Course announcement

- Homework set 3 will be posted on eLearn today. It will be due on 11/18 Friday at 5PM.
- Solution of homework set 3 will be posted tonight.

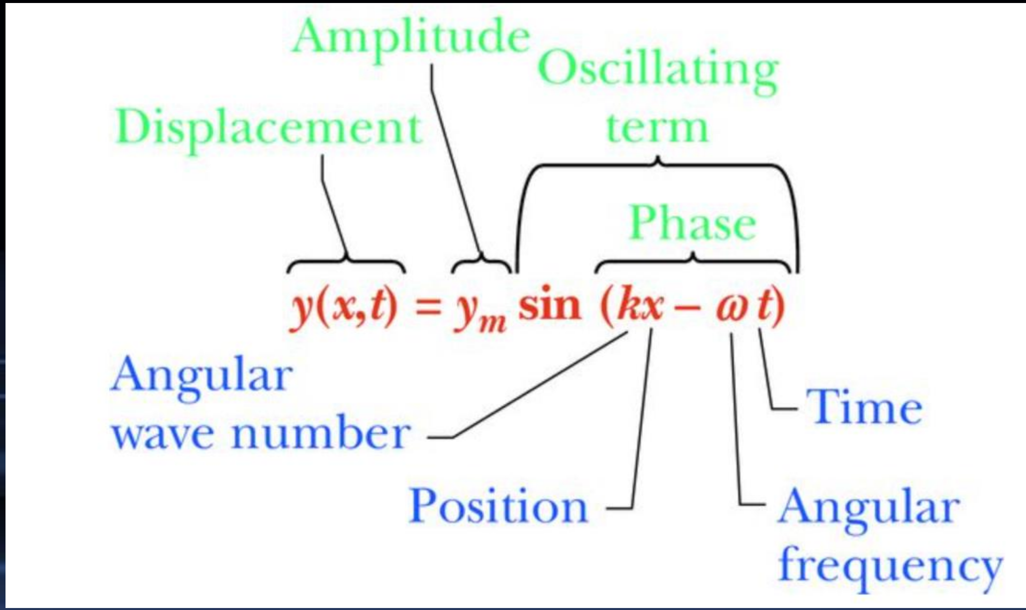
10	11/15(Tue.)	**Oscillation and Waves:** description of waves
10	11/18(Fri.)	**Oscillation and Waves:** interference of waves
11	11/22(Tue.)	**Oscillation and Waves:** propagation of waves
11	11/25(Fri.)	**Fluid Motion:** Density, Pressure, and Hydrostatic Equilibrium (Homework4)

GENERAL PHYSICS B1
OSCILLATION & WAVE

Interference of Waves
2022/11/11

Description of wave

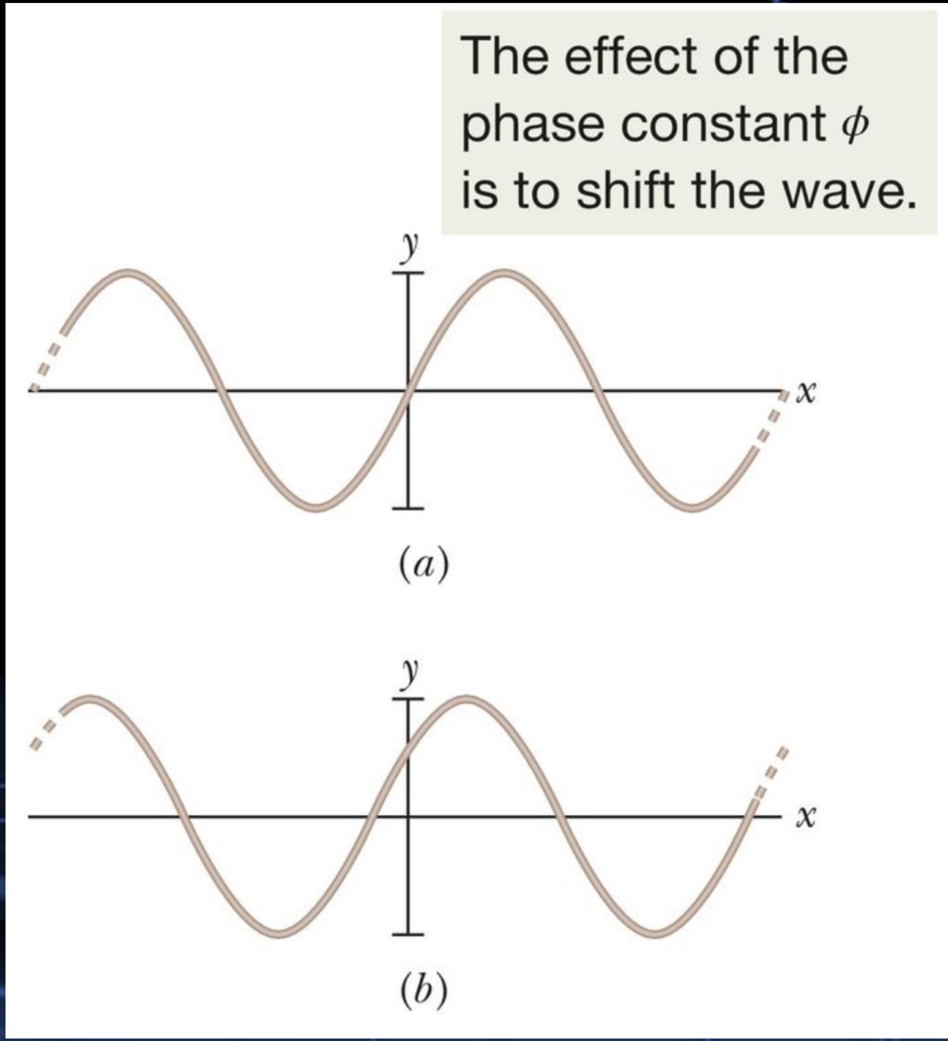
- Use wave on a string as an example. Imagine a sinusoidal wave traveling in the positive direction of an x axis. The elements oscillate parallel to the y axis. At time t, the displacement y(x,t) of the element located at position x is given by



The phase constant

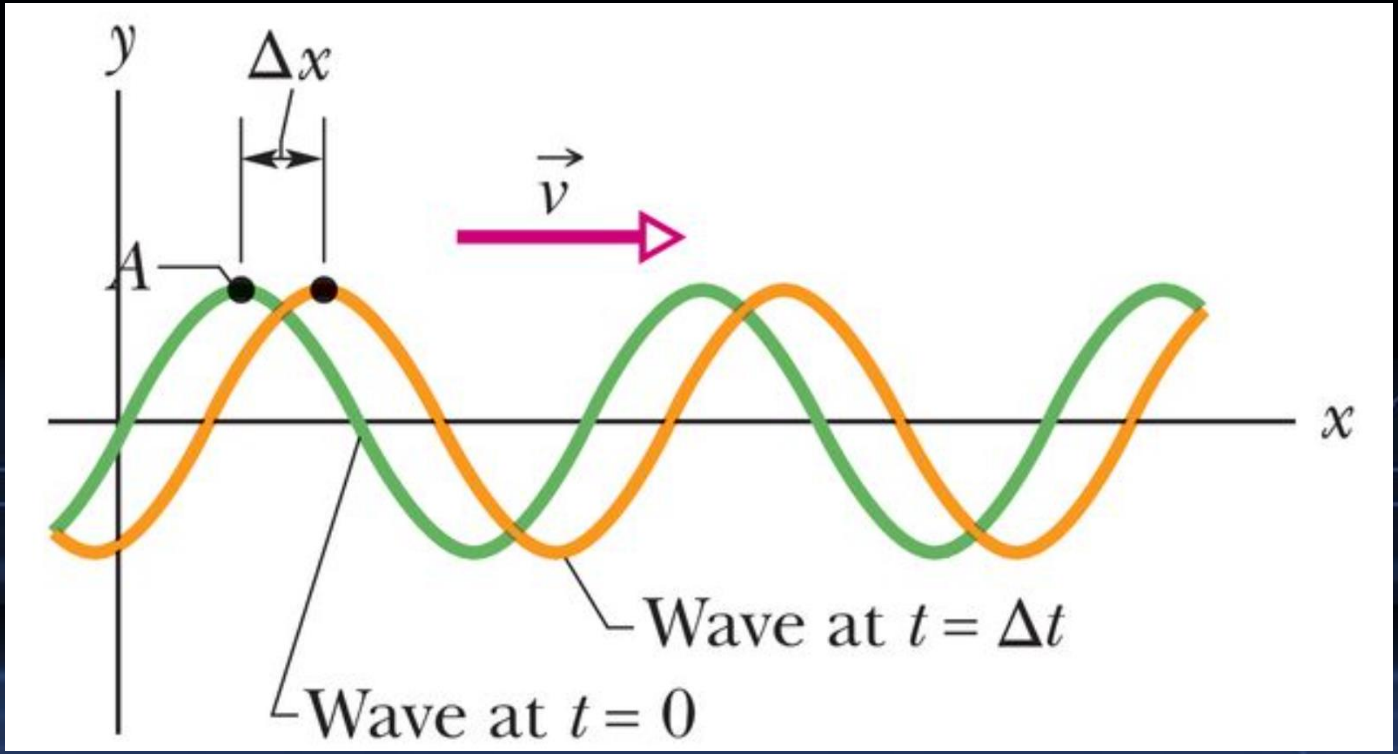
- A value of ϕ can be chosen so that the function gives some other displacement and slope at x = 0 when t = 0.

$$y = y_m \sin(kx - \omega t + \phi)$$



The speed of a traveling wave

- If point A retains its displacement as it moves, the phase must remain a constant: $kx - \omega t = constant$.
- Thus the wave speed is $\frac{dx}{dt} = v = \frac{\omega}{k} = \frac{\lambda}{T} = \lambda f$



Direction of wave propagation

- With the concept of wave speed, one can find that:

$$y(x,t) = y_m \sin(kx - \omega t + \phi)$$

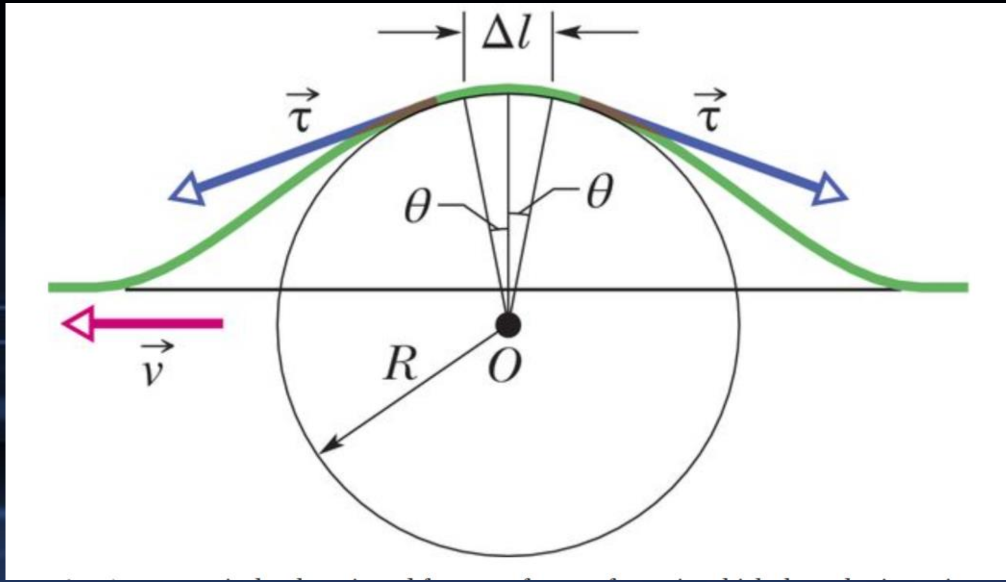
is a wave traveling to positive x direction with positive wave speed $v = \frac{\omega}{k}$.

$$y(x,t) = y_m \sin(kx + \omega t + \phi)$$

is a wave traveling to negative x direction with negative wave speed $v = -\frac{\omega}{k}$.

Wave speed on a stretched string

- $v = \sqrt{\frac{\tau}{\mu}}$, which is the wave speed of the stretched string.
- The power of total energy transfer is $P_{avg} = \frac{1}{2}\mu v {y_m}^2 \omega^2$



Today’s topic

- Wave equation
- Interference of wave
- Standing wave

The wave equation

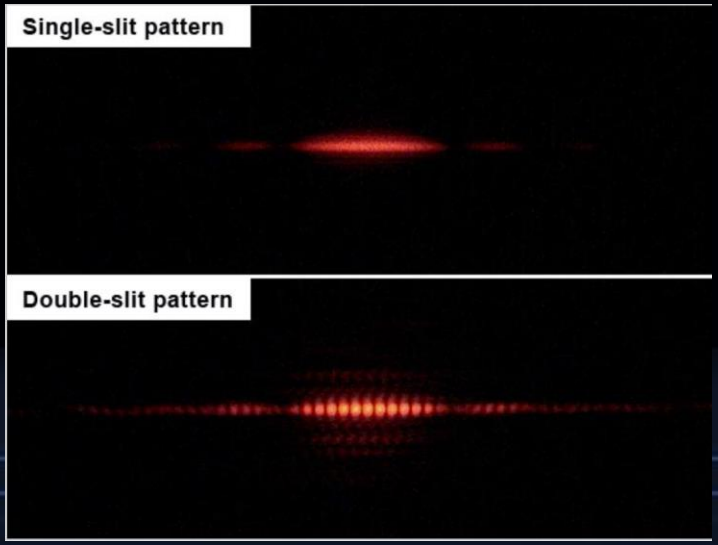
- By applying Newton's second law to the element's motion, we can derive a general differential equation, called the wave equation, that governs the travel of waves of any type:

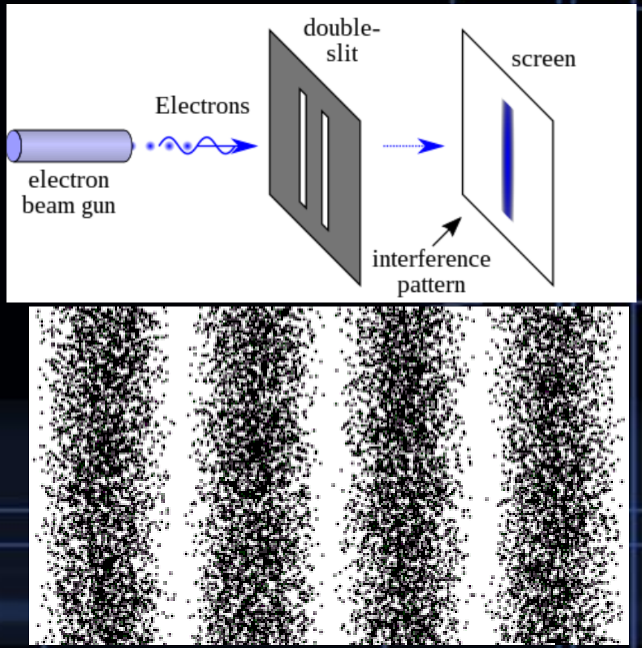
$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

- The description $y(x,t) = y_m \sin(kx - \omega t + \phi)$ is the solution of this general differential equation.

Interference of wave

- It often happens that two or more waves pass simultaneously through the same region and have interference to each others





https://gfycat.com/gifs/search/destructive+waves

https://en.wikipedia.org/wiki/Double-slit_experiment

Principle of superposition

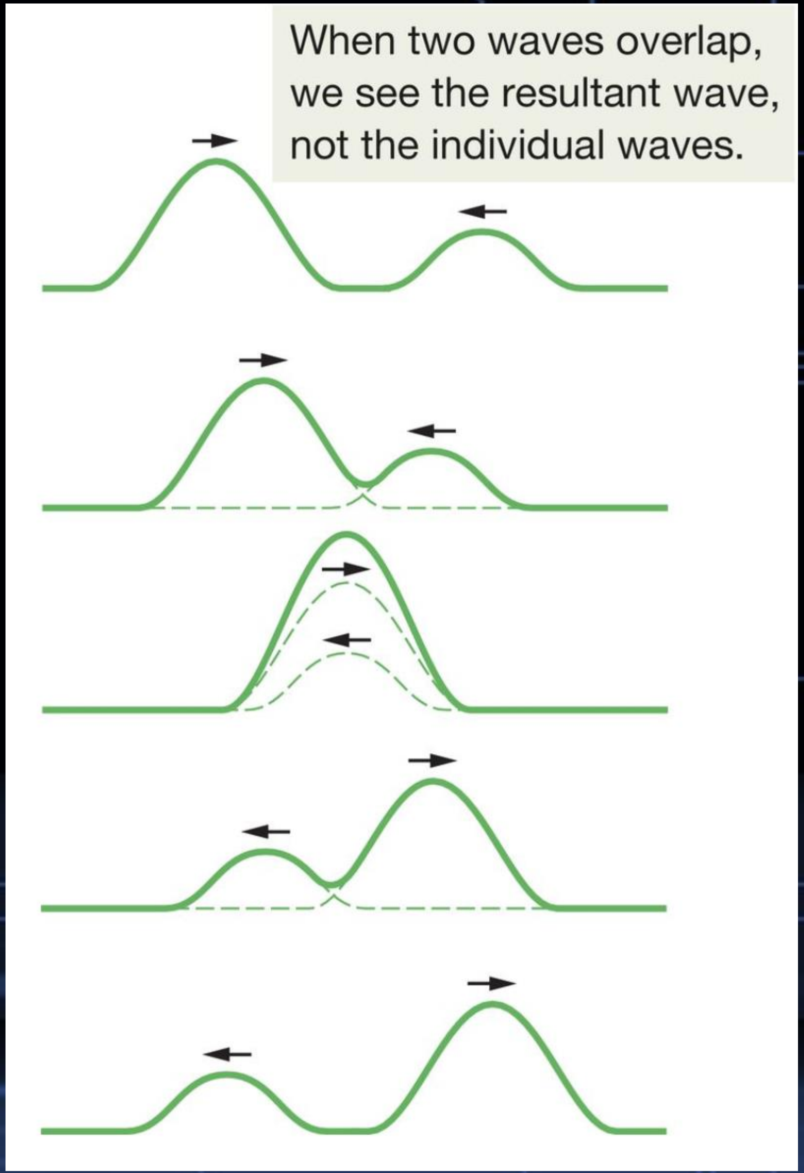
- Principle of superposition: when several effects occur simultaneously, their net effect is the sum of the individual effects.
- Suppose that two waves travel simultaneously along the same stretched string. Let y1(x, t) and y2(x, t) be the displacements that the string would experience if each wave traveled alone. The displacement of the string when the waves overlap is then the algebraic sum:

$$y'(x,t) = y_1(x,t) + y_2(x,t)$$

Principle of superposition

$$y'(x,t) = y_1(x,t) + y_2(x,t)$$

- Overlapping waves algebraically add to produce a resultant wave (or net wave).

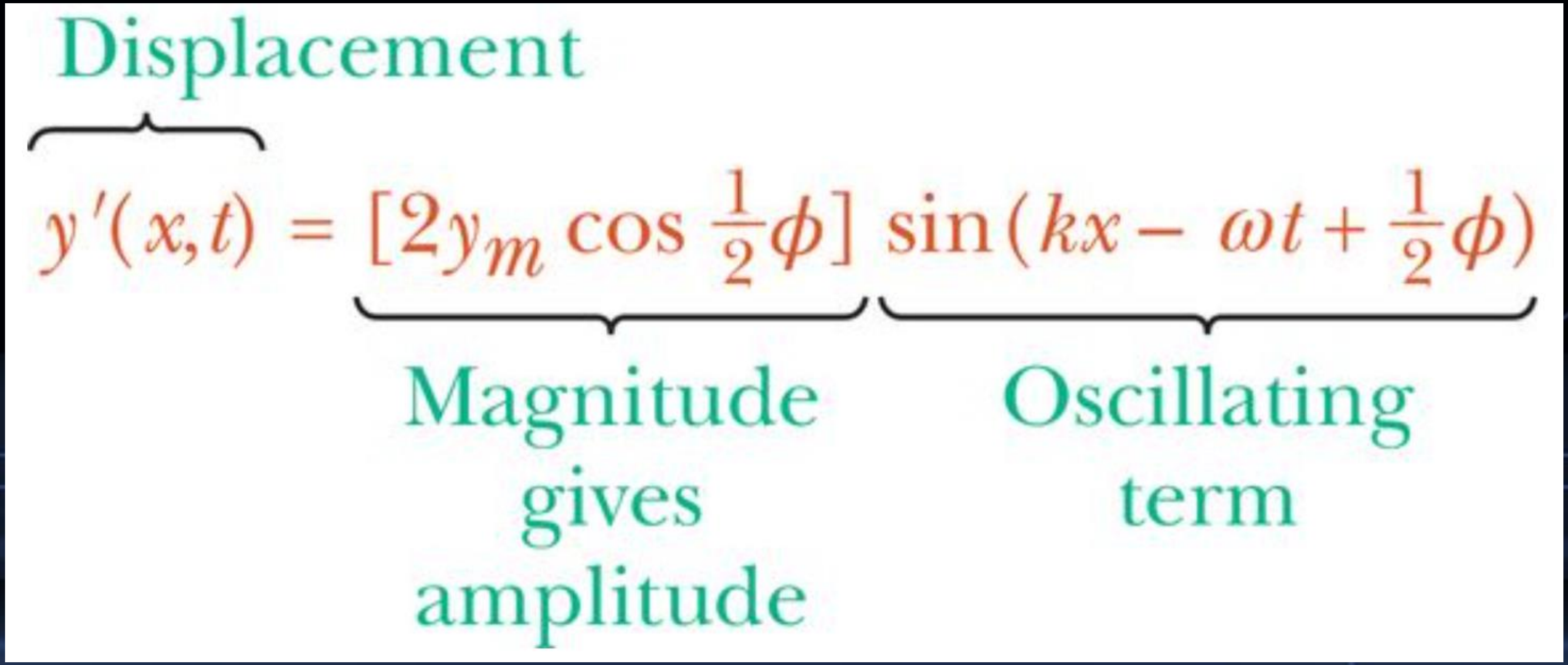


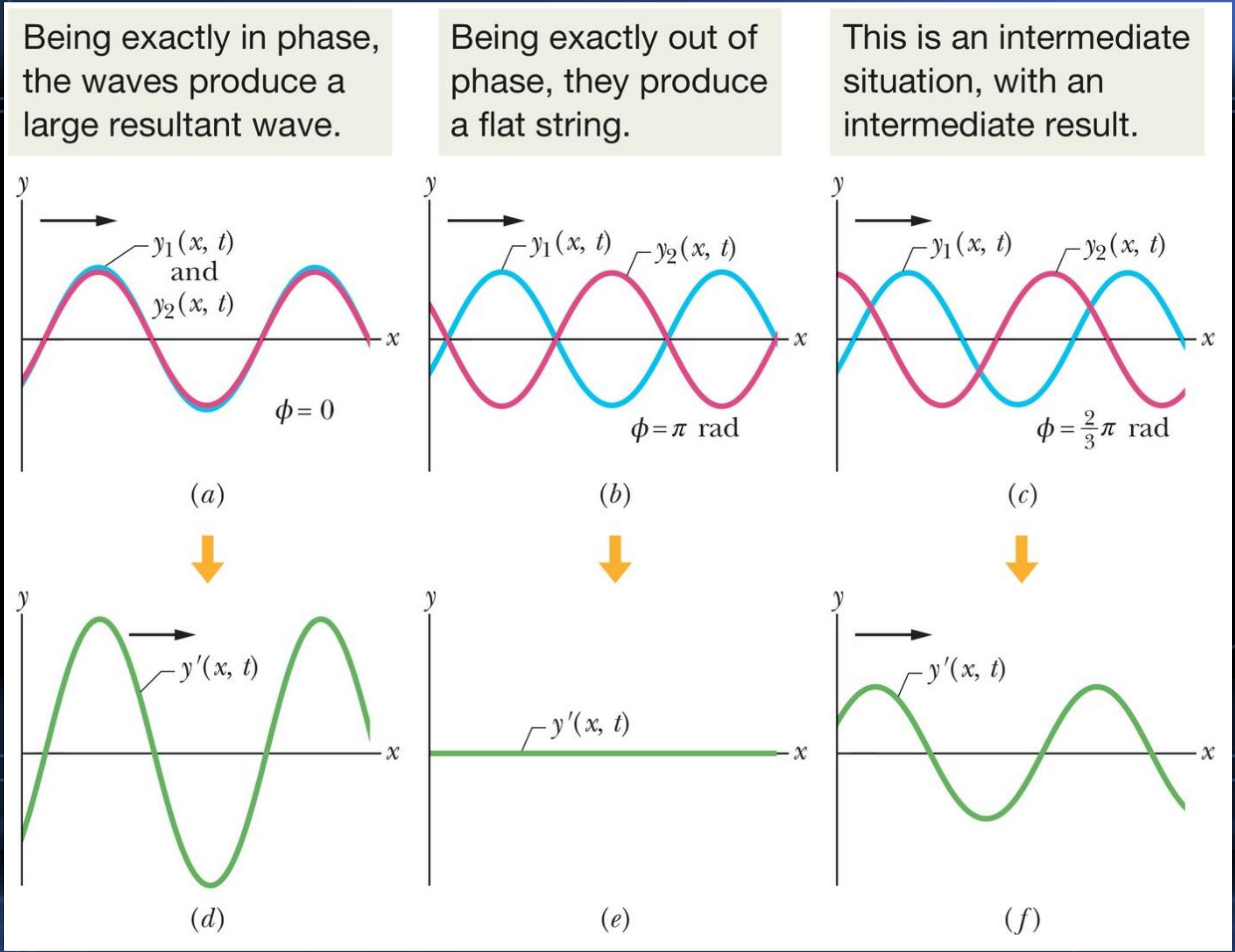
Math description: interference of waves

- Let one wave traveling along a stretched string be given by $y_1(x,t) = y_m \sin(kx - \omega t)$.
- Another wave is $y_2(x,t) = y_m \sin(kx - \omega t + \phi)$
- They differ only by a constant angle ϕ, the phase constant. These waves are said to be out of phase by ϕ or to have a phase difference of ϕ, or one wave is said to be phase-shifted from the other by ϕ.

Math description: interference of waves

- The resultant wave: $y'(x,t) = y_1(x,t) + y_2(x,t) = y_m \sin(kx - \omega t) + y_m \sin(kx - \omega t + \phi)$





Being exactly in phase, the waves produce a large resultant wave.
Being exactly out of phase, they produce a flat string.
This is an intermediate situation, with an intermediate result.
y
x
y1(x, t) and y2(x, t)
φ = 0
(a)
y1(x, t)
y2(x, t)
φ = π rad
(b)
y1(x, t)
y2(x, t)
φ = 2/3 π rad
(c)
y′(x, t)
(d)
y′(x, t)
(e)
y′(x, t)
(f)

Phase Difference, in			Amplitude of Resultant Wave	Type of Interference
Degrees	Radians	Wavelengths		
0	0	0	$2y_m$	Fully constructive
120	$\frac{2}{3}\pi$	0.33	y_m	Intermediate
180	π	0.50	0	Fully destructive
240	$\frac{4}{3}\pi$	0.67	y_m	Intermediate
360	2π	1.00	$2y_m$	Fully constructive
865	15.1	2.40	$0.60y_m$	Intermediate

Standing Wave

- we discussed two sinusoidal waves of the same wavelength and amplitude traveling in the same direction along a stretched string. What if they travel in opposite directions?
- We again use principle of superposition to treat the problem.

Superposition of two waves in opposite directions

- Consider two waves: $y_1(x,t) = y_m \sin(kx - \omega t)$ and $y_2(x,t) = y_m \sin(kx + \omega t)$.
- By principle of superposition:

$$y'(x,t) = y_1(x,t) + y_2(x,t)$$

- The resultant wave:

$$y'(x,t) = [2y_m \sin(kx)] cos\omega t$$

Mathematical form of standing wave

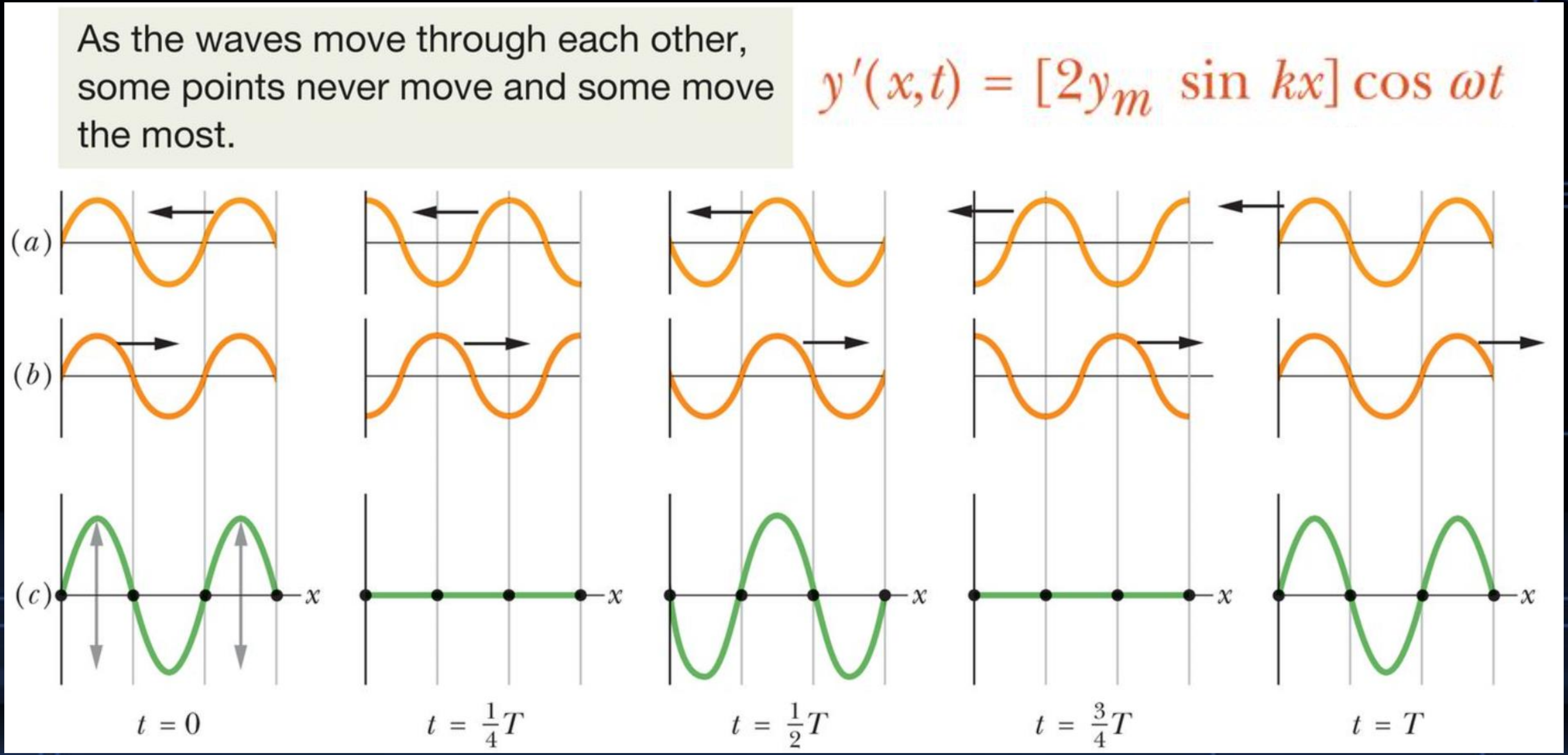
Displacement

$$y'(x,t) = [2y_m \sin kx] \cos \omega t$$

$[2y_m \sin kx]$: Magnitude gives amplitude at position x

$\cos \omega t$: Oscillating term

Visualization of standing wave



Node and antinode of standing wave

- The amplitude is zero for values of kx that give sin kx = 0:

$$kx = n\pi, where\ n = 0,1,2, \ldots$$

By insert $k = 2\pi/\lambda$, we find the nodes of the standing wave:

$$x = n\frac{\lambda}{2}, where\ n = 0,1,2, \ldots$$

- The amplitude is maximum for values $2y_m$ when:

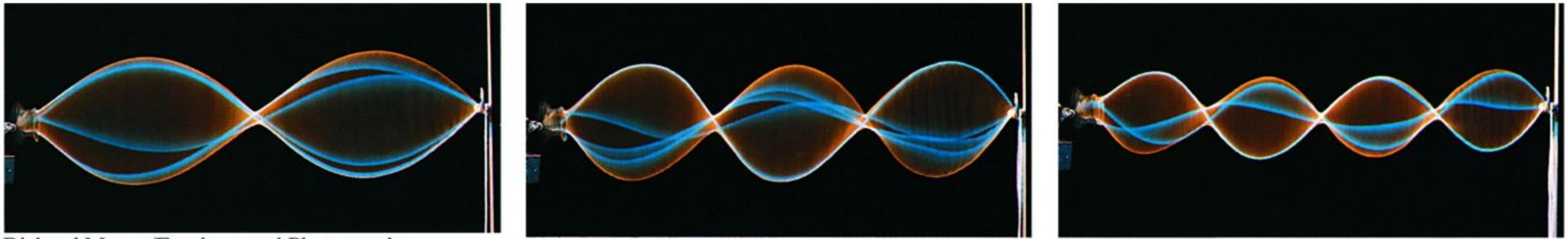
$$kx = (n + \frac{1}{2})\pi, where\ n = 0,1,2, \ldots$$

By insert $k = 2\pi/\lambda$, we find the antinodes of the standing wave:

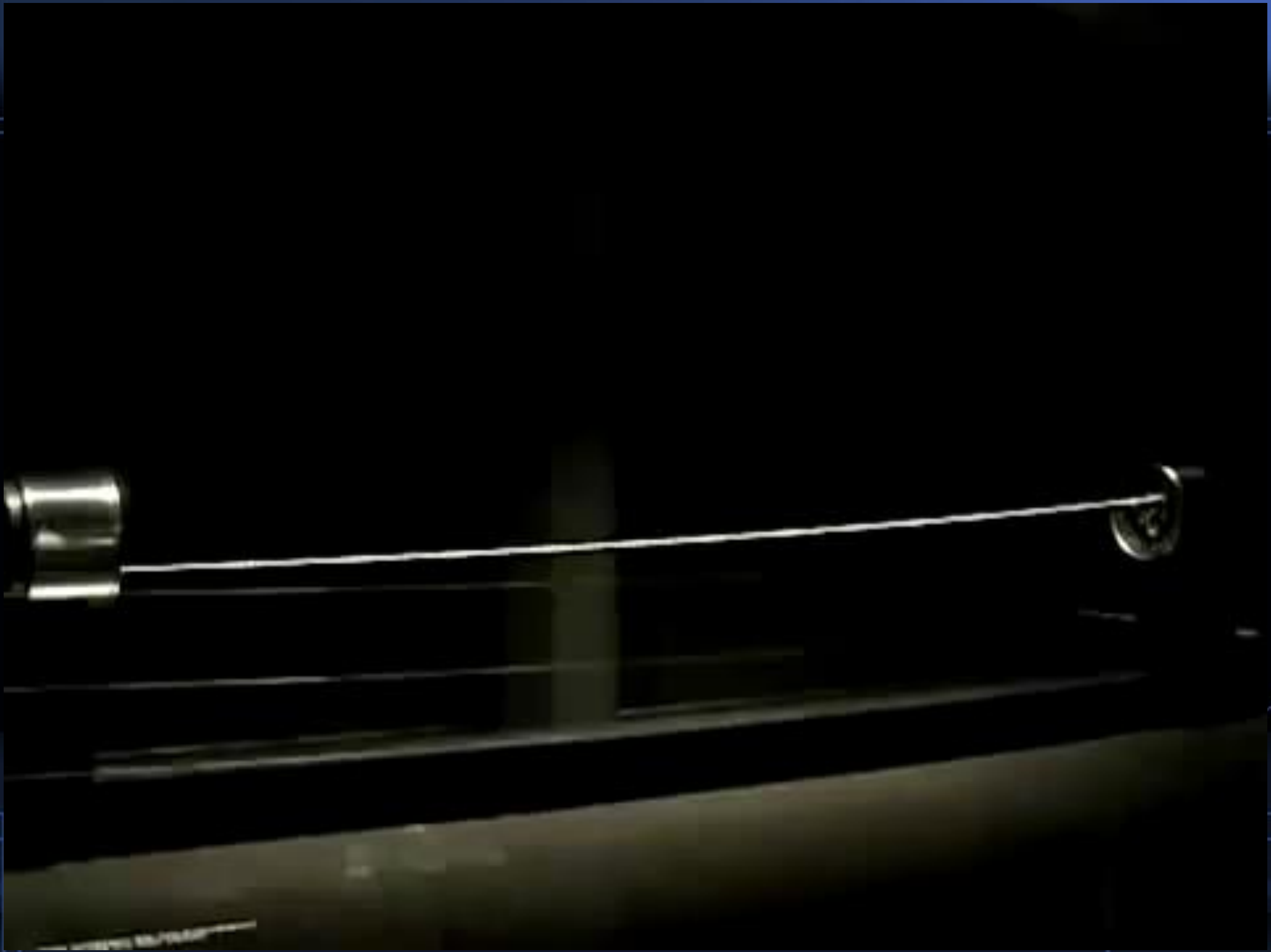
$$x = (n + \frac{1}{2})\frac{\lambda}{2}, where\ n = 0,1,2, \ldots$$

Standing wave and resonance

- Consider a string, such as a guitar string, that is stretched between two clamps. Suppose we send a continuous sinusoidal wave of a certain frequency along the string,
- Resonance: for certain frequencies, the interference produces a standing wave pattern (or oscillation mode) with nodes and large antinodes.



Richard Megna/Fundamental Photographs



https://www.youtube.com/watch?v=no7ZPPqtZEg

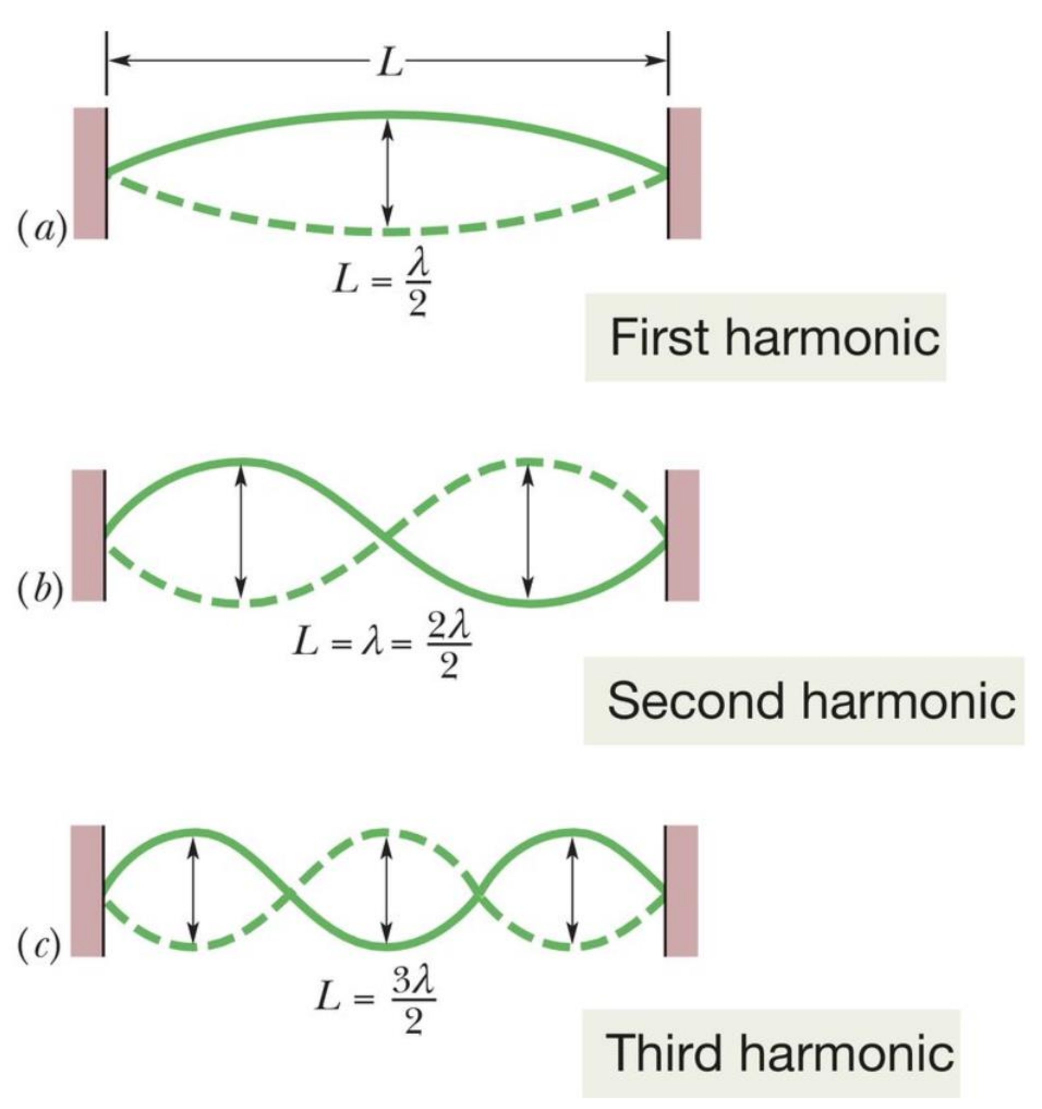
Resonance frequency

- A standing wave can be set up on a string of length L by a wave with a wavelength equal to one of the values:

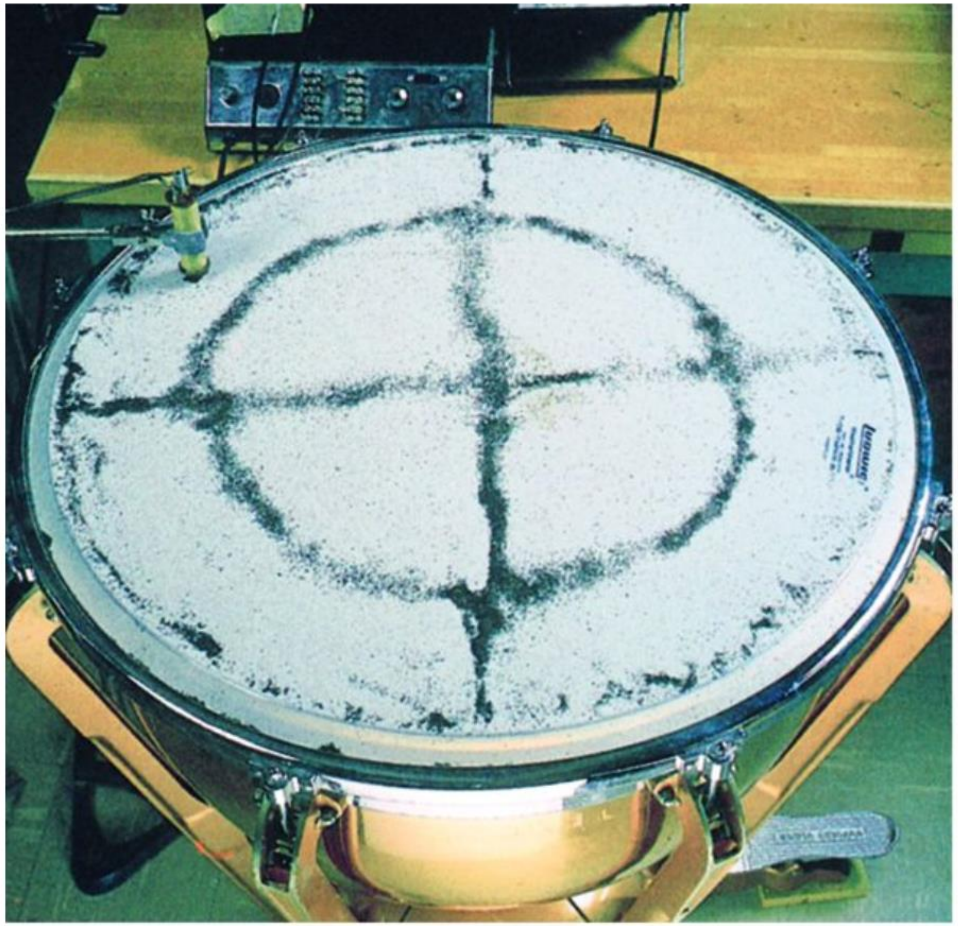
$$\lambda = \frac{2L}{n}, n = 1,2,3, \ldots$$

- Therefore, the resonance frequency is:

$$f = \frac{v}{\lambda} = n\frac{v}{2L}, n = 1,2,3, \ldots$$



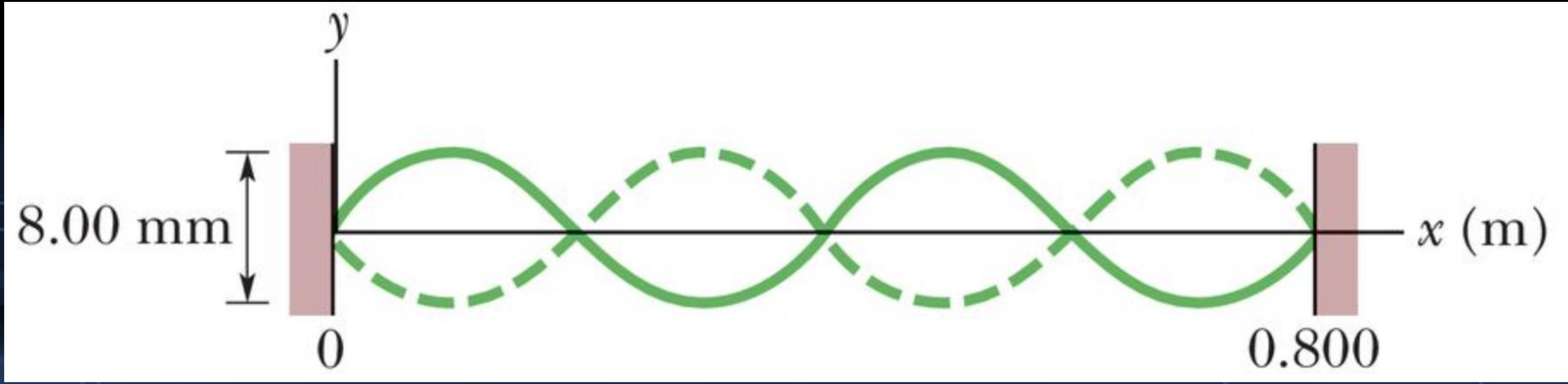
Standing wave on a drum



Courtesy Thomas D. Rossing, Northern Illinois University

Example

resonant oscillation of a string of mass m = 2.500 g and length L = 0.800 m and that is under tension τ = 325.0 N. (a)What is the wavelength λ of the transverse waves producing the standing wave pattern, and what is the harmonic number n?

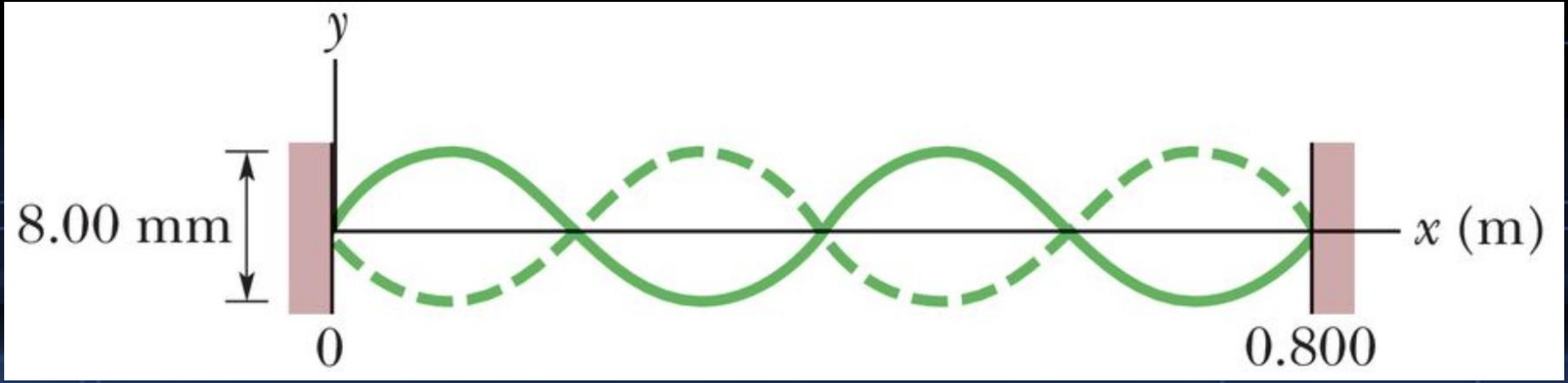


Example

$$y'(x,t) = [2y_m \sin(kx)]cos\omega t$$

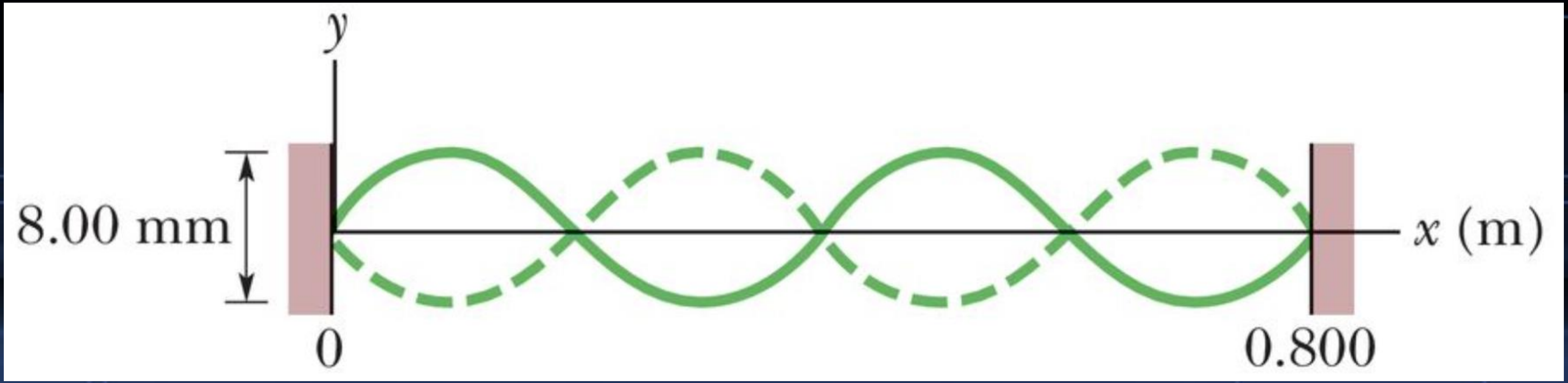
We can find in the plot that $L = 2\lambda$. Thus $\lambda = 0.4m$

By counting the or half-wavelengths, the correspond harmonic is n=4



Example

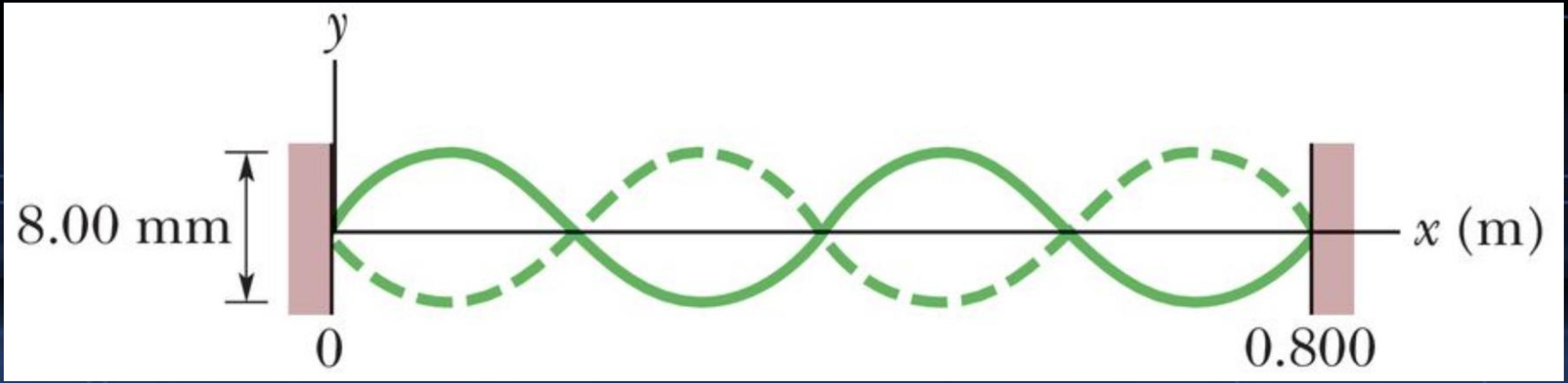
(b)What is the frequency f of the transverse waves and of the oscillations of the moving string elements?



Example

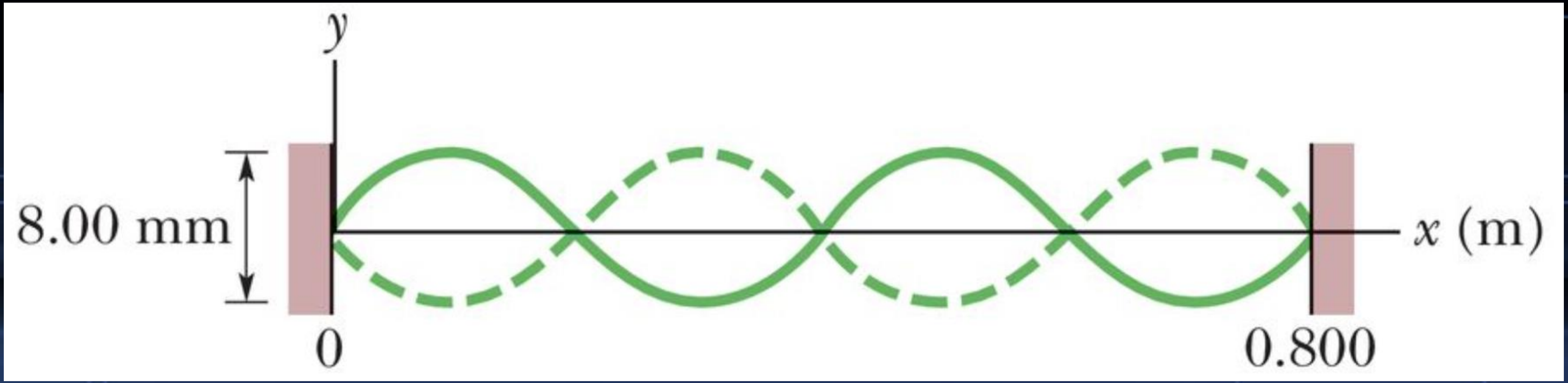
With the wave speed on string: $v = \sqrt{\frac{\tau}{\mu}} = \sqrt{\frac{\tau L}{m}} = 322.49 m/s$

The resonance frequency $f = \frac{v}{\lambda} = 806.2 Hz$



Example

(c) What is the maximum magnitude of the transverse velocity um of the element oscillating at coordinate x = 0.180 m?



Example

By taking derivative of displacement respect to time, we have: $u(t) = \frac{\partial}{\partial t}[2y_m sinkx]cos\omega t = [-2y_m \omega sinkx]sin\omega t$

Therefore, at x=0.18m, the maximum $u = 6.26 m/s$