Course announcement

- Homework set 3 will be posted on eLearn today. It will be due on 11/18 Friday at 5PM.

10	11/15(Tue.)	**Oscillation and Waves:** description of waves
10	11/18(Fri.)	**Oscillation and Waves:** interference of waves
11	11/22(Tue.)	**Oscillation and Waves:** propagation of waves
11	11/25(Fri.)	**Fluid Motion:** Density, Pressure, and Hydrostatic Equilibrium (Homework4)

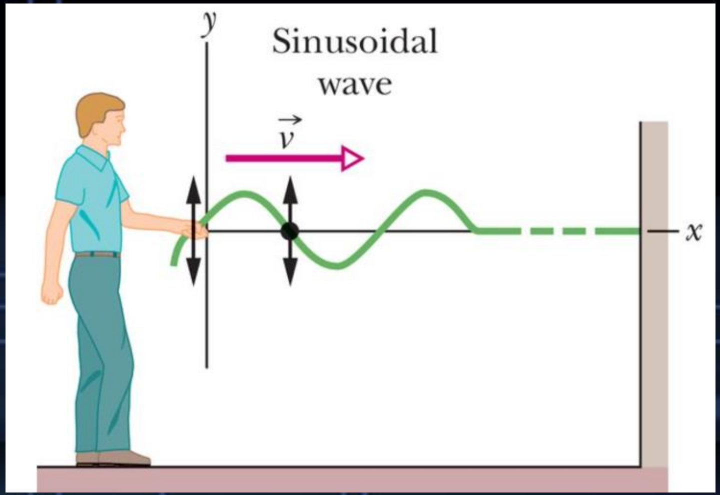
Different types of waves

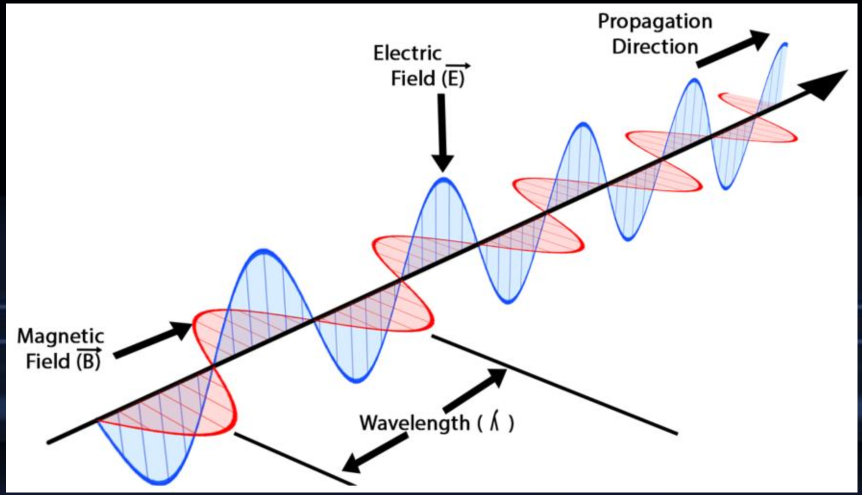
- Waves are of three main types:

Mechanical waves.

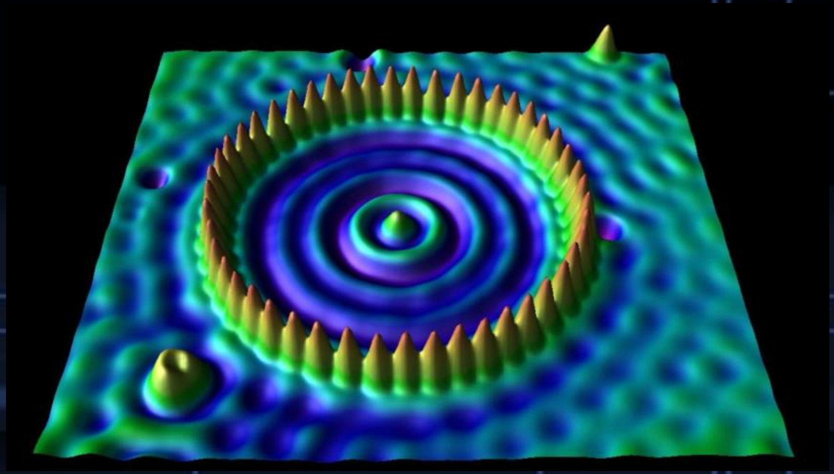
Electromagnetic waves.

Matter waves.





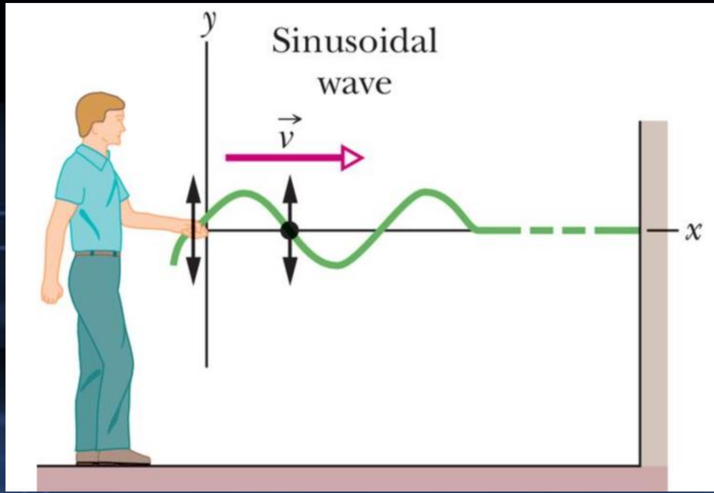
https://commons.wikimedia.org/wiki/File:Electromagnetic_waves.png

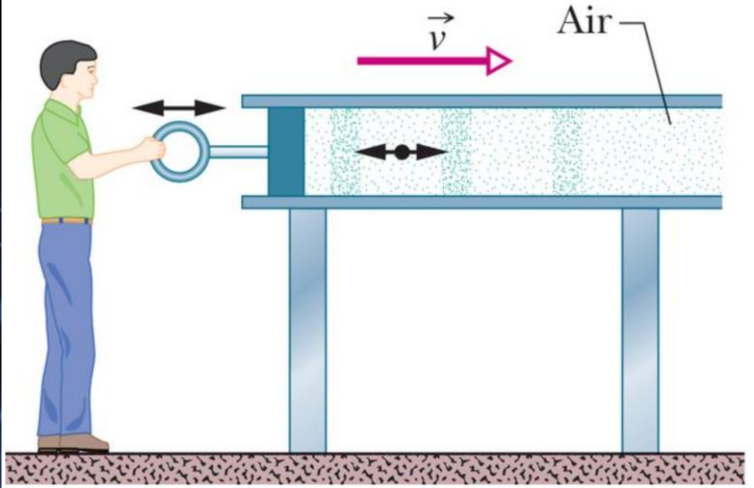


https://www.nisenet.org/catalog/scientific-image-quantum-corral-top-view

Transverse and Longitudinal Waves

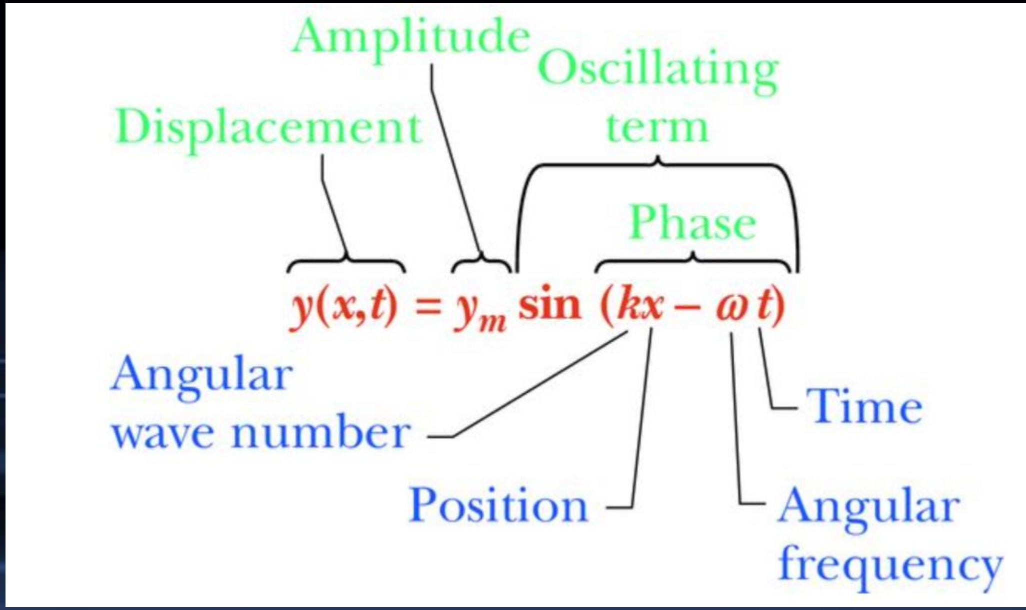
- Transverse waves: the displacement of every oscillating element is perpendicular to the direction of travel of the wave.
- Longitudinal waves: the displacement of the oscillating elements is parallel to the direction of the wave's travel.





Description of wave

- Use wave on a string as an example. Imagine a sinusoidal wave traveling in the positive direction of an x axis. The elements oscillate parallel to the y axis. At time t, the displacement y(x,t) of the element located at position x is given by



GENERAL PHYSICS B1
OSCILLATION & WAVE

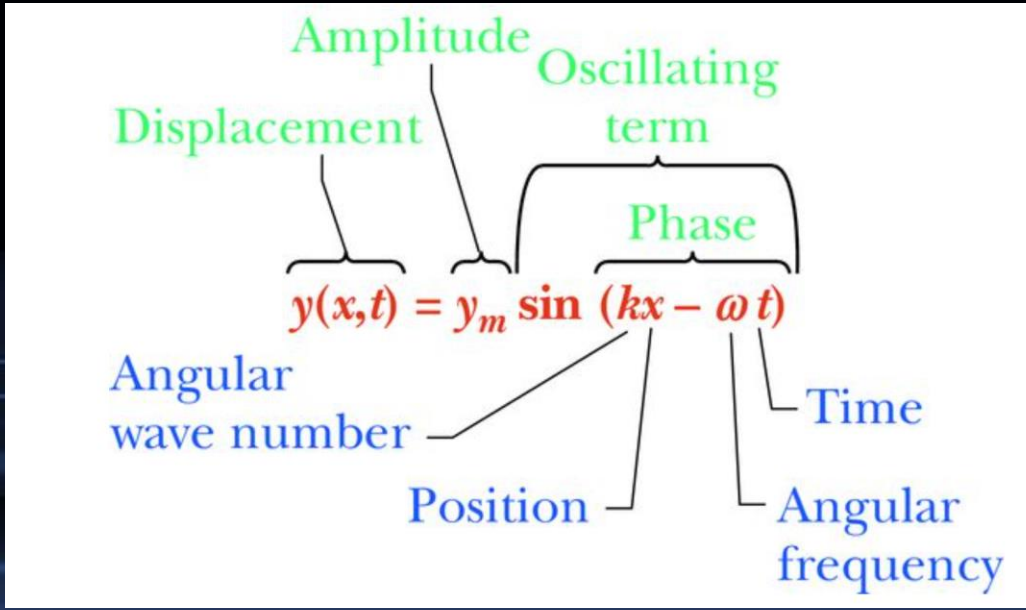
Damped, Forced Oscillation, and Wave
2022/11/11

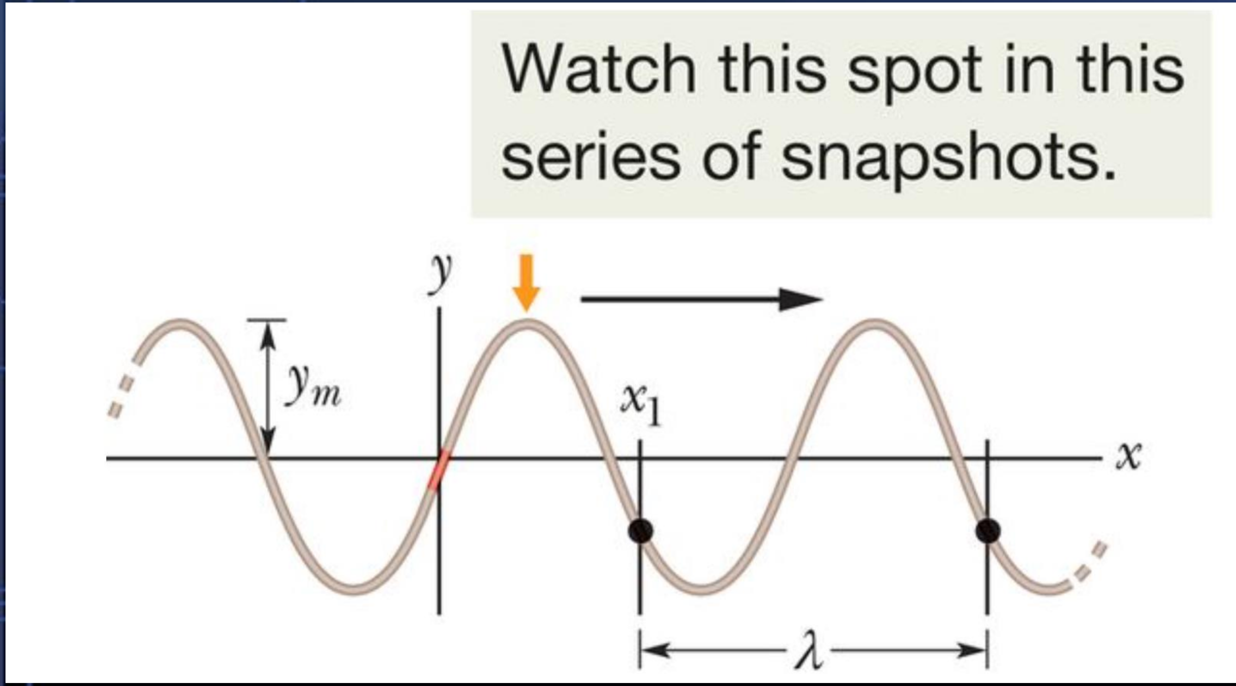
Today's topic

- Description of Wave
- Wave speed on a stretched string
- Wave equation

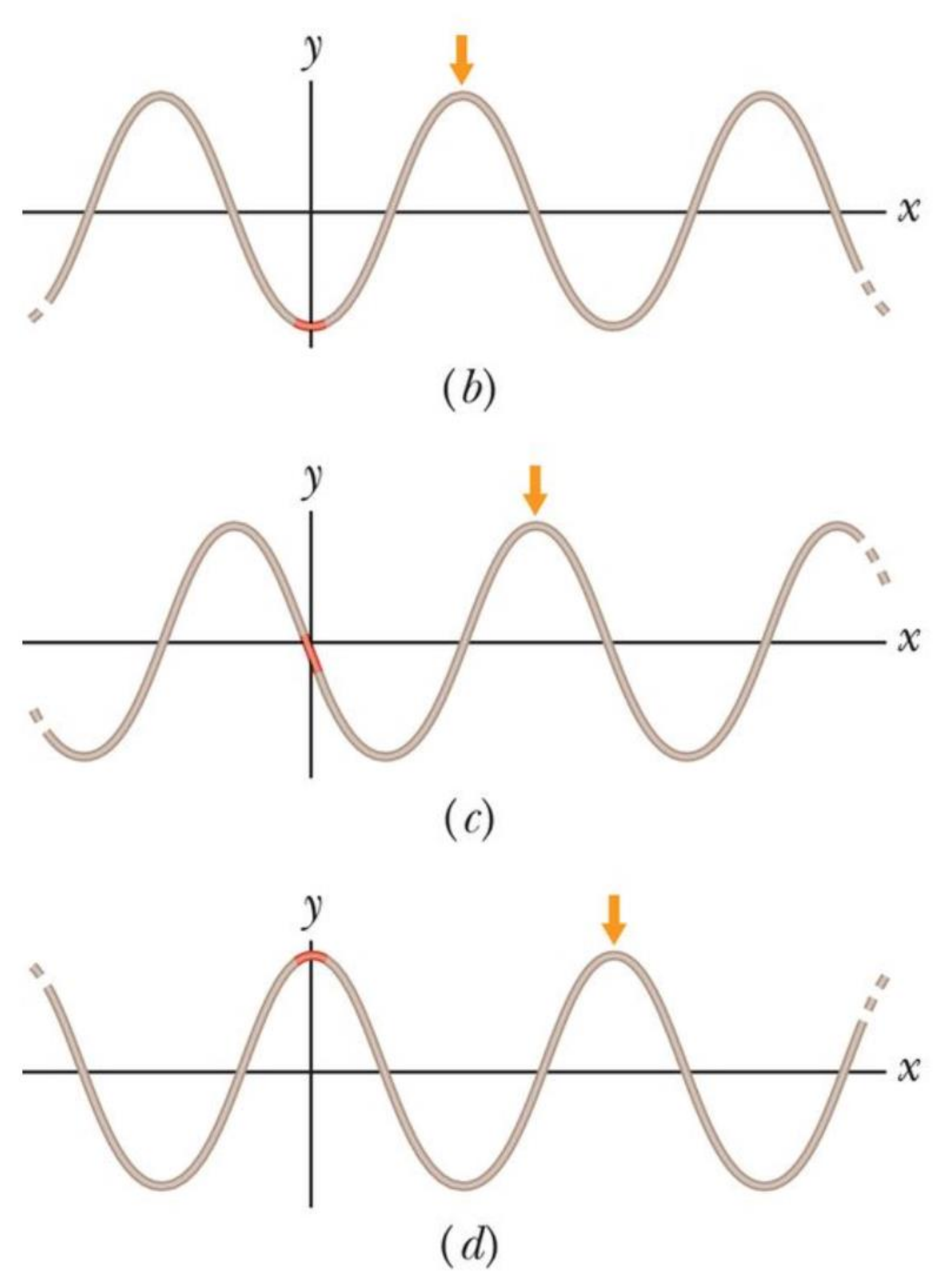
Description of wave

- Use wave on a string as an example. Imagine a sinusoidal wave traveling in the positive direction of an x axis. The elements oscillate parallel to the y axis. At time t, the displacement y(x,t) of the element located at position x is given by

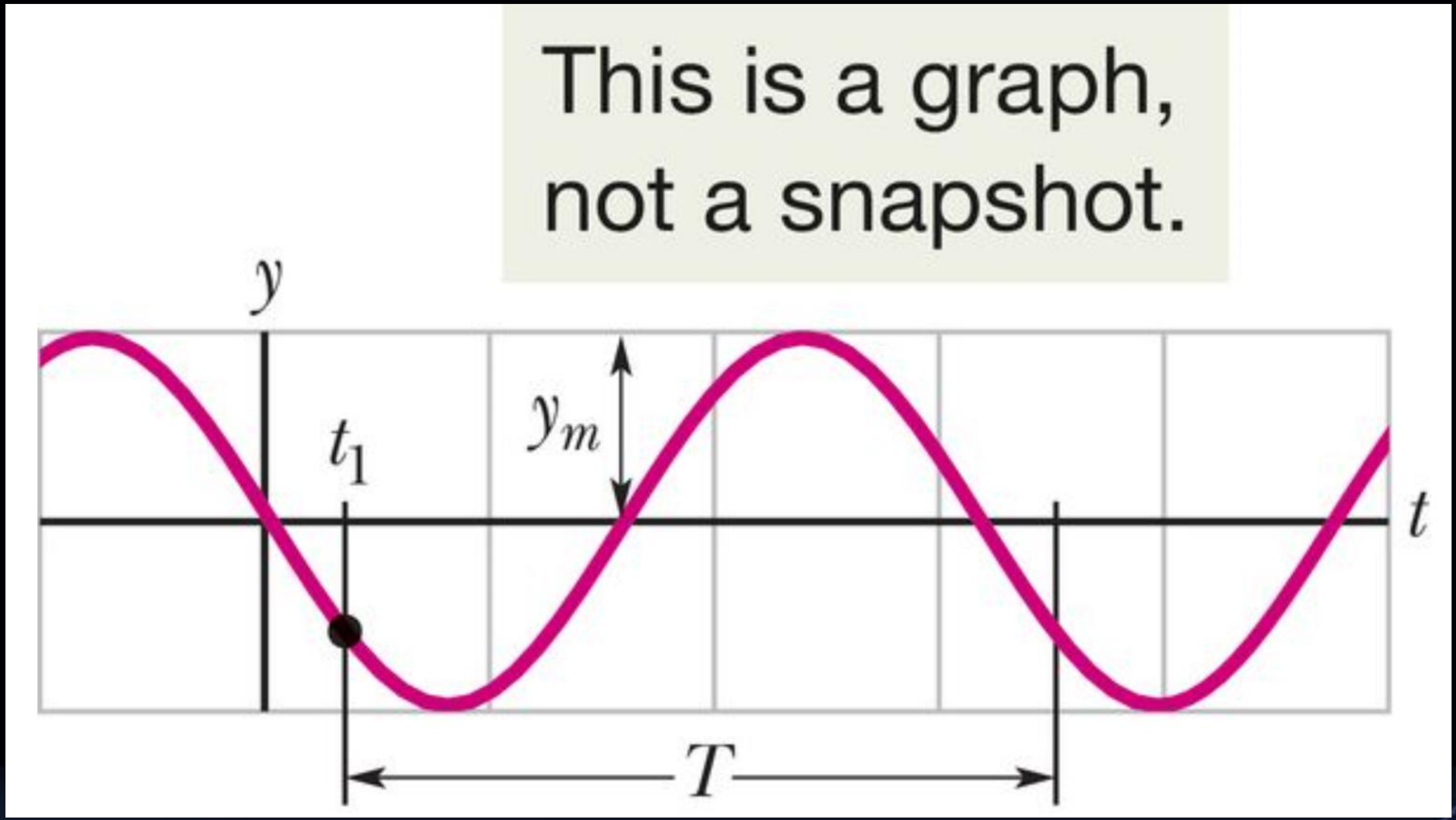




- $k = \frac{2\pi}{\lambda}$ (angular wave number)
- phase of the wave is the *argument* $(kx - \omega t)$ of the sine: a wave traveling to positive x direction



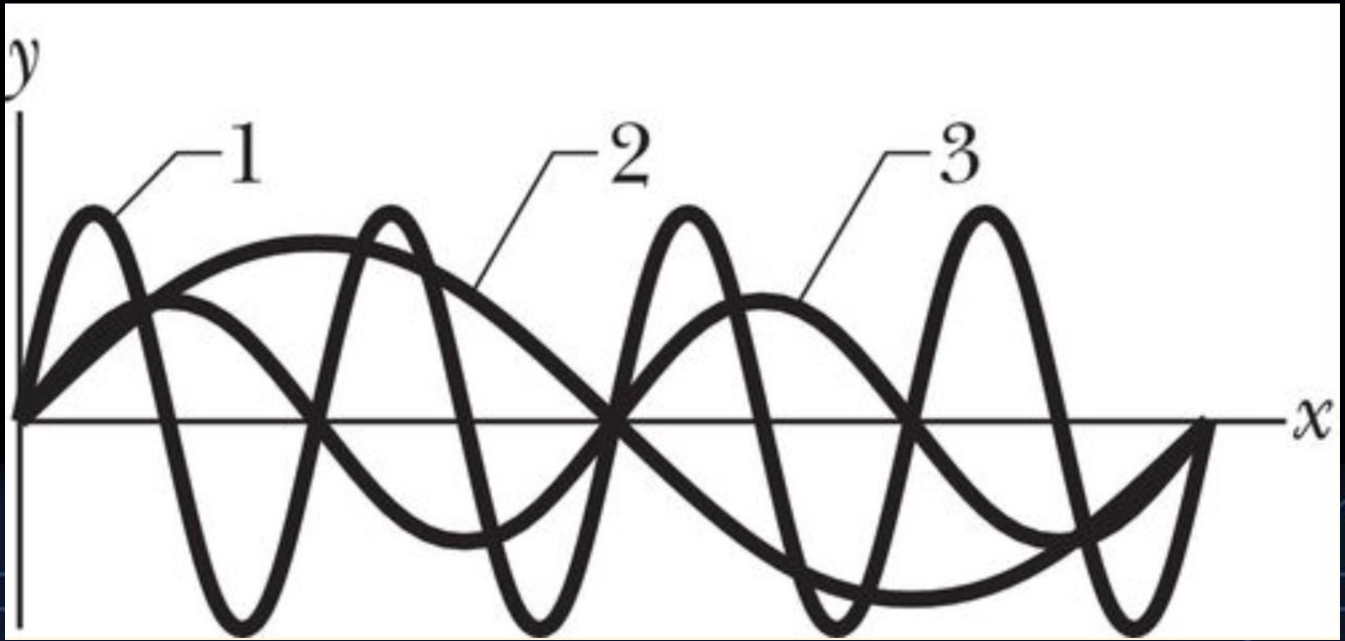
Angular frequency and frequency of wave



- $\omega = \frac{2\pi}{T}$ (angular wave frequency)
- $f = \frac{1}{T} = \frac{\omega}{2\pi}$ (frequency)

Example

The phases for the waves are given by (a) $2x - 4t$, (b) $4x - 8t$, and (c) $8x - 16t$. Which phase corresponds to which wave in the figure?



Example

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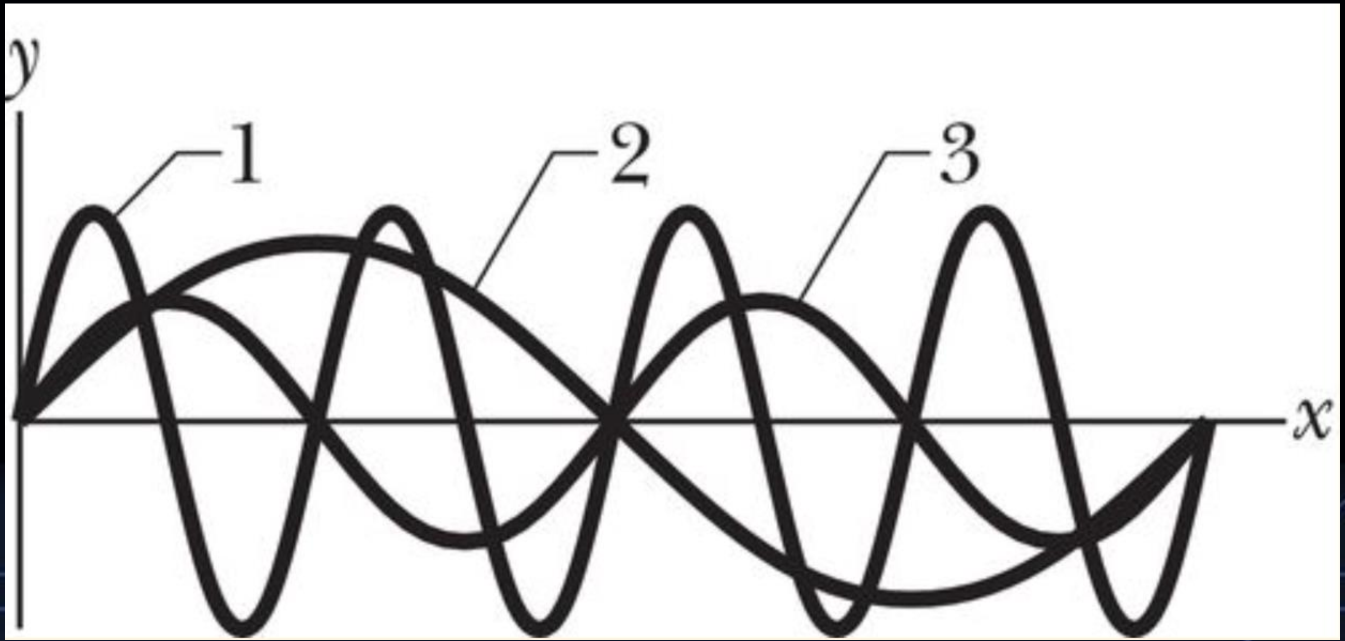
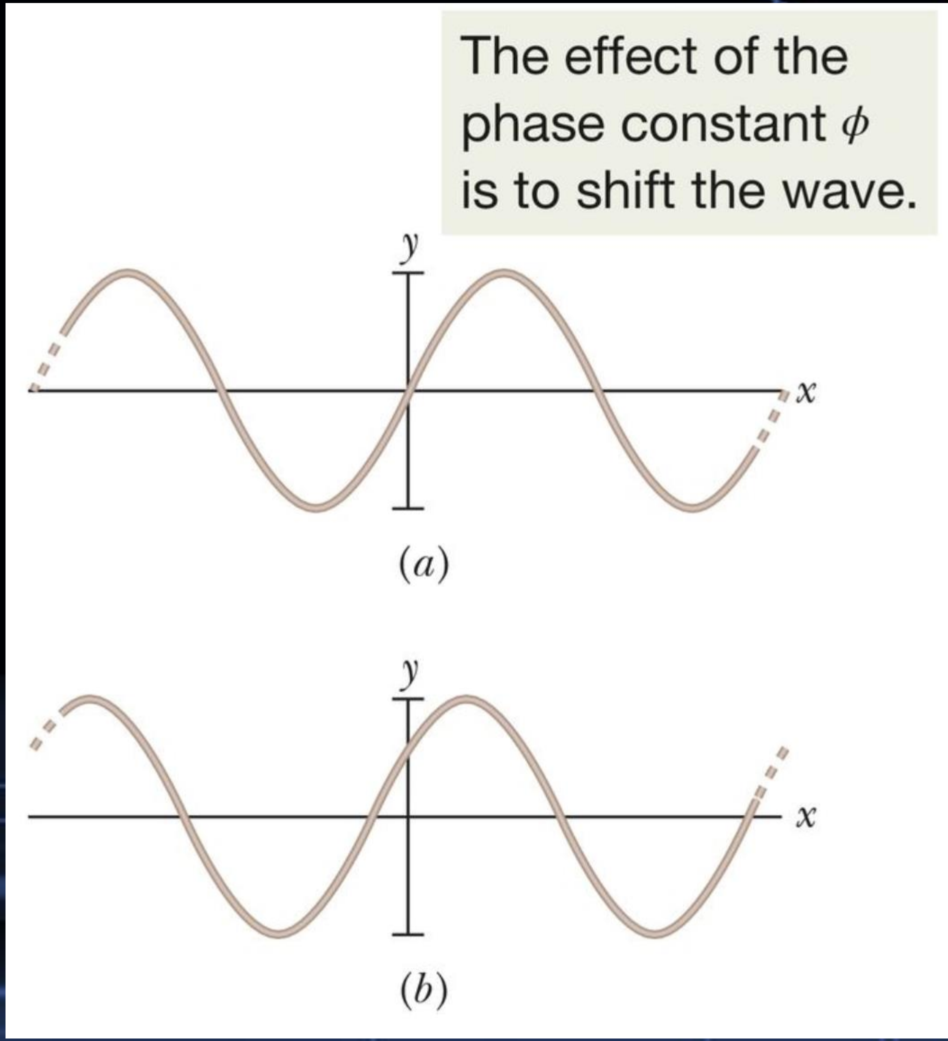


Ans: 1-(c), 2-(a), and 3-(b). $k = \frac{2\pi}{\lambda}$:longer wavelength, smaller k

The phase constant

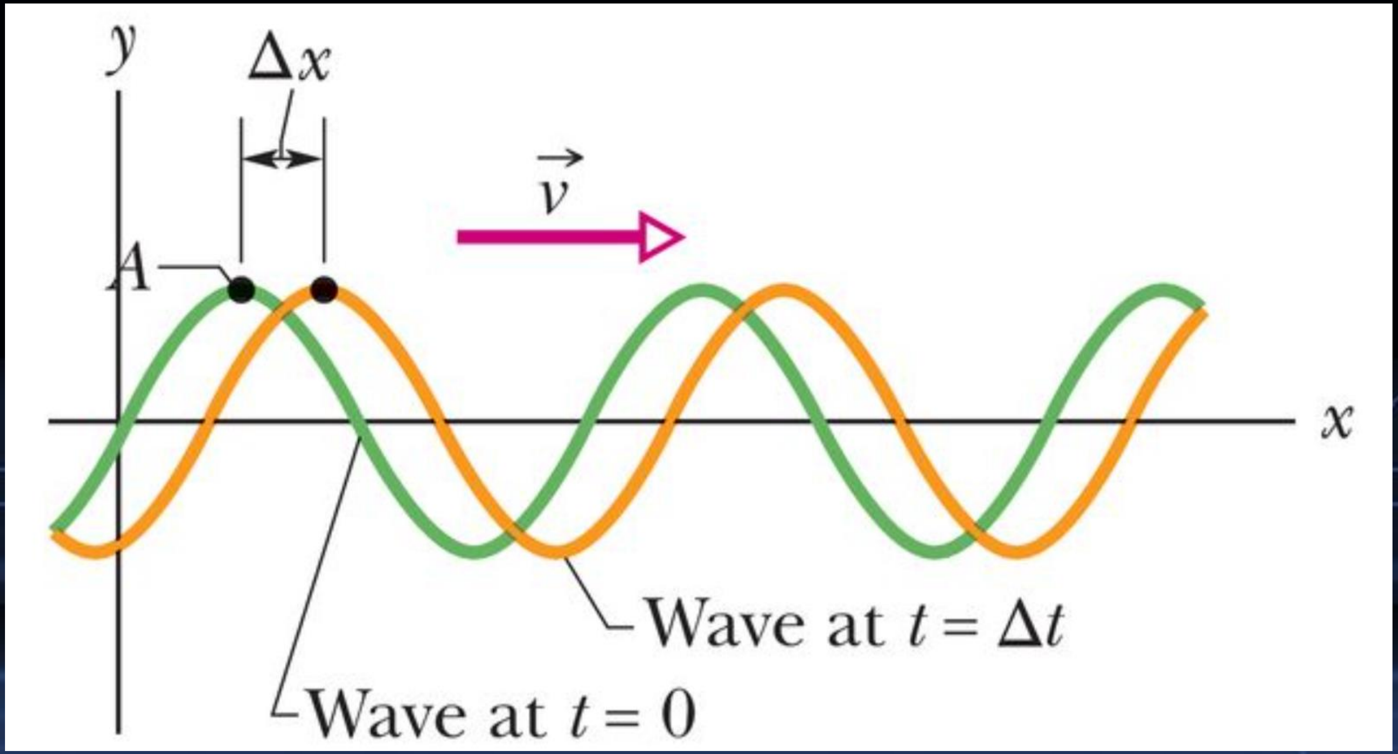
- A value of ϕ can be chosen so that the function gives some other displacement and slope at x = 0 when t = 0.

$$y = y_m \sin(kx - \omega t + \phi)$$



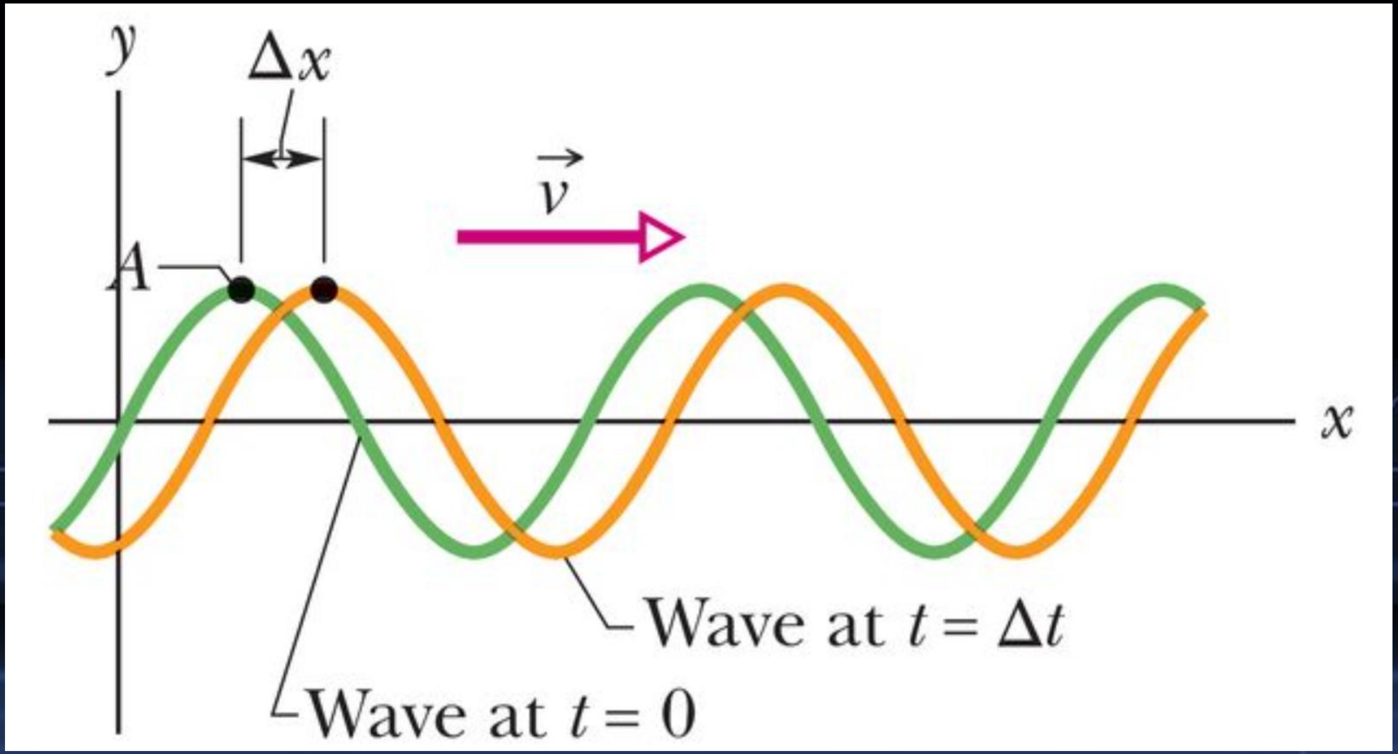
The speed of a traveling wave

- The wave is traveling in the positive direction of x, the entire wave pattern moving a distance Δx in that direction during the interval Δt. The ratio Δx/Δt (or, in the differential limit, dx/dt) is the wave speed v.



The speed of a traveling wave

- If point A retains its displacement as it moves, the phase must remain a constant: $kx - \omega t = constant$.
- Thus the wave speed is $\frac{dx}{dt} = v = \frac{\omega}{k} = \frac{\lambda}{T} = \lambda f$



Direction of wave propagation

- With the concept of wave speed, one can find that:

$$y(x,t) = y_m \sin(kx - \omega t + \phi)$$

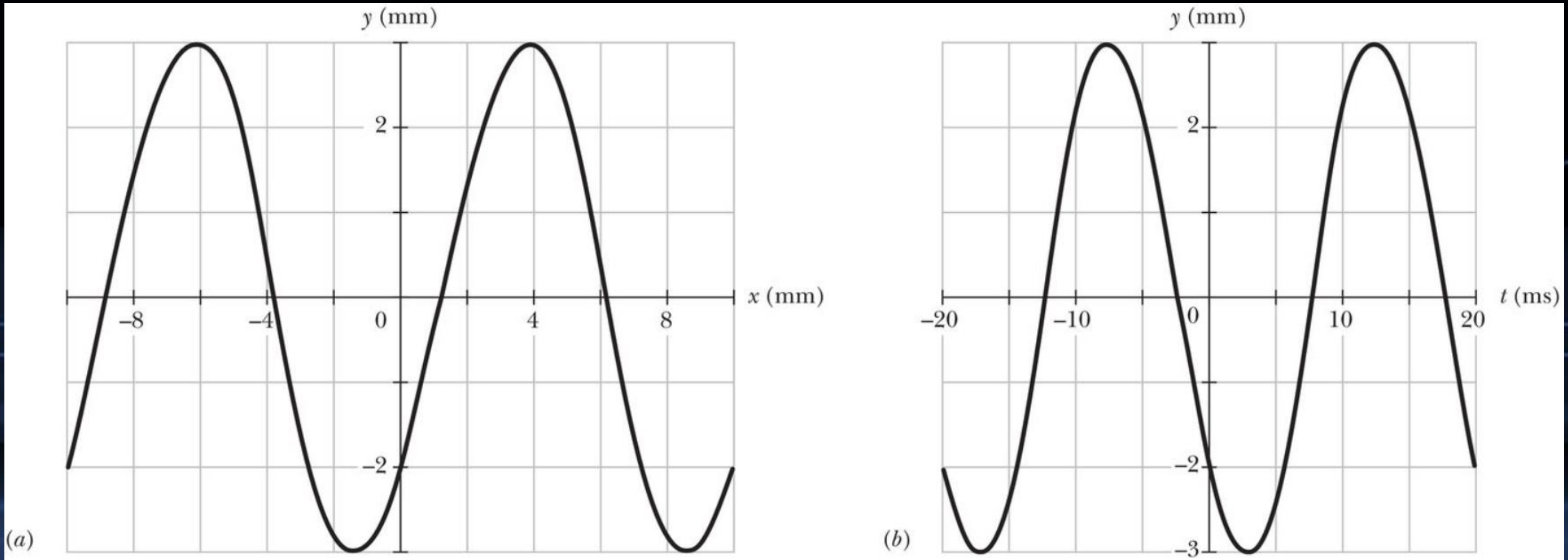
is a wave traveling to positive x direction with positive wave speed $v = \frac{\omega}{k}$.

$$y(x,t) = y_m \sin(kx + \omega t + \phi)$$

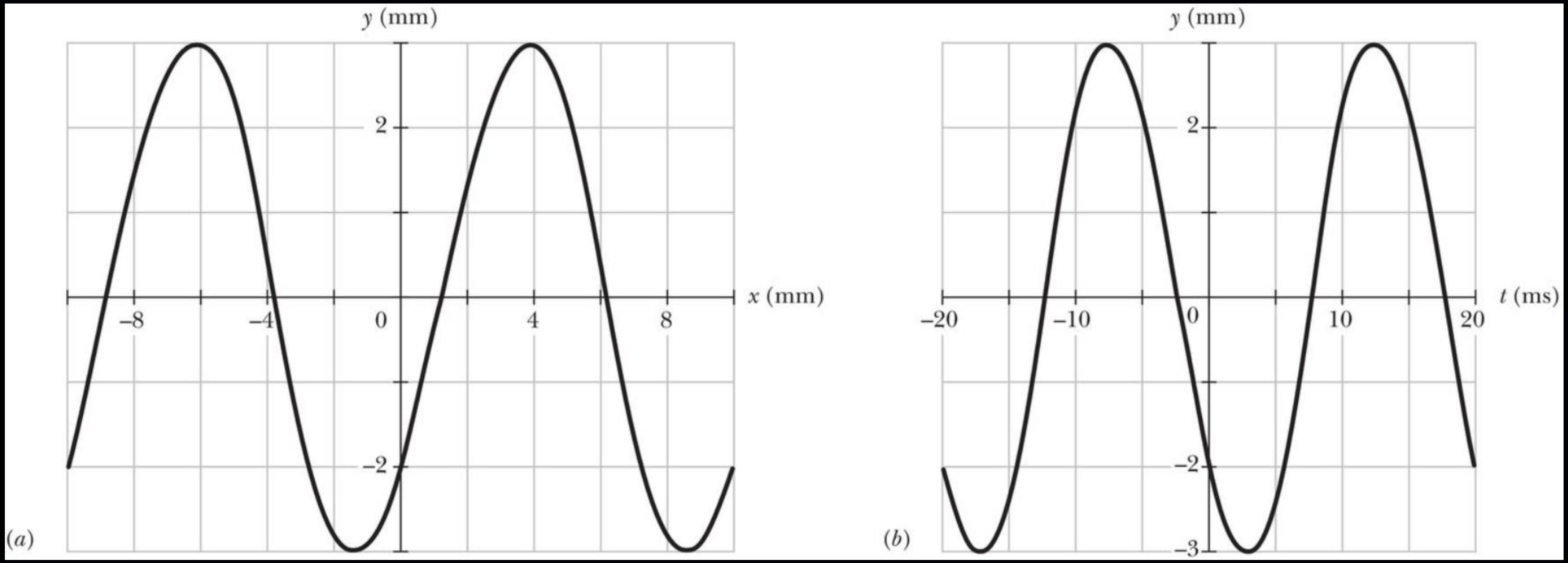
is a wave traveling to negative x direction with negative wave speed $v = -\frac{\omega}{k}$.

Example

A transverse wave traveling along an x axis has the form given by $y(x,t) = y_m \sin(kx \pm \omega t + \phi)$. Figure (a) gives the displacements of string elements as a function of x, all at time t = 0. Figure (b) gives the displacements of the element at x = 0 as a function of t. Find the values of the quantities .

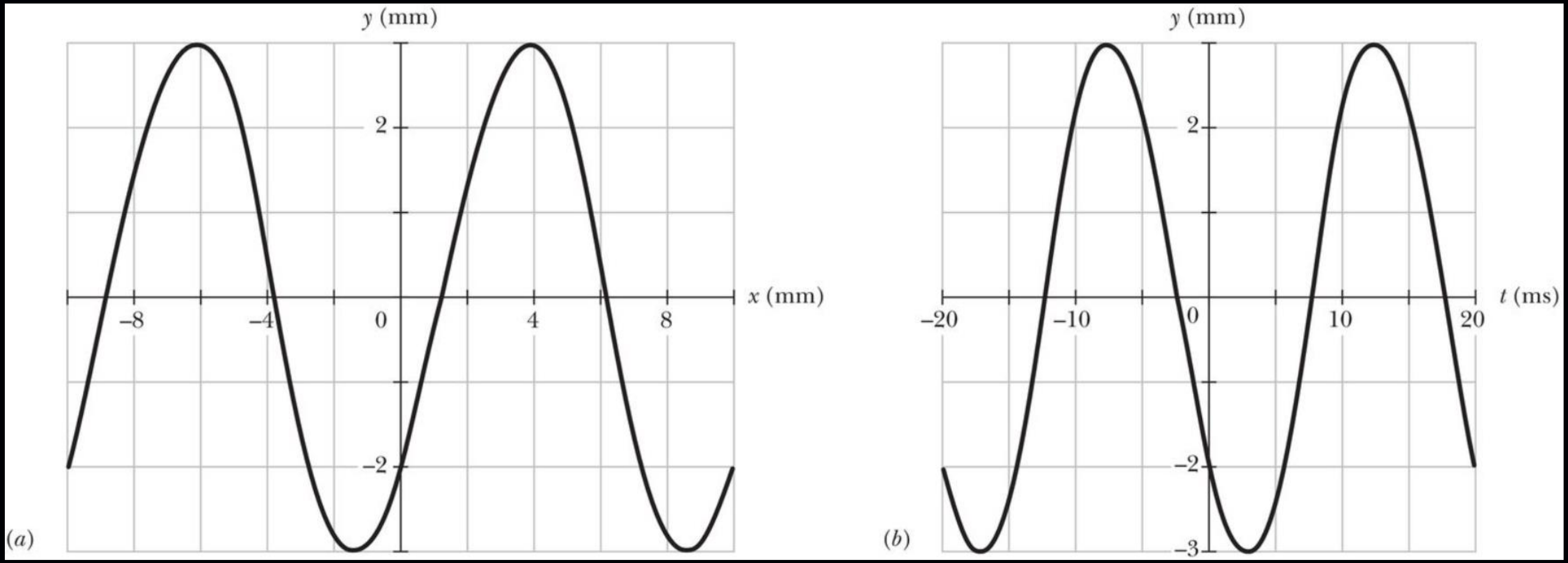


Example



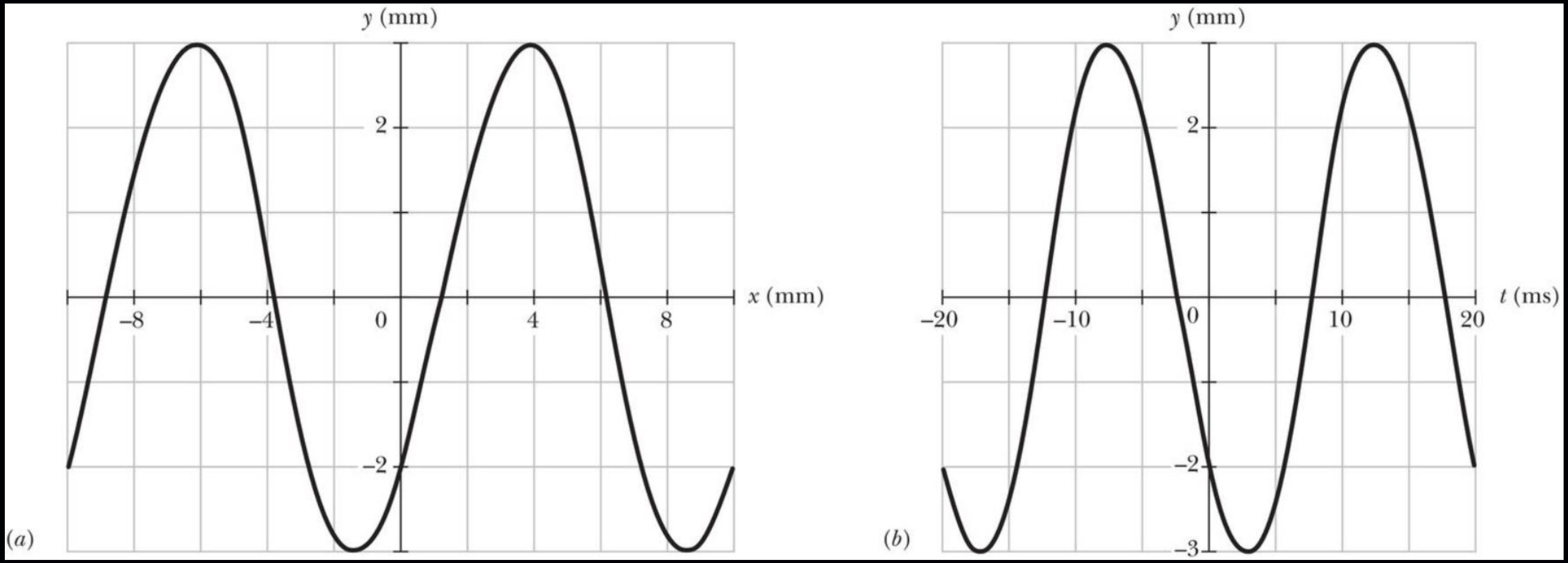
- Amplitude of wave y_m: The wave is oscillating between $+y_m$ and $-y_m$. In both figure, the wave oscillating between +3mm and -3mm. Therefore $y_m = 3mm$

Example



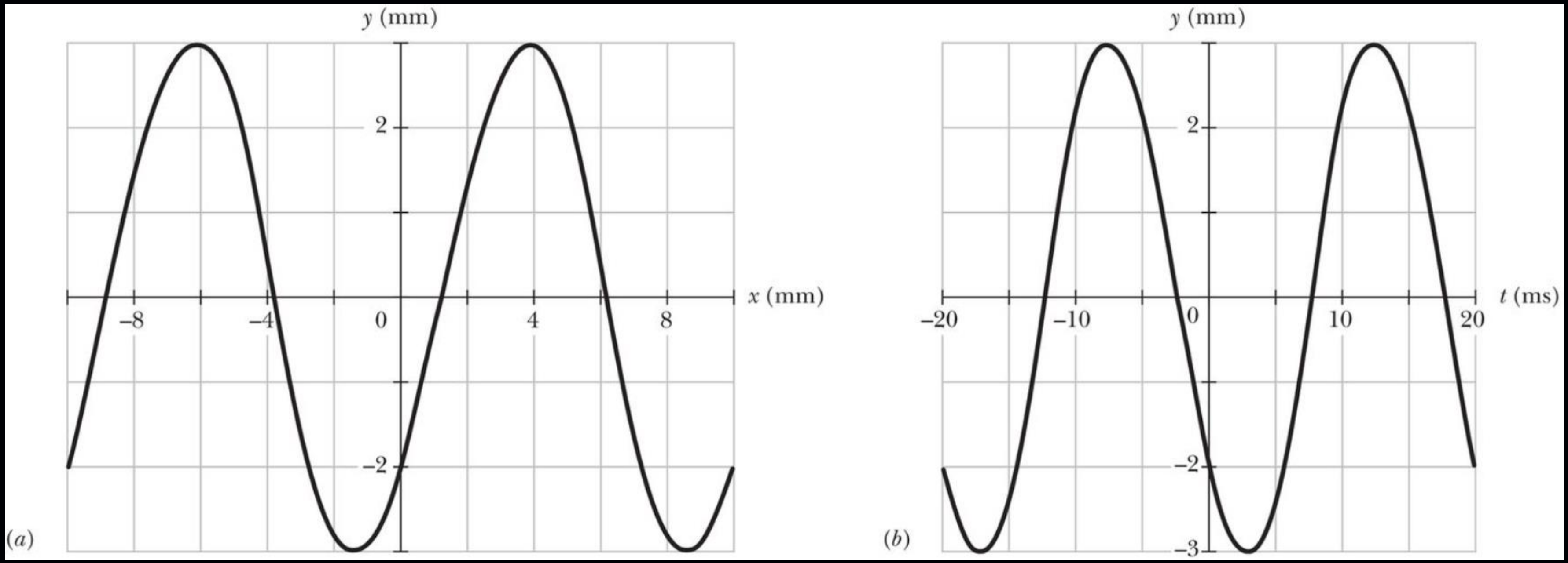
- Angular wave number k: We will find out wavelength λ first and use $k = \frac{2\pi}{\lambda}$ to find out angular wave number. From figure (a), we can find out $\lambda = 10mm$. Therefore $k = 200\pi\ rad/m$

Example



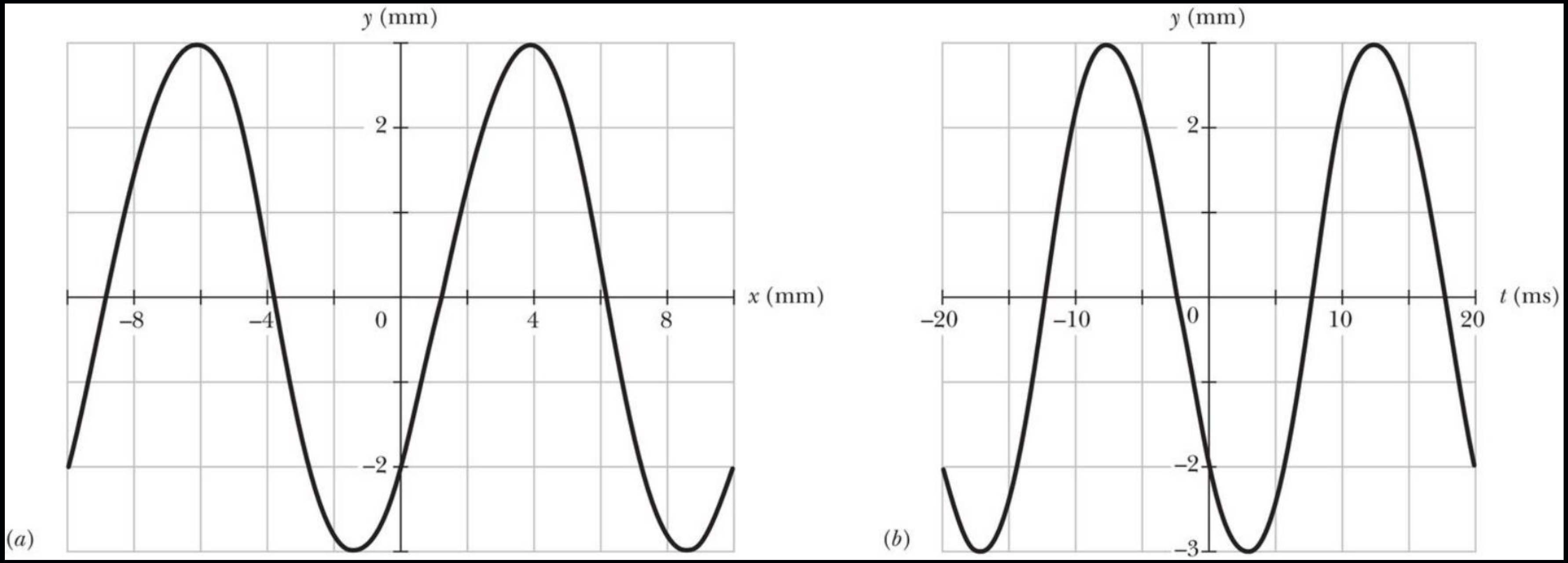
- Angular wave frequency ω: We will find out period T first and use $\omega = \frac{2\pi}{T}$ to find out angular wave frequency. From figure (b), we can find out $\mathrm{T} = 20ms$. Therefore $\omega = 100\pi\ rad/s$

Example



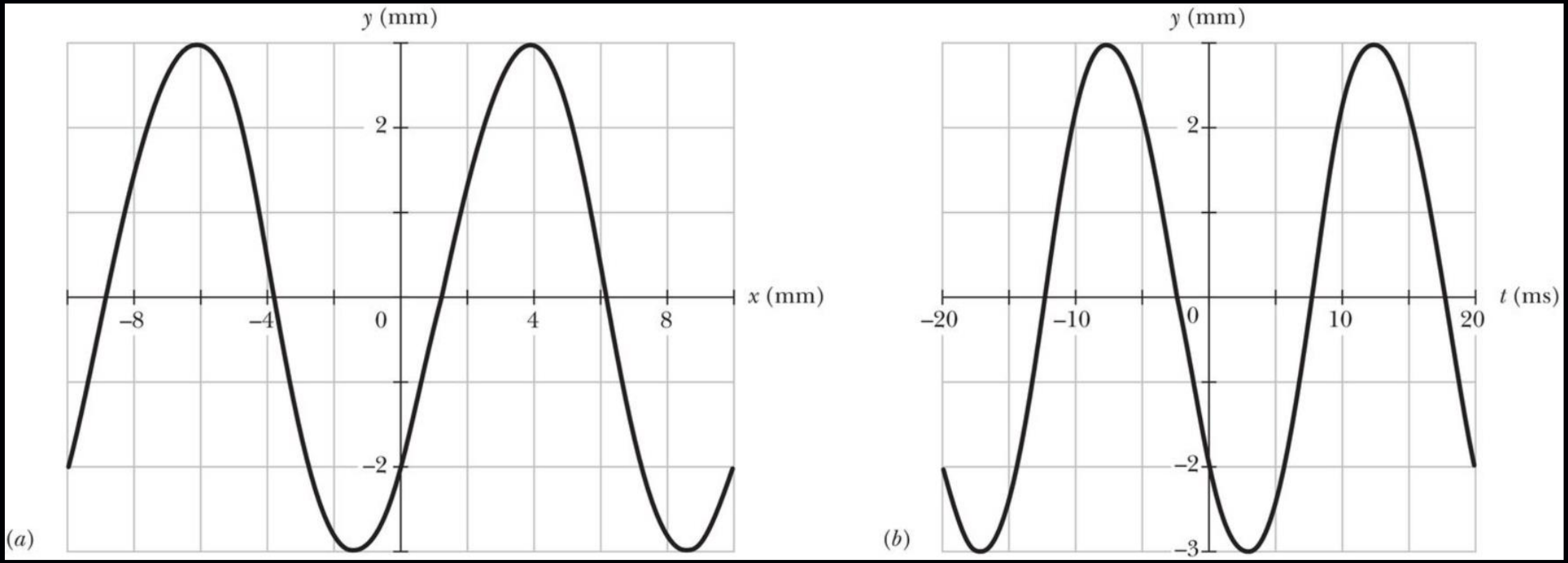
- Direction of travel: From (b), we know that the point x=0 is oscillating toward negative first when t is increasing from 0. This can only happen when the wave is moving to the right in figure (a). Thus, it should be $kx - \omega t$

Example



- Phase constant ϕ: From (a), we know that at $y(x = 0, t = 0) = 3\sin(\phi) = -2mm$. This gives $\phi = sin^{-1}\left(-\frac{2}{3}\right) = -0.73rad$

Example

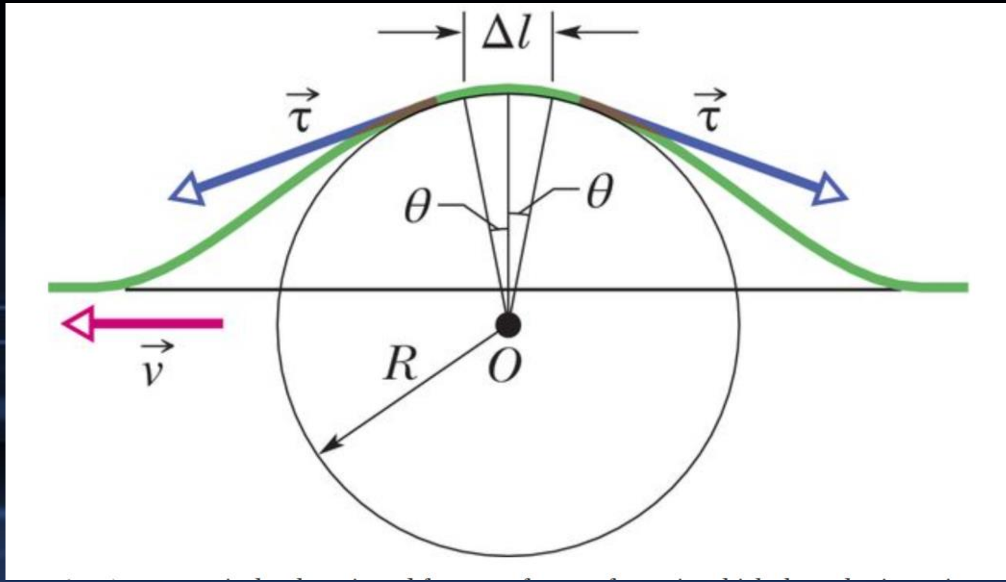


- Therefore we have $y(x,t) = 3mm\,\sin(200\pi x - 100\pi t - 0.73)$

Wave speed on a stretched string

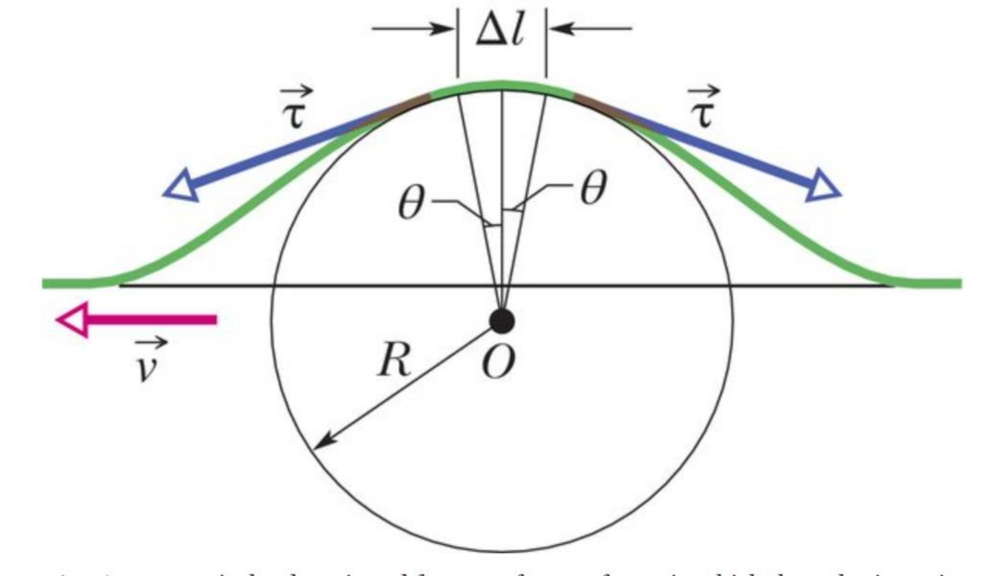
- What is a wave speed of a stretched string? Let's analyze in the following way:

consider a single symmetrical pulse moving from left to right along a string with speed v.



Wave speed on a stretched string

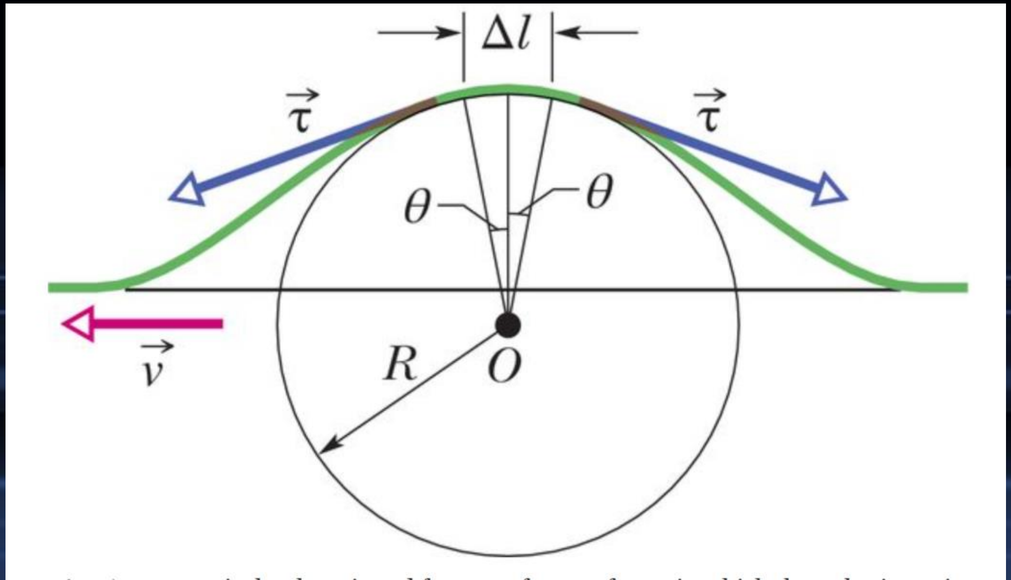
Consider a small string element of length Δl within the pulse, an element that forms an arc of a circle of radius R and subtending an angle 2θ at the center of that circle. A force $\vec{\tau}$ with a magnitude equal to the tension in the string pulls tangentially on this element at each end.



Wave speed on a stretched string

A radial restoring force with magnitude $F = 2\tau sin\theta \approx \tau \frac{\Delta l}{R}$

The mass of the element is $\Delta m = \mu\Delta l$, where μ is the unit mass of the string.



Wave speed on a stretched string

Considering it as a uniform circular motion, the force provide a centripetal acceleration:$a = \frac{v^2}{R}$ to the mass $\Delta m = \mu \Delta l$.

Thus, we have $\tau \frac{\Delta l}{R} = \mu \Delta l a = \mu \Delta l \frac{v^2}{R}$

There fore we can get $v = \sqrt{\frac{\tau}{\mu}}$, which is the wave speed of the stretched string.

Energy of a wave traveling along string

- The kinetic energy dK associated with a string element of mass $dm = \mu dx$ is given by $dK = \frac{1}{2} dm u^2$, where u is the transverse speed of the oscillating string element.
- $dK = \frac{1}{2} dm (\frac{dy}{dt})^2 = \frac{1}{2} dm (\frac{d y_m \sin(kx - \omega t + \phi)}{dt})^2 = \frac{1}{2} \mu dx {y_m}^2 \omega^2 cos^2(kx - \omega t + \phi)$
- Therefore, $\frac{dK}{dt} = \frac{1}{2} \mu v {y_m}^2 \omega^2 \, cos^2(kx - \omega t + \phi)$, where $v = \frac{dx}{dt}$ is the wave speed

Energy of a wave traveling along string

- The average kinetic energy transfer

$$\left(\frac{dK}{dt}\right)_{avg} = \frac{1}{2}\mu v {y_m}^2 \omega^2 \left[cos^2(kx - \omega t + \phi)\right]_{avg} = \frac{1}{4}\mu v {y_m}^2 \omega^2$$

- The average potential energy transfer will be the same amount because the nature of harmonic oscillation. Therefore, the power of the total energy transfer will be $P_{avg} = \frac{1}{2}\mu v {y_m}^2 \omega^2$

The wave equation

- By applying Newton’s second law to the element’s motion, we can derive a general differential equation, called the wave equation, that governs the travel of waves of any type:

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

- The description $y(x,t) = y_m \sin(kx - \omega t + \phi)$ is the solution of this general differential equation.