

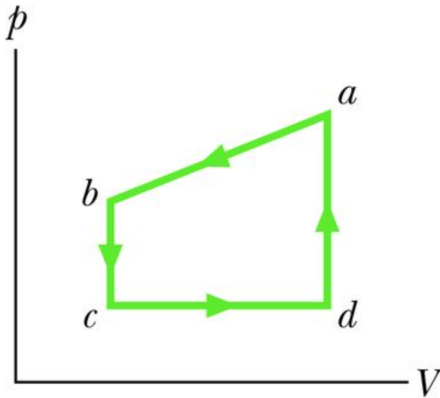
## General Physics B1 - Homework Set 5

Due on 01/03/2023, 5:00PM sharp. Please hand in your homework via eLearn.

1 points for each problem. Total:5 points

### 1. The First Law of Thermodynamics

The bottom figure represents a closed cycle for a gas (the figure is not drawn to scale). The change in the internal energy of the gas as it moves from a to c along the path abc is  $-200\text{ J}$ . As it moves from c to d,  $180\text{ J}$  must be transferred to it as heat. An additional transfer of  $80\text{ J}$  to it as heat is needed as it moves from d to a. How much work is done on the gas as it moves from c to d?



Solution:

Since process abcda is a close cycle, the change of internal energy is zero. Given the internal energy change of process abc is  $-200\text{ J}$ , we can conclude that the internal energy change of process cda is  $200\text{ J}$ . In process c to d, assume the work done on the gas is  $W_{cd}$  and the heat absorbed by gas is  $180\text{ J}$ . In process d to a, the work done is zero (no volume change) and the heat absorbed by gas is  $80\text{ J}$ .

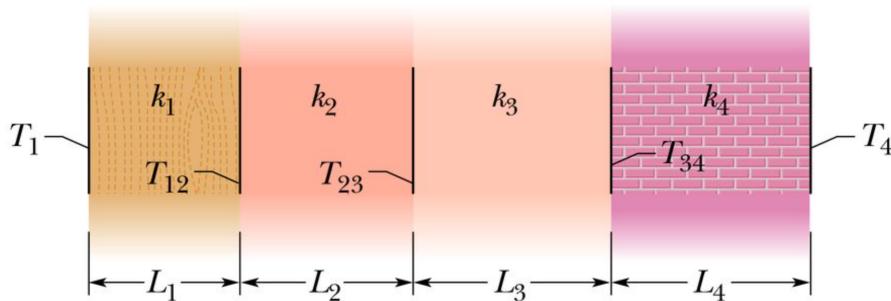
According to the first law of thermodynamics:  $\Delta E = Q + W$ , we can write down:

$$\Delta E_{cda} = 200\text{ J} = Q_{cd} + Q_{da} + W_{cd} + W_{da} = 180\text{ J} + 80\text{ J} + W_{cd} + 0\text{ J}$$

$$\text{Thus } W_{cd} = 200\text{ J} - 260\text{ J} = -60\text{ J} \text{ (Answer)}$$

### 2. Thermal Conduction of Multilayer Insulation

As in the figure, a wall consisting of four layers, with thermal conductivities  $k_1 = 0.060\text{ W/m}\cdot\text{K}$ ,  $k_3 = 0.040\text{ W/m}\cdot\text{K}$ , and  $k_4 = 0.12\text{ W/m}\cdot\text{K}$  ( $k_2$  is not known). The layer thicknesses are  $L_1 = 1.5\text{ cm}$ ,  $L_3 = 2.8\text{ cm}$ , and  $L_4 = 3.5\text{ cm}$  ( $L_2$  is not known). The known temperatures are  $T_1 = 30^\circ\text{C}$ ,  $T_12 = 25^\circ\text{C}$ , and  $T_4 = -10^\circ\text{C}$ . Energy transfer through the wall is steady. What is interface temperature  $T_{34}$ ?



Assuming the conduction rate of each section is  $P_1, P_2, P_3, P_4$ . Since the heat flow is steady, then we have  $P_1 = P_2 = P_3 = P_4$ .

Assuming the unit area of the wall is  $A$ , according to thermal conduction formula:

$$k_1 \frac{A}{L_1} (T_1 - T_{12}) = k_2 \frac{A}{L_2} (T_{12} - T_{23}) = k_3 \frac{A}{L_3} (T_{23} - T_{34}) = k_4 \frac{A}{L_4} (T_{34} - T_4).$$

We can further simplified it:

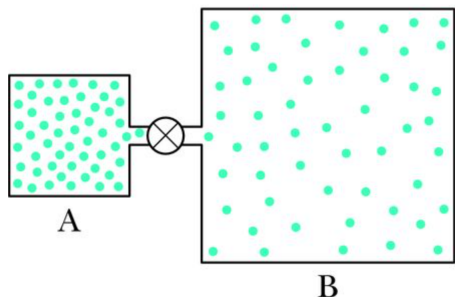
$$\frac{k_1}{L_1}(T_1 - T_{12}) = \frac{k_4}{L_4}(T_{34} - T_4)$$

Thus  $\frac{0.060(W/m \cdot K)}{0.015(m)}(30 - 25) = \frac{0.12(W/m \cdot K)}{0.035(m)}(T_{34} + 10)$

One can find  $T_{34} = -4.16^\circ C$  (Answer)

### 3. Ideal Gas

Container A in the following figure holds an ideal gas at a pressure of  $5.0 \times 10^5$  Pa and a temperature of 300 K. It is connected by a thin tube (and a closed valve) to container B, with four times the volume of A. Container B holds the same ideal gas at a pressure of  $1.0 \times 10^5$  Pa and a temperature of 400 K. The valve is opened to allow the pressures to equalize, but the temperature of each container is maintained. What then is the pressure?



Solution

Before the valve open, the ideal gases in the two containers follows:

$$p_A V_A = n_A R T_A \text{ and } p_B V_B = n_B R T_B$$

From the given condition, we have  $p_A = 5.0 \times 10^5 Pa$ ,  $p_B = 1.0 \times 10^5 Pa = \frac{1}{5} p_A$ ,  $T_A = 300K$ ,  $T_B = 400K = \frac{4}{3} T_A$ , and  $V_B = 4V_A$ .

$$\text{Thus, we have } n_A + n_B = \frac{p_A V_A}{R T_A} + \frac{p_B V_B}{R T_B} = \frac{p_A V_A}{R T_A} + \frac{3}{5} \frac{p_A V_A}{R T_A} = \frac{8}{5} \frac{p_A V_A}{R T_A}.$$

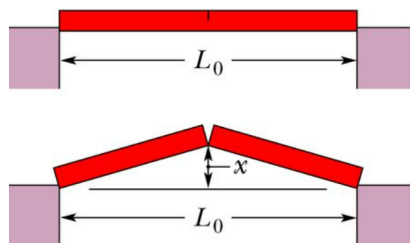
After the valve is opened, the container will reach new pressure  $p'$  for both side while the volume, temperature, and the total number of molecules are the same.

$$\text{Therefore, } \frac{p' V_A}{R T_A} + \frac{p' V_B}{R T_B} = p' \left( \frac{V_A}{R T_A} + \frac{4 V_A}{R \frac{4}{3} T_A} \right) = p' \left( 4 \frac{V_A}{R T_A} \right) = n_A + n_B = \frac{8}{5} \frac{p_A V_A}{R T_A}.$$

$$\text{Then } p' = \frac{2}{5} p_A = 2.0 \times 10^5 Pa \text{ (Answer)}$$

### 4. Thermal expansion

As a result of a temperature rise of  $32^\circ C$ , a bar with a crack at its center buckles upward as shown in the following figure. The fixed distance  $L_0$  is 3.77 m and the coefficient of linear expansion of the bar is  $25 \times 10^{-6}/^\circ C$ . Find the rise  $x$  of the center.



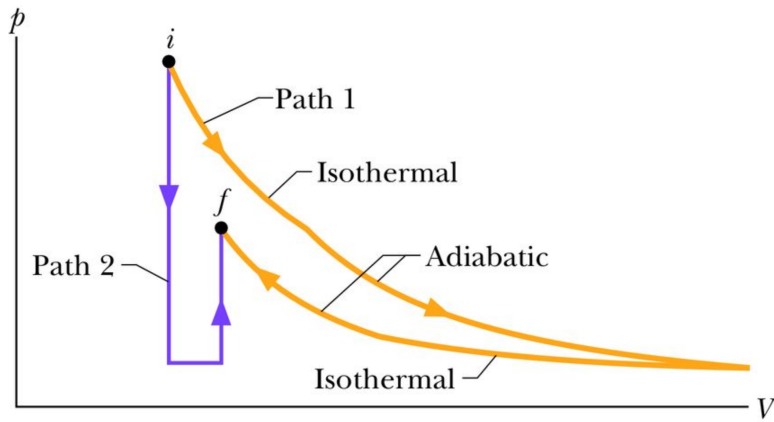
Solution:

After the temperature rise of  $32^\circ C$ , the new total length is  $L' = \alpha L_0 \Delta T$

$$\text{Thus, } x = \sqrt{\left(\frac{L'}{2}\right)^2 - \left(\frac{L_0}{2}\right)^2} = \frac{L_0 \sqrt{(1 + \alpha \Delta T)^2 - 1}}{2} = 7.5 cm \text{ (Answer)}$$

### 5. The Adiabatic and Isothermal Expansion of an Ideal Gas

The following figure, shows two paths that may be taken by a gas from an initial point i to a final point f. Path 1 consists of an isothermal expansion (work is 50 J in magnitude), an adiabatic expansion (work is 40 J in magnitude), an isothermal compression (work is 30 J in magnitude), and then an adiabatic compression (work is 25 J in magnitude). What is the change in the internal energy of the gas if the gas goes from point i to point f along path 2?



Solution

Since internal energy only depends temperature, the change in the internal energy of the gas goes from point i to point f along path 2 will be the same along path one. For path one, there is no change in internal energy in all the isothermal process and the work is equal to heat according to the first law of thermal dynamics. There is heat exchange in all the adiabatic process and the work is equal to change of internal energy. Thus, only the two adiabatic processes have change in internal energy and therefore  $\Delta E_{i \rightarrow f} = (-40J) + (+25J) = -15J$  (Answer)