{(ideal gas law)}$$

p: the absolute pressure.

V: volume of the gas.

n: the number of moles of gas present.

T: the temperature in kelvins.

R: gas constant $R = 8.31\ \mathrm{J/mol \cdot K}$

# Ideal Gas Law

- A different form of idea gas law:

$$pV = NkT \quad \text{(ideal gas law)}$$

where:

N: the number of gas molecules.

k: Boltzmann constant

$$k = \frac{R}{N_A} = \frac{8.31\ \text{J/mol·K}}{6.02\times10^{23}\ \ \text{mol}^{-1}} = 1.38 \times 10^{-23}\ \text{J/K}$$

# Idea gas law

- The ideal gas law universally governs macroscopic properties of gases. It can be used to described a state of an gas
- All real gases approach the ideal state at low enough densities—that is, under conditions in which their molecules are far enough apart that they do not interact with one another.

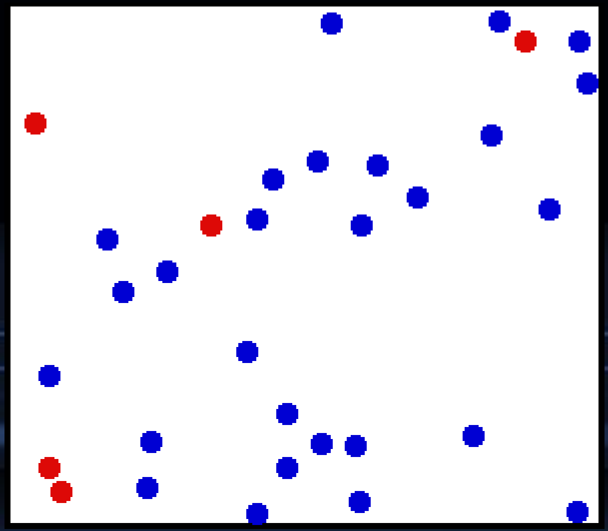
# Example of ideal gas

- A stainless-steel tank was filled with steam at a temperature of 110°C through a valve at one end. Then cool it down. Within less than a minute, the enormously sturdy tank was crushed.

$$pV = NkT \quad \text{(ideal gas law)}$$

# Molecular dynamics to macroscopic quantities

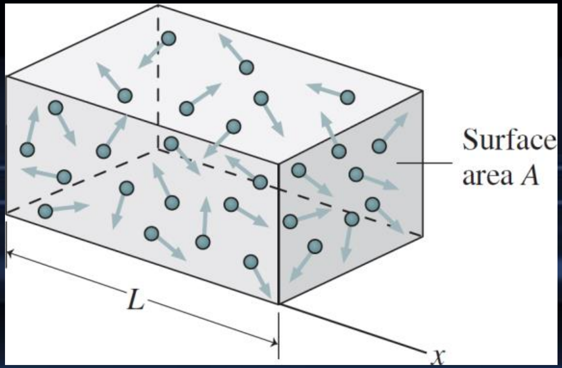
Let n moles of an ideal gas be confined in a cubical box of volume V ($V=L^3$). The walls of the box are held at temperature T. What is the connection between the pressure p exerted by the gas on the walls and the speeds of the molecules?

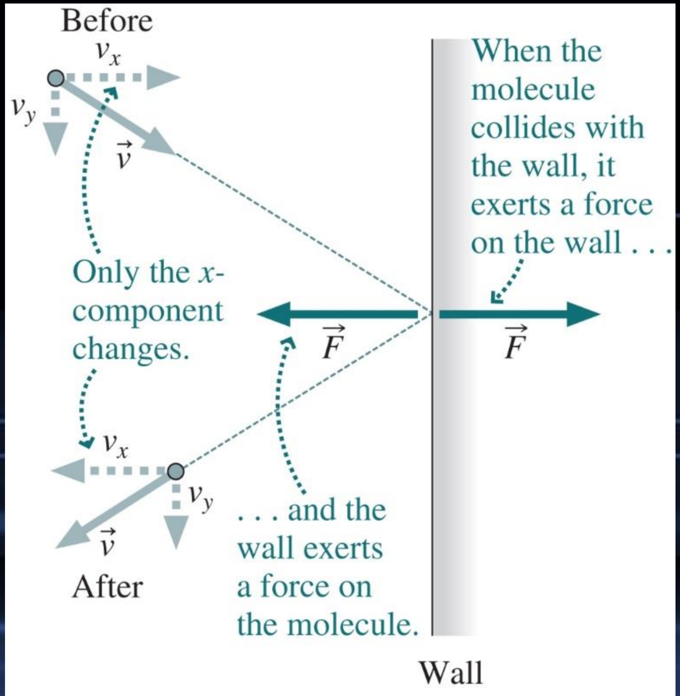


# Pressure due to change of momentum

- A typical gas molecule, of mass m and velocity $\vec{v}$ , that is about to collide with the shaded wall.
- The only change in the particle's momentum is along the x axis, and that change is

$$\Delta p_x = (-mv_x) - (mv_x) = -2mv_x$$

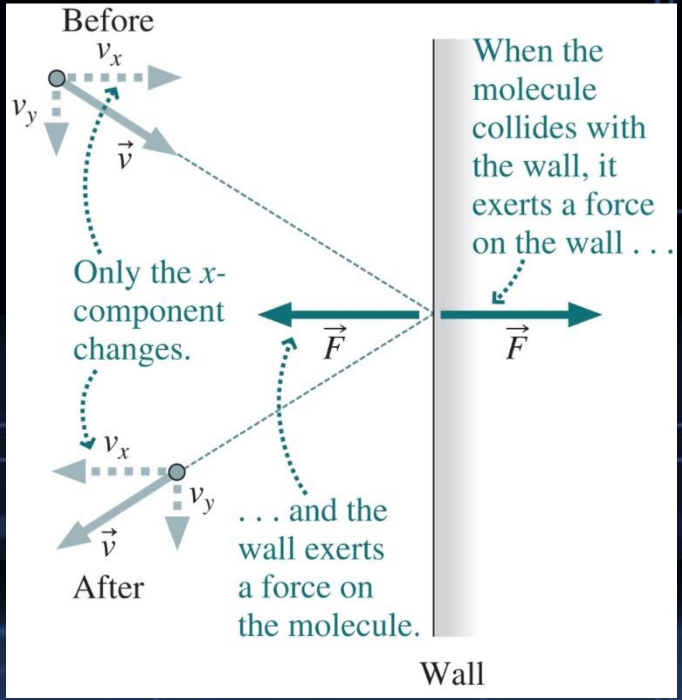


# Pressure due to change of momentum

- The time Δt between collisions is the time the molecule takes to travel to the opposite wall and back again (a distance 2L) at speed $v_x$. Thus, Δt is equal to $2L/v_x$.

$$\frac{\Delta p_x}{\Delta t} = \frac{2mv_x}{2L/v_x} = \frac{mv_x^2}{L}$$



# Pressure due to change of momentum

- From Newton’s second law, the rate at which momentum is delivered to the wall is the force acting on the wall. Using the expression for $\Delta p_x/\Delta t$, we can write this pressure p as at wall due to N molecules is:

$$
\begin{aligned}
p &= \frac{F_x}{L^2} = \frac{mv_{x1}^2/L + mv_{x2}^2/L + \cdots + mv_{xN}^2/L}{L^2} \\
&= \left(\frac{m}{L^3}\right)\left(v_{x1}^2 + v_{x2}^2 + \cdots + v_{xN}^2\right),
\end{aligned}
$$

# Pressure due to change of momentum

- There are N terms in the second set of parentheses. We can replace that quantity by $N\overline{v_x^2}$, where $\overline{v_x^2}$ is the average value of the square of the x components of all the molecular speeds. The total volume V is $L^3$. Thus, we have:

$$p = \frac{mN}{V}\overline{v_x^2}$$

# Equal contribution of average velocity from each directions

- For any molecule, $v^2 = v_x^2 + v_y^2 + v_z^2$. Because there are many molecules and they are all moving in random directions, the average values of the squares of their velocity components are equal, so that $\overline{v_x^2} = \frac{1}{3}\overline{v^2}$. Therefore:

$$pV = \frac{mN}{3}\overline{v^2}$$

# Average kinetic energy vs temperature

- We can further modify the previous result to be:

$$pV = \frac{mN}{3}\overline{v^2} = \frac{2}{3}N\left(\frac{1}{2}m\overline{v^2}\right)$$

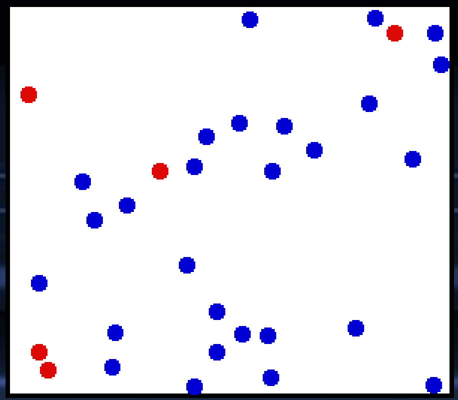
- Comparing to ideal gas law, we have:

$$\frac{2}{3}N\left(\frac{1}{2}m\overline{v^2}\right) = NkT$$

$$\Rightarrow \frac{1}{2}m\overline{v^2} = \frac{3}{2}kT$$

# Meaning of temperature for idea gas molecules

- At a given temperature T, all ideal gas molecules—no matter what their mass—have the same average translational kinetic energy—namely, $\frac{3}{2}kT$. When we measure the temperature of a gas, we are also measuring the average translational kinetic energy of its molecules' random motion.



# Thermal speed speed and temperature

- We introduce thermal speed or root-mean-square speed of the molecules and symbolized by $v_{th}$ or $v_{rms}$. And we have $v_{th} = \sqrt{\overline{v^2}}$.
- By the previous results, we can find

$$v_{th} = \sqrt{\frac{3kT}{m}}$$

- The microscopic quantity $v_{th}$ is linked to macroscopic quantity T

# Example of RMS speed

**Table 19-1 Some RMS Speeds at Room Temperature ($T$ = 300 K)**

| Gas | Molar Mass ($10^{-3}$ kg/mol) | $v_{rms}$ (m/s) |
|---|---|---|
| Hydrogen ($H_2$) | 2.02 | 1920 |
| Helium (He) | 4.0 | 1370 |
| Water vapor ($H_2O$) | 18.0 | 645 |
| Nitrogen ($N_2$) | 28.0 | 517 |
| Oxygen ($O_2$) | 32.0 | 483 |
| Carbon dioxide ($CO_2$) | 44.0 | 412 |
| Sulfur dioxide ($SO_2$) | 64.1 | 342 |

# Example

A gas mixture consists of molecules of Helium (He, atomic mass=4), Neon (Ne, atomic mass=20), and Argon (Ar, atomic mass=40). The gas is at a status of equilibrium. Rank the three types according to (a) average kinetic energy (b) Thermal speed

# Example

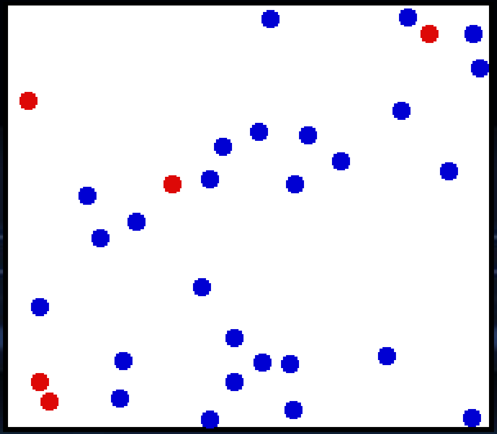
A gas mixture consists of molecules of Helium (He, atomic mass=4), Neon (Ne, atomic mass=20), and Argon (Ar, atomic mass=40). The gas is at a status of equilibrium. Rank the three types according to (a) average kinetic energy (b) Thermal speed

(a) The kinetic energy is the same($K_{He} = K_{Ne} = K_{Ar}$) because it is only related to temperature: $\frac{1}{2}m\overline{v^2} = \frac{3}{2}kT$

(b) The $V_{th,He} > V_{th,Ne} > V_{th,Ar}$ because $v_{th} = \sqrt{\frac{3kT}{m}}$

# The distribution of molecular speed

- The thermal speed $V_{th}$ of gas give us a general idea of typical molecular speed. However, Molecules in a gas exhibit a range of speeds that result from random collisions among the molecules. What is the distribution of the speed?



# Maxwell's speed distribution law

- In 1852, Scottish physicist James Clerk Maxwell first solved the problem of finding the speed distribution of gas molecules. His result, known as Maxwell's speed distribution law, is

$$P(v) = 4\pi \left( \frac{M}{2\pi RT} \right)^{3/2} v^2 \, e^{-Mv^2/2RT}$$

M:the molar mass of the gas,

R:the gas constant,

T: the gas temperature

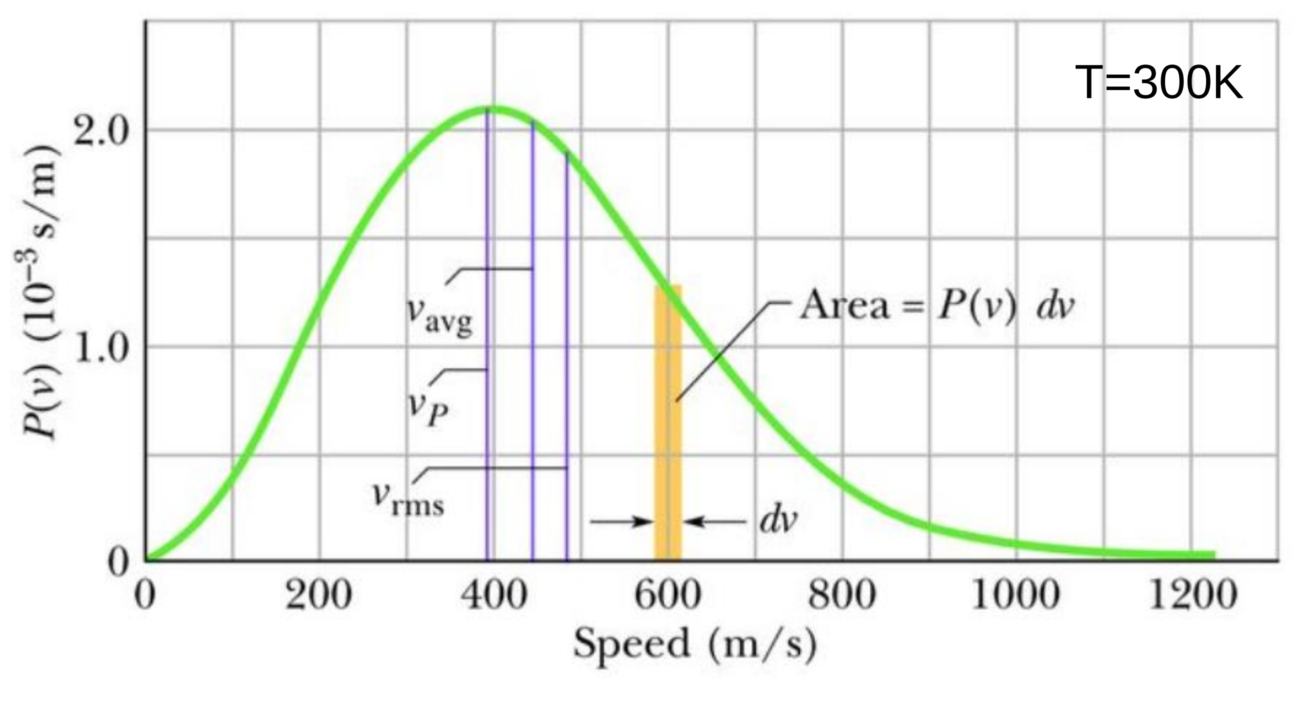
v :the molecular speed.

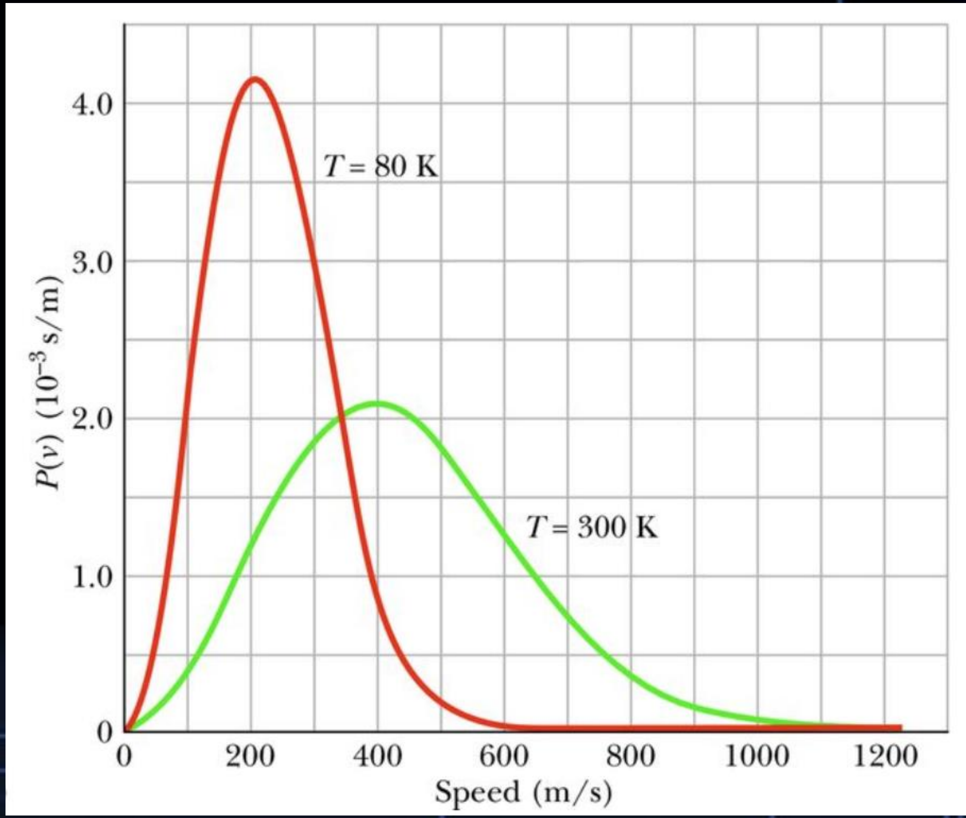
# Maxwell's speed distribution law

$$P\left(v\right) = 4\pi\left(\frac{M}{2\pi RT}\right)^{3/2}\ v^{2}\ e^{-Mv^{2}/2RT}$$

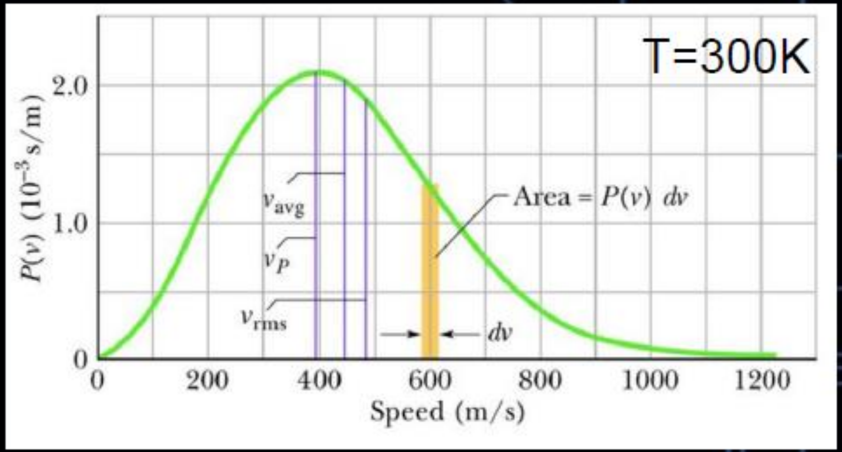
- The quantity P(v) is a probability distribution function: For any speed v, the product P(v) dv (a dimensionless quantity) is the fraction of molecules with speeds in the interval dv centered on speed v.

# Maxwell's speed distribution law



# Usage of speed distribution




- Normalization condition

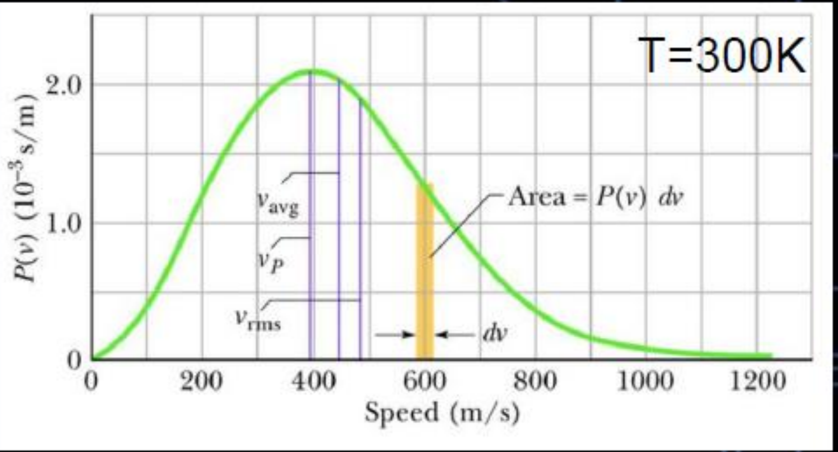
$$\int_0^\infty P(v)dv = 1$$

- fraction (frac) of molecules with speeds in an interval:

$$\text{frac} = \int_{v_1}^{v_2} P(v)\ dv$$

# Average speed of gas molecules

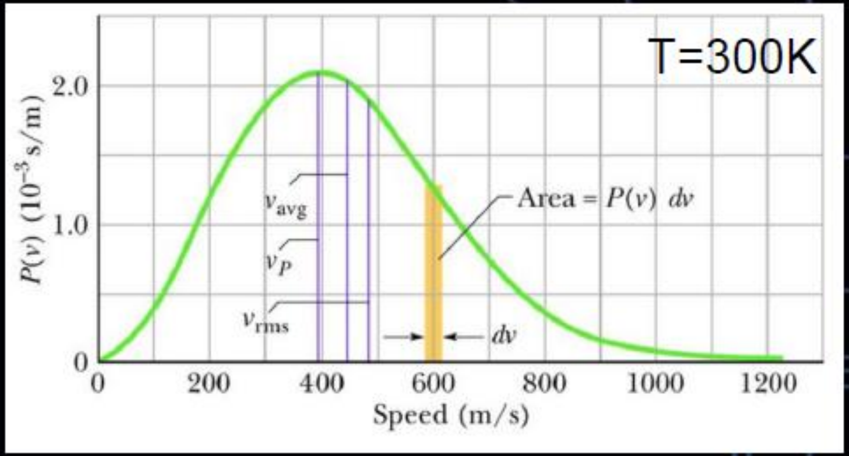
- In principle, we can find the average speed $v_{avg}$ of the molecules in a gas with the following procedure: We weight each value of v in the distribution




$$v_{\text{avg}} = \int_0^\infty v\,P(v)\,dv$$

$$v_{\text{avg}} = \sqrt{\frac{8RT}{\pi M}} \quad \text{(average speed)}$$

# RMS speed of gas molecules




- Similarly, we can find the average of the square of the speeds $(v^2)_{avg}$ with

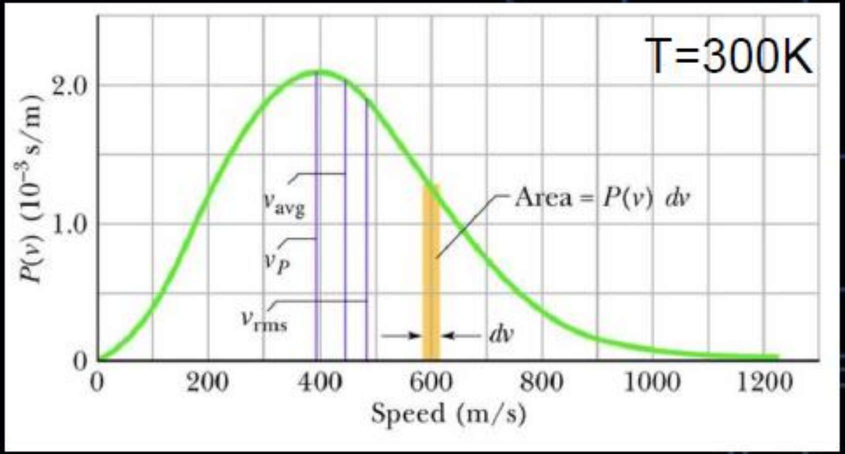
$$\left(v^2\right)_{\text{avg}} = \int_0^\infty v^2 \; P(v) \; dv.$$

- One can get:

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}} \quad \text{(rms speed)}$$

# Most possible speed

- The most probable speed $v_P$ is the speed at which P(v) is maximum. To calculate $v_P$, we set dP/dv = 0




$$v_P = \sqrt{\frac{2RT}{M}} \quad \text{(most probable speed)}$$