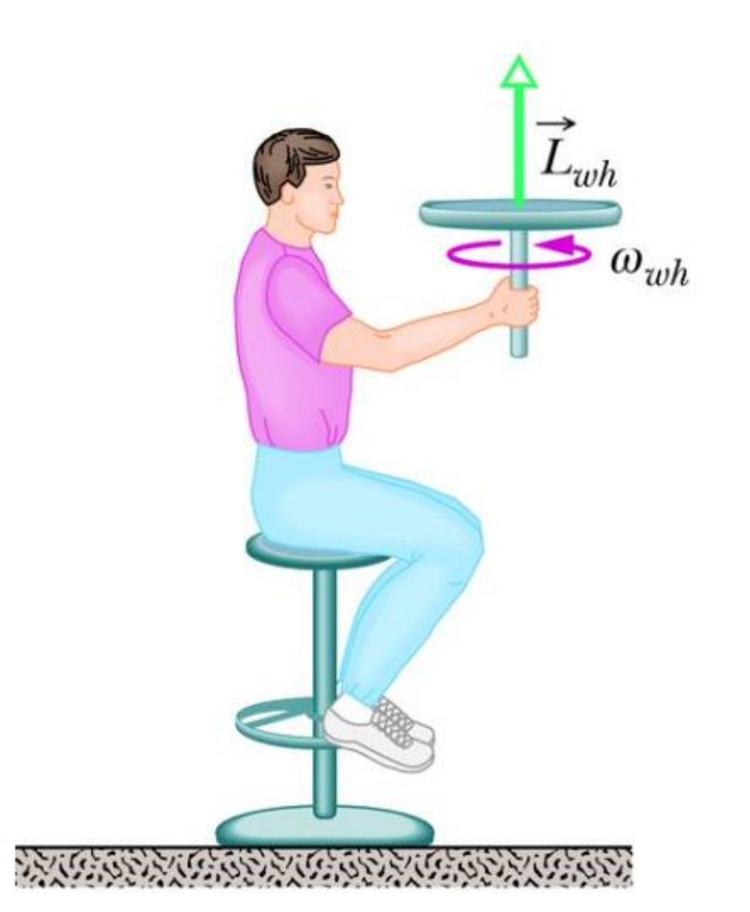
Course announcement

- Final midterm score is posted on eLearn. If you have any questions about score, please directly contact me (yhlin@phys.nthu.edu.tw).
- Review midterm exam: 11/8, 11/11 after class, 11/14 office hours
- Midterm warning has been sent out.

Think about it…

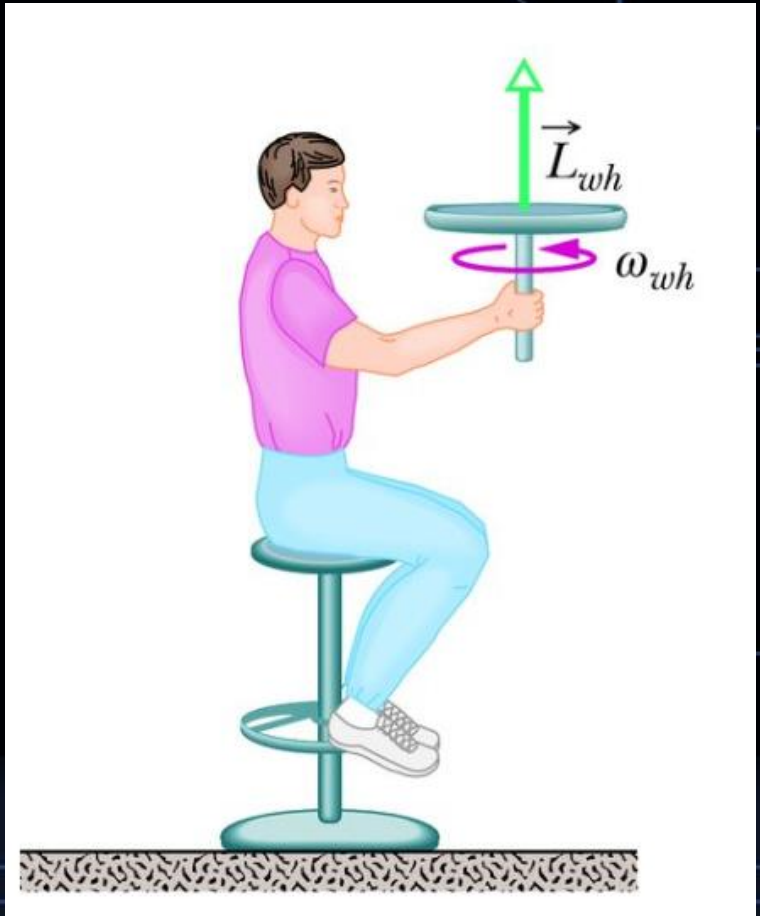
As shown in the figure, a student is sitting on a stool that can rotate freely about a vertical axis. The student, initially at rest, is holding a bicycle wheel whose rim is loaded with lead. The wheel is rotating as seen from overhead, the rotation is counterclockwise. The student now inverts the wheel. What will happen?



Think about it…

A. The student will still be at rest.
B. The student will rotate clockwise as seen from over head.
C. The student will rotate counterclockwise as seen from over head.
D. Others.

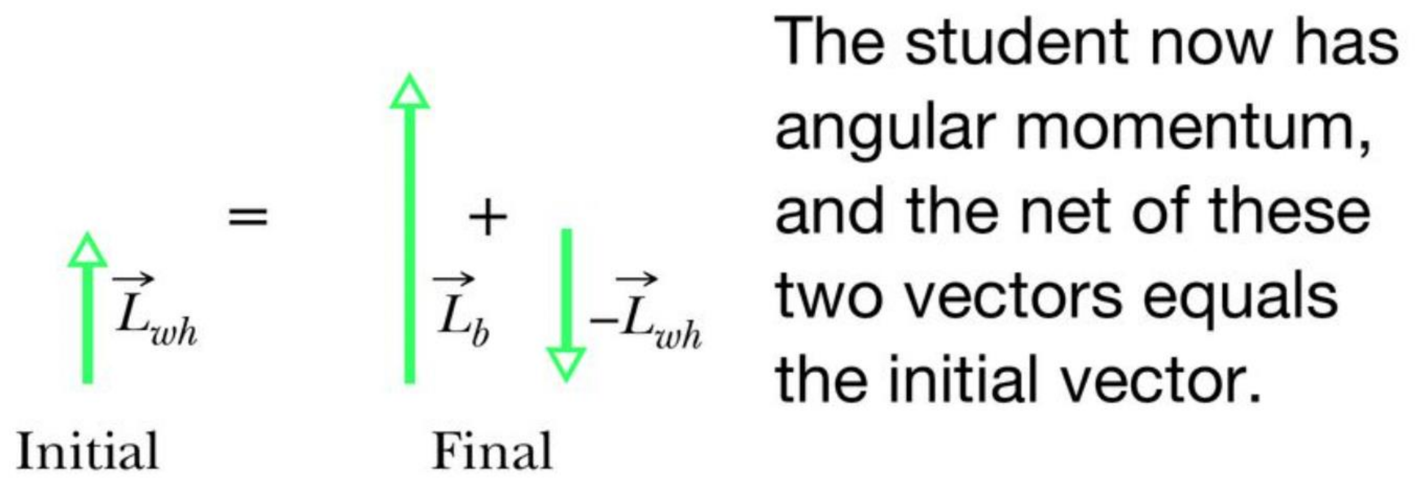
Why?

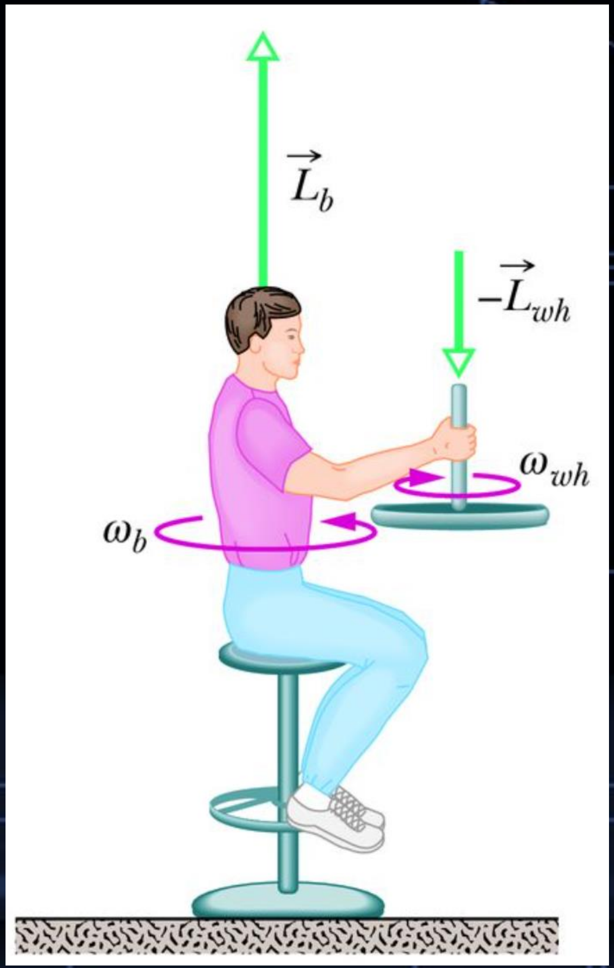


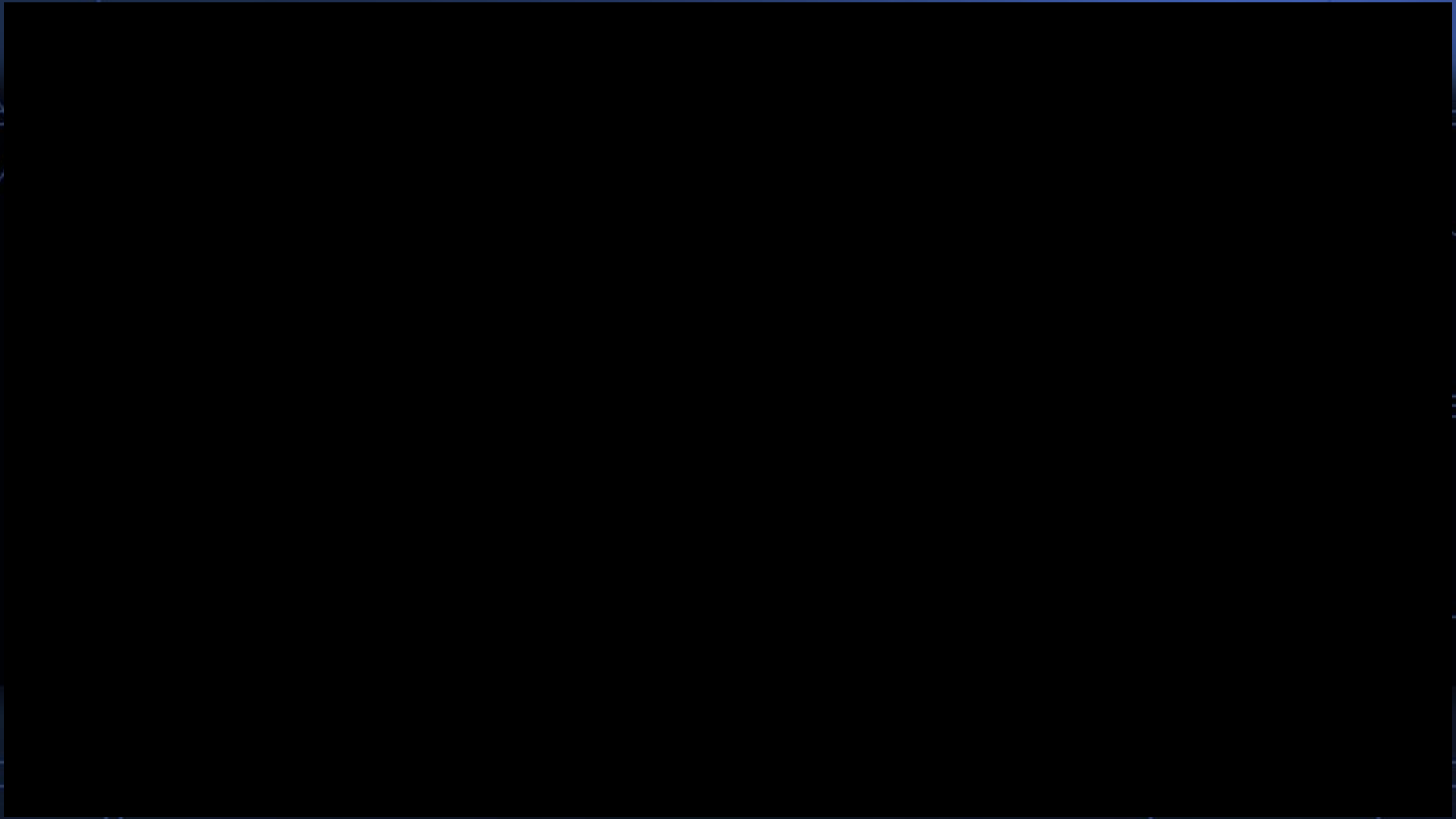
Conservation of angular momentum

C. The student will rotate counterclockwise as seen from over head.

Why?



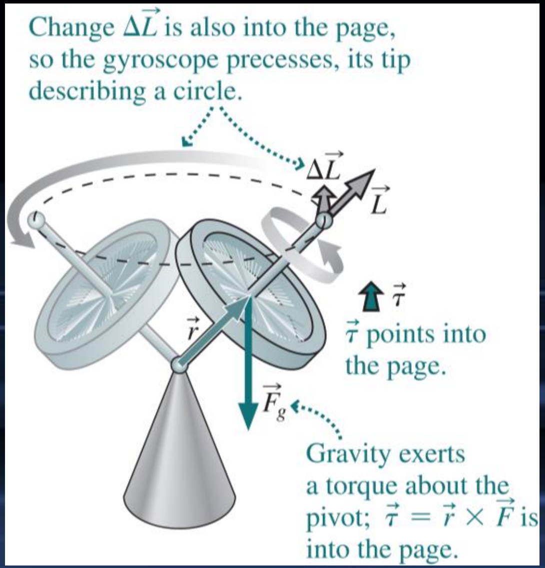


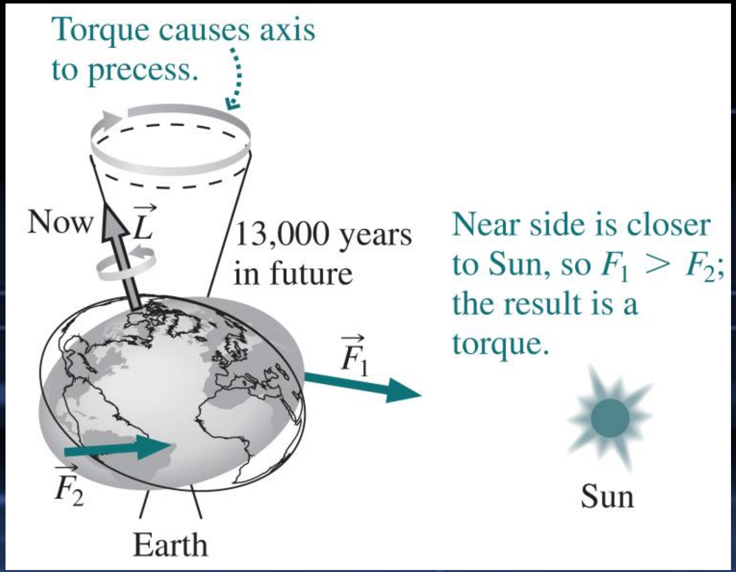


https://www.youtube.com/watch?v=iaauRiRX4do&t

Gyroscopes and Precession

- Precession occurs when there is a continual change in the rotational axis of a rotating body:
  - Precession can occur in a rotating gyroscope when an external torque acts on it, changing the direction (but not necessarily the magnitude) of its angular momentum vector.
  - As a result, the rotation axis undergoes circular motion:





Summary I

Pure Translation (Fixed Direction)		**Pure Rotation (Fixed Axis)**	
Position	x	Angular position	θ
Velocity	$v = dx/dt$	Angular velocity	$\omega = d\theta/dt$
Acceleration	$a = dv/dt$	Angular acceleration	$\alpha = d\omega/dt$
Mass	m	Rotational inertia	I
Newton's second law	$F_{net} = ma$	Newton's second law	$\tau_{net} = I\alpha$
Work	$W = \int F\, dx$	Work	$W = \int \tau\, d\theta$
Kinetic energy	$K = \frac{1}{2}mv^2$	Kinetic energy	$K = \frac{1}{2}I\omega^2$
Power (constant force)	$P = Fv$	Power (constant torque)	$P = \tau\omega$
Work–kinetic energy theorem	$W = \Delta K$	Work–kinetic energy theorem	$W = \Delta K$

Summary II

Translational		Rotational	
Force	$\vec{F}$	Torque	$\vec{\tau}\left(=\vec{r}\times\vec{F}\right)$
Linear momentum	$\vec{p}$	Angular momentum	$\vec{\ell}\left(=\vec{r}\times\vec{p}\right)$
Linear momentum[b]	$\vec{P}\left(=\Sigma\vec{p}_i\right)$	Angular momentum[b]	$\vec{L}\left(=\Sigma\vec{\ell}_i\right)$
Linear momentum[b]	$\vec{P}=M\vec{v}_{\text{com}}$	Angular momentum[c]	$L=I\omega$
Newton's second law[b]	$\vec{F}_{\text{net}}=\frac{d\vec{P}}{dt}$	Newton's second law[b]	$\vec{\tau}_{\text{net}}=\frac{d\vec{L}}{dt}$
Conservation law[d]	$\vec{P}=\text{a constant}$	Conservation law[d]	$\vec{L}=\text{a constant}$

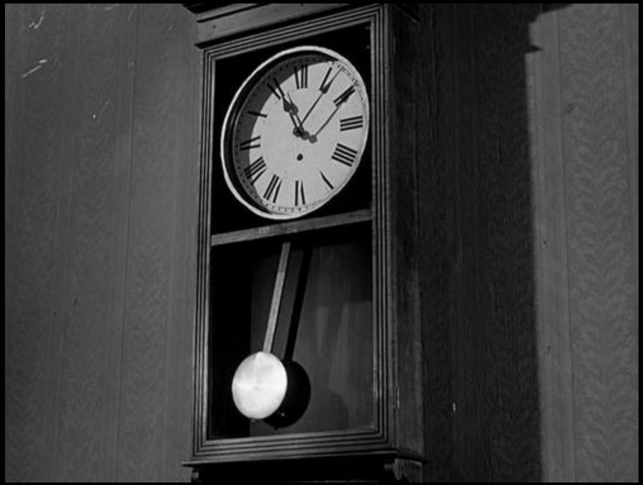
GENERAL PHYSICS B1
OSCILLATION & WAVE

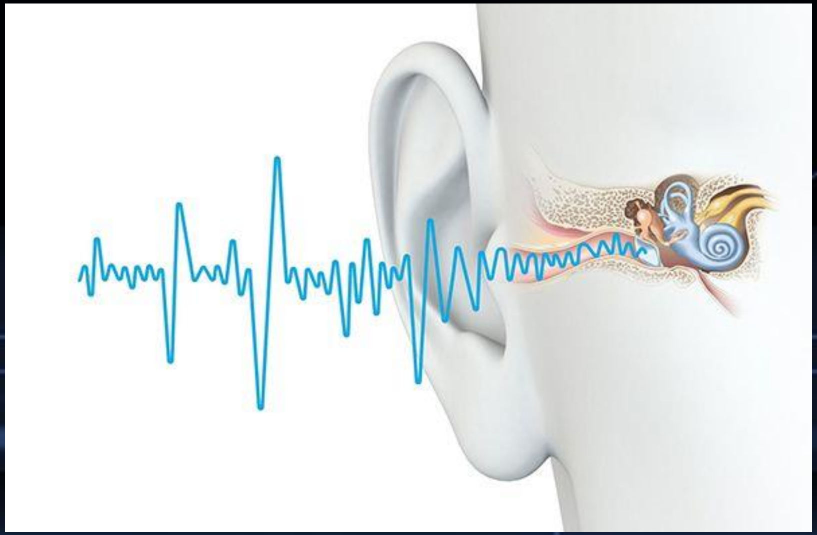
Simple Harmonic Oscillation
2022/11/08

Today’s topic

- Harmonic Oscillation
- Energy in Simple Harmonic Oscillation
- Pendulum

Examples of oscillation





https://studiousguy.com/simple-harmonic-motion-examples-everyday-life/

Simple Harmonic Motion (SHM)

- A particle that is oscillating about the origin of an x axis, repeatedly going left and right by identical amounts. A simple harmonic motion (SHM) is that the displacement can be described as a sinusoidal function of time t:

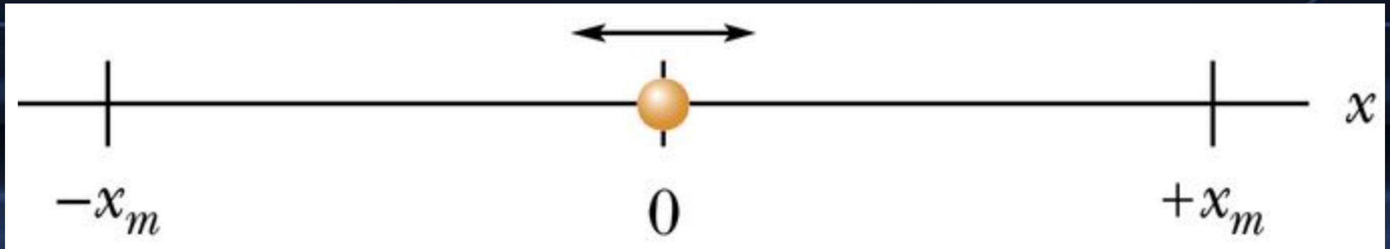
$$x(t) = x_m \cos(\omega t + \phi)$$ Phase

Displacement as a function of time t

Amplitude

Angular frequency

Phase constant

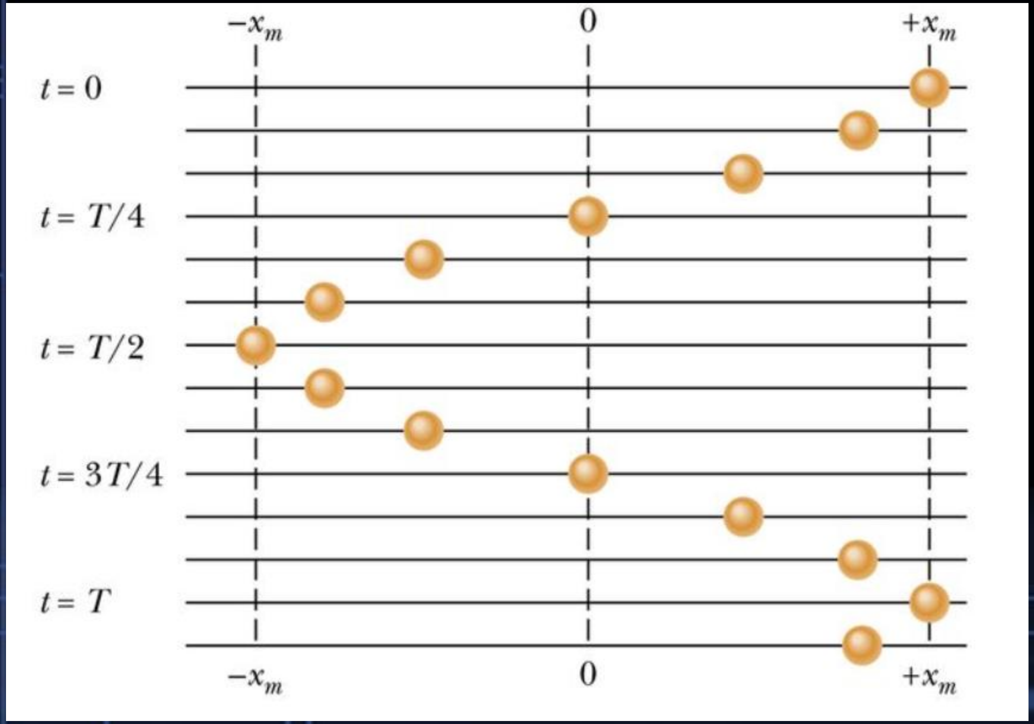


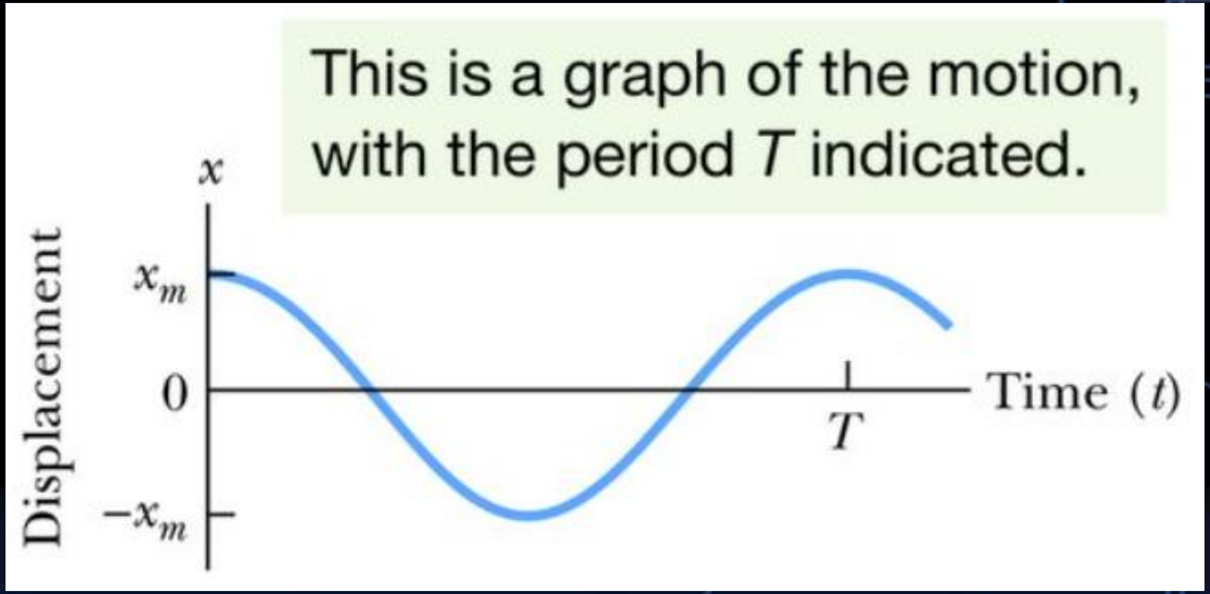
Simple Harmonic Motion

- The frequency f of the oscillation is the number of times per second that it completes a full oscillation (a cycle) and has the unit of hertz (abbreviated Hz)
- The time for one full cycle is the period T of the oscillation, which is: $T = \frac{1}{f}$

Simple Harmonic Motion

$$x(t) = x_m \cos(\omega t + \phi)$$





Think about it…

- A typical human heart rate is about 65 beats per minute. The corresponding period and frequency are:

a) period just over 1 s and frequency just under 1 Hz.

b) period just under 1 s and frequency just under 1 Hz.

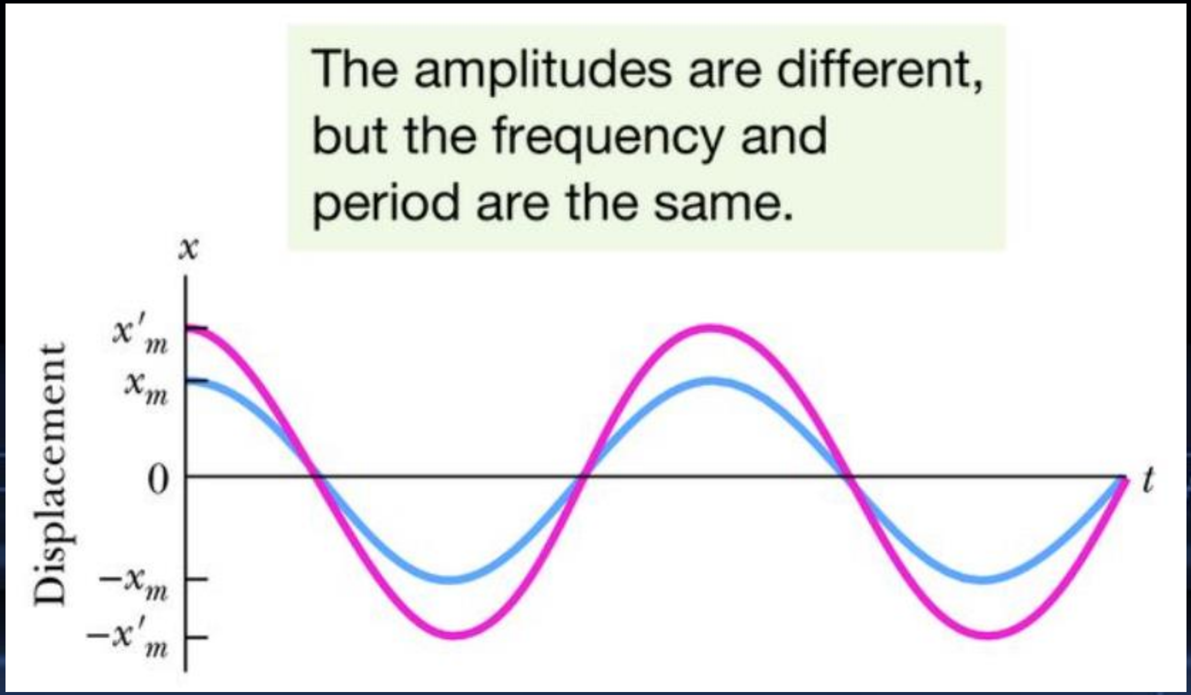
c) period just under 1 s and frequency just over 1 Hz.

d) period just over 1 minute and frequency of 70 Hz.

Amplitude

$$x(t) = x_m \cos(\omega t + \phi)$$

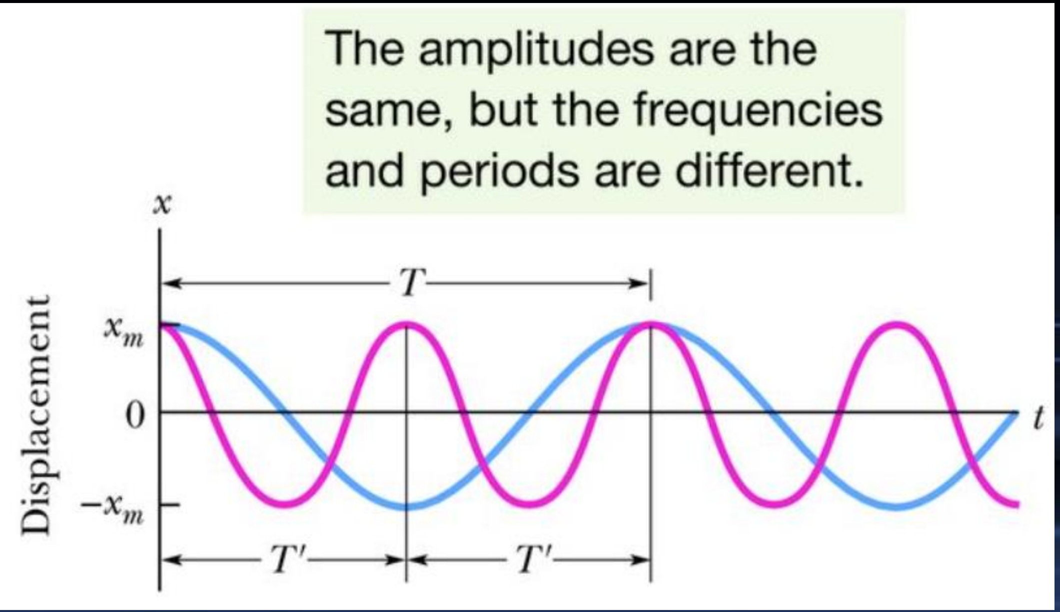
- The x_m determines how far the particle moves in its oscillations and is called the amplitude of the oscillations.



Angular frequency

$$x(t) = x_m \cos(\omega t + \phi)$$

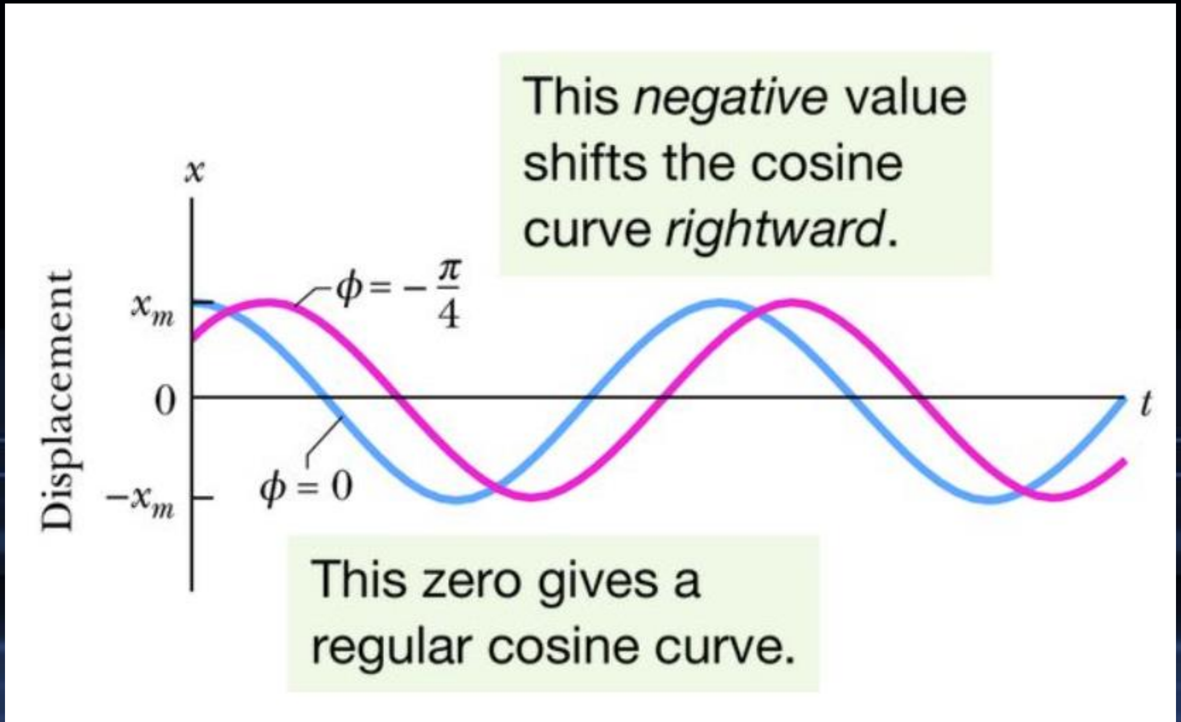
- The ω is the angular frequency of the motion. It is how fast the object change phase during the motion and related to the period and frequency by: $\omega = \frac{2\pi}{T} = 2\pi f$



Phase constant

$$x(t) = x_m \cos(\omega t + \phi)$$

- The ϕ is the phase constant of the motion, which represents the starting phase when t=0.

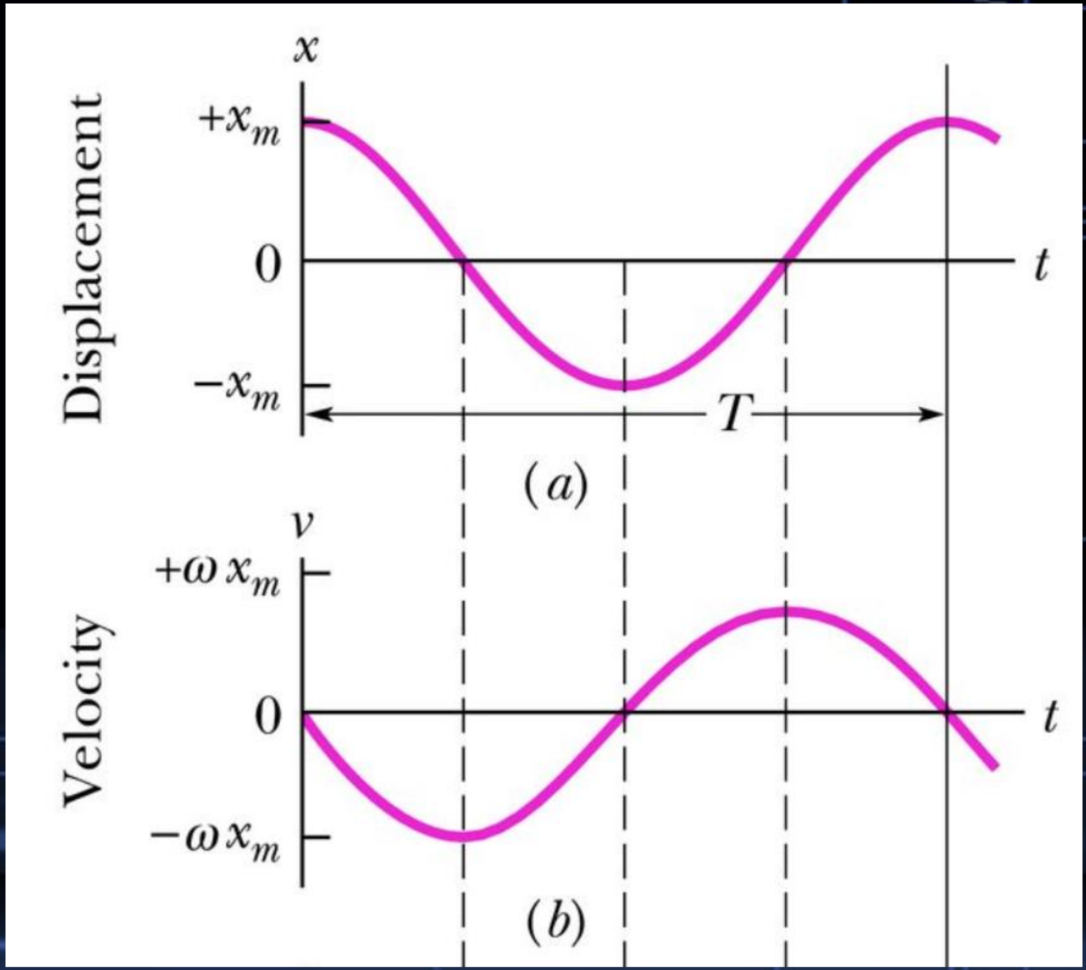


The Velocity of SHM

- By taking derivative of displacement respect to time, one can get velocity of SHM:

$$v(t) = \frac{dx(t)}{dt} = \frac{d}{dt}\left[x_m \cos(\omega t + \phi)\right]$$

$$v(t) = -\omega x_m \sin(\omega t + \phi) \quad \text{(velocity)}$$



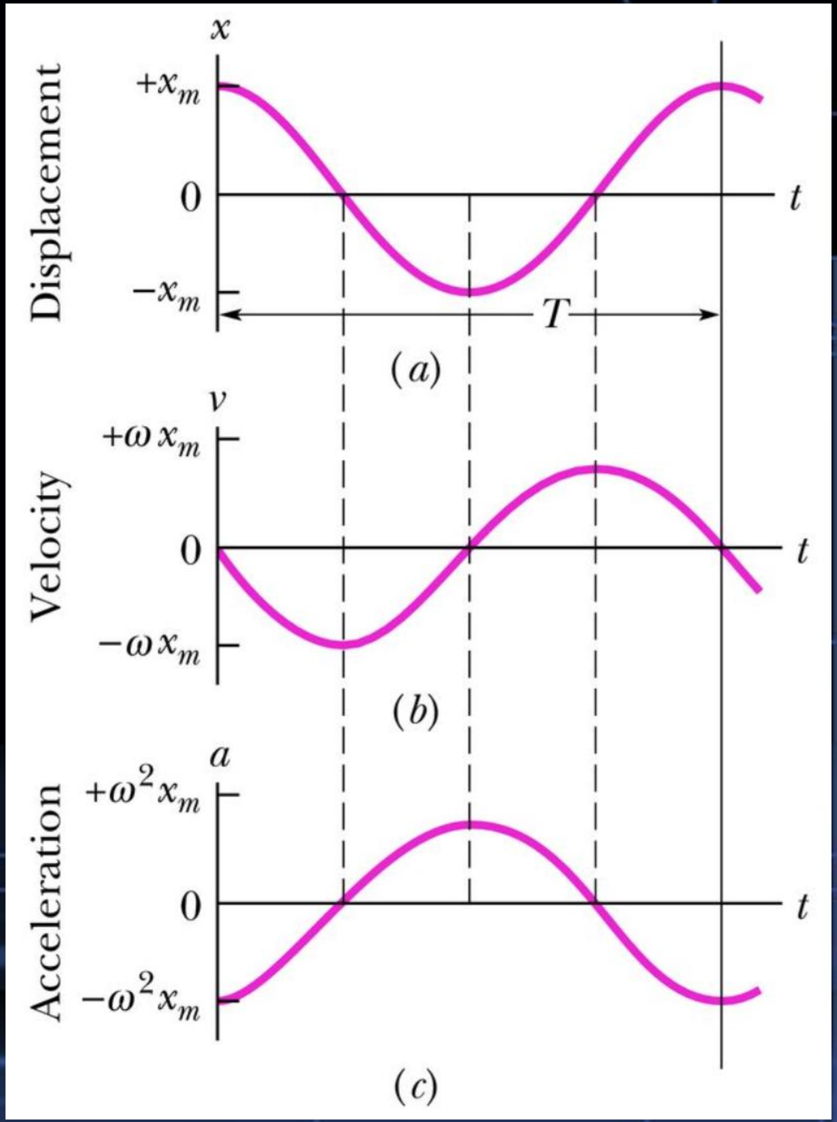
The acceleration of SHM

- By taking derivative of velocity respect to time, one can get velocity of SHM:

$$a(t) = -\omega^2 x_m \cos(\omega t + \phi) \quad \text{(acceleration)}$$

- We can also find that:

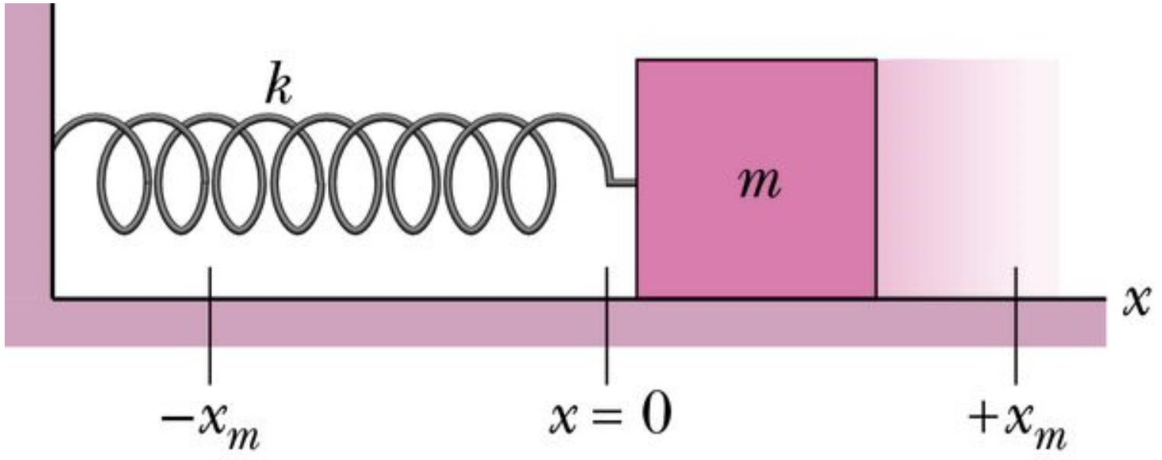
$$a(t) = -\omega^2 x(t)$$



The force law of simple harmonic oscillation

- From Newton's second law we can know that the force in the SHM should have the form of:

$$F = ma = m\left(-\omega^2 x\right) = -\left(m\omega^2\right)x$$



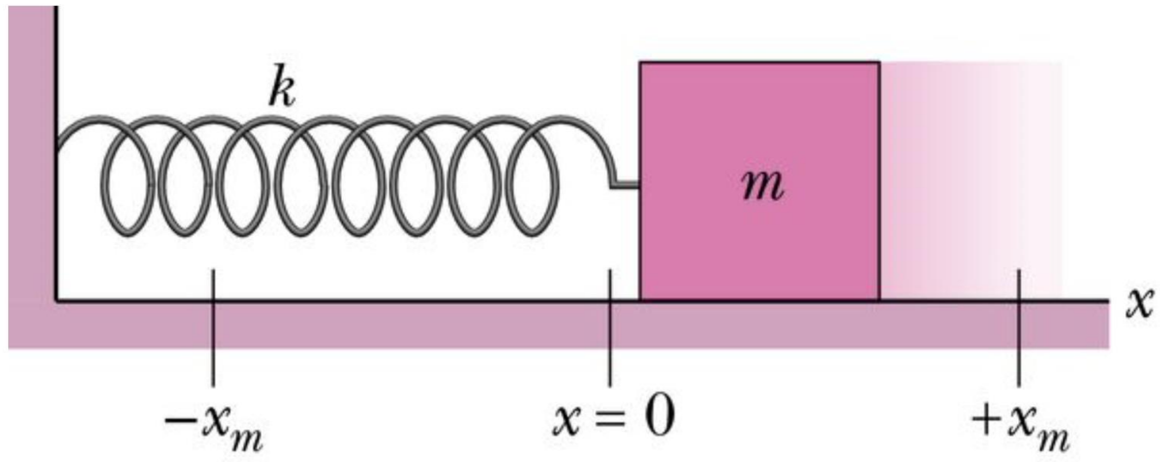
- By applying Hooke's law($F = -kx$), we have :

$$k = m\omega^2$$

The force law of simple harmonic oscillation

- From Newton's second law we can know that the force in the SHM should have the form of:

$$F = ma = m\left(-\omega^2 x\right) = -\left(m\omega^2\right)x$$



- Thus:

$$\omega = \sqrt{\frac{k}{m}} \quad \text{(angular frequency)}$$

$$T = 2\pi\sqrt{\frac{m}{k}} \quad \text{(period)}$$

Energy in SHM

- When we discussed about spring force, we know the potential energy due to spring force is:

$$U(t) = \frac{1}{2}kx^2 = \frac{1}{2}kx_m^2 \cos^2(\omega t + \phi)$$

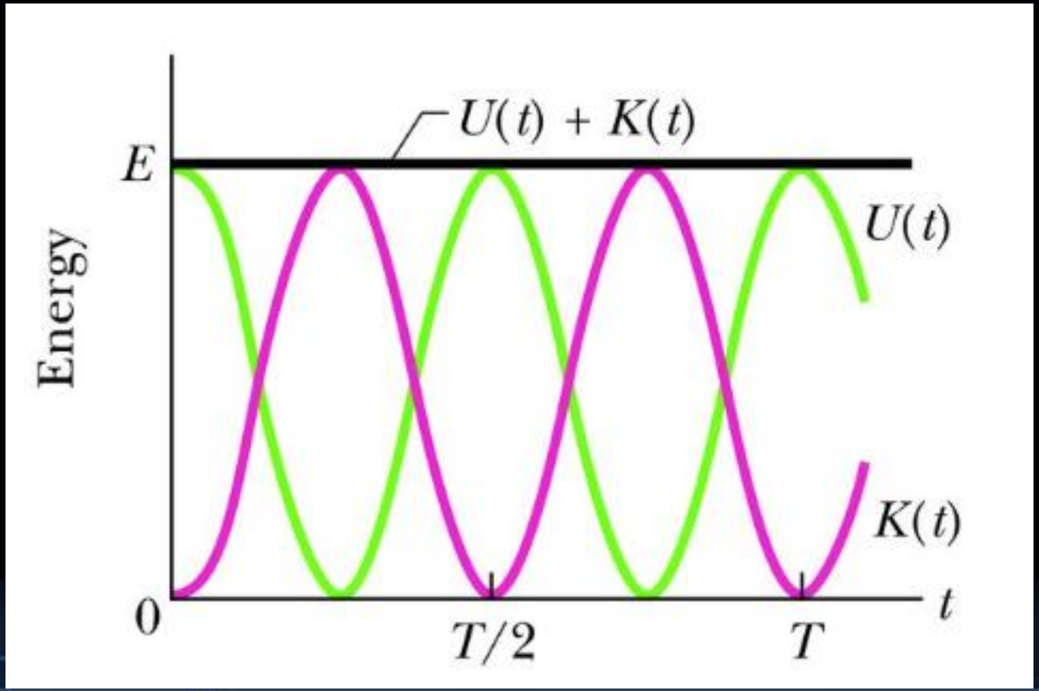
- The Kinetic energy:

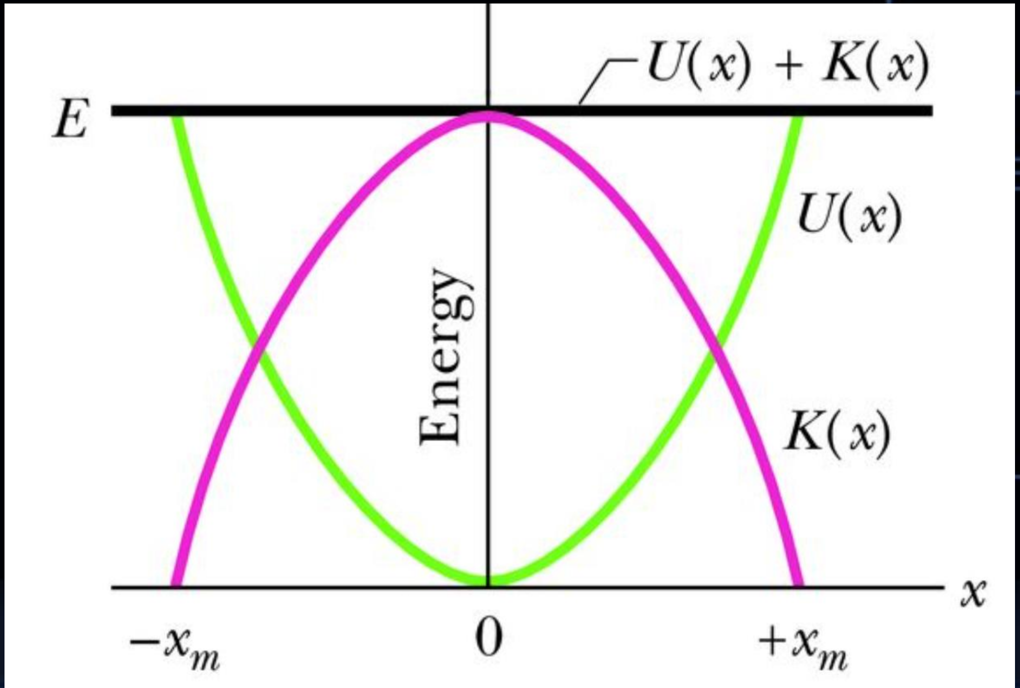
$$K(t) = \frac{1}{2}mv^2 = \frac{1}{2}kx_m^2 \sin^2(\omega t + \phi)$$

- The total energy is:

$$E = U + K = \frac{1}{2}kx_m^2$$

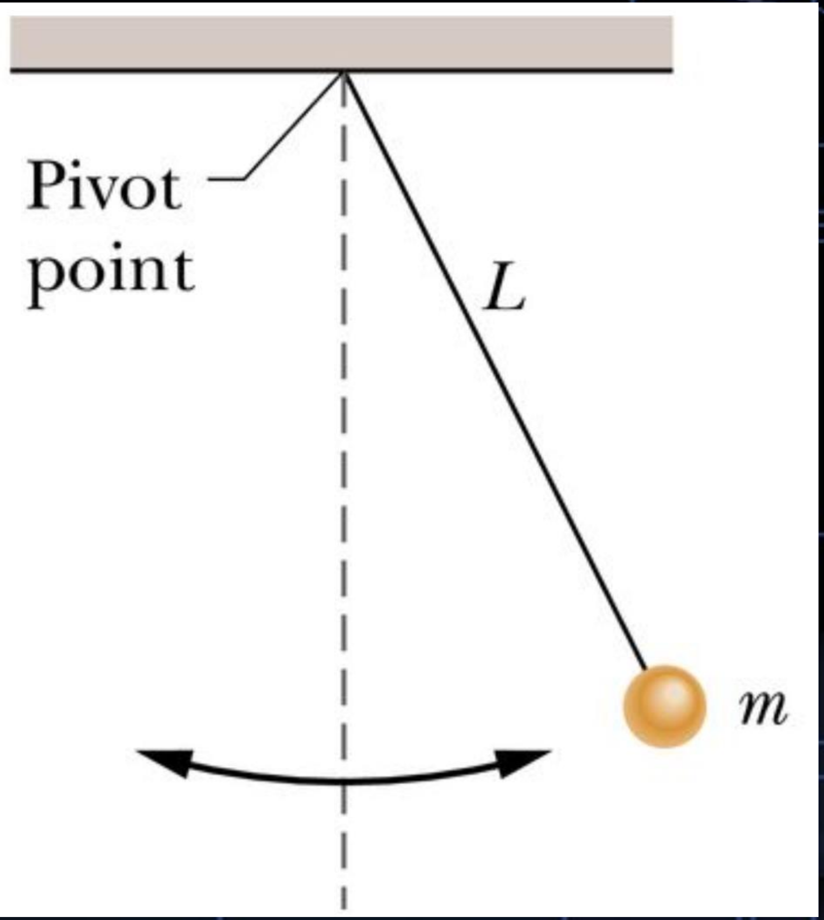
Energy in SHM





Simple pendulums

Consider a simple pendulum, which consists of a particle of mass m suspended from one end of an unstretchable, massless string of length L that is fixed at the other end. The mass is free to swing back and forth in the plane of the page, to the left and right of a vertical line through the pendulum's pivot point



Simple pendulums

- We can find the torque on m:

$$\tau = -L\left(F_g \sin \theta\right)$$

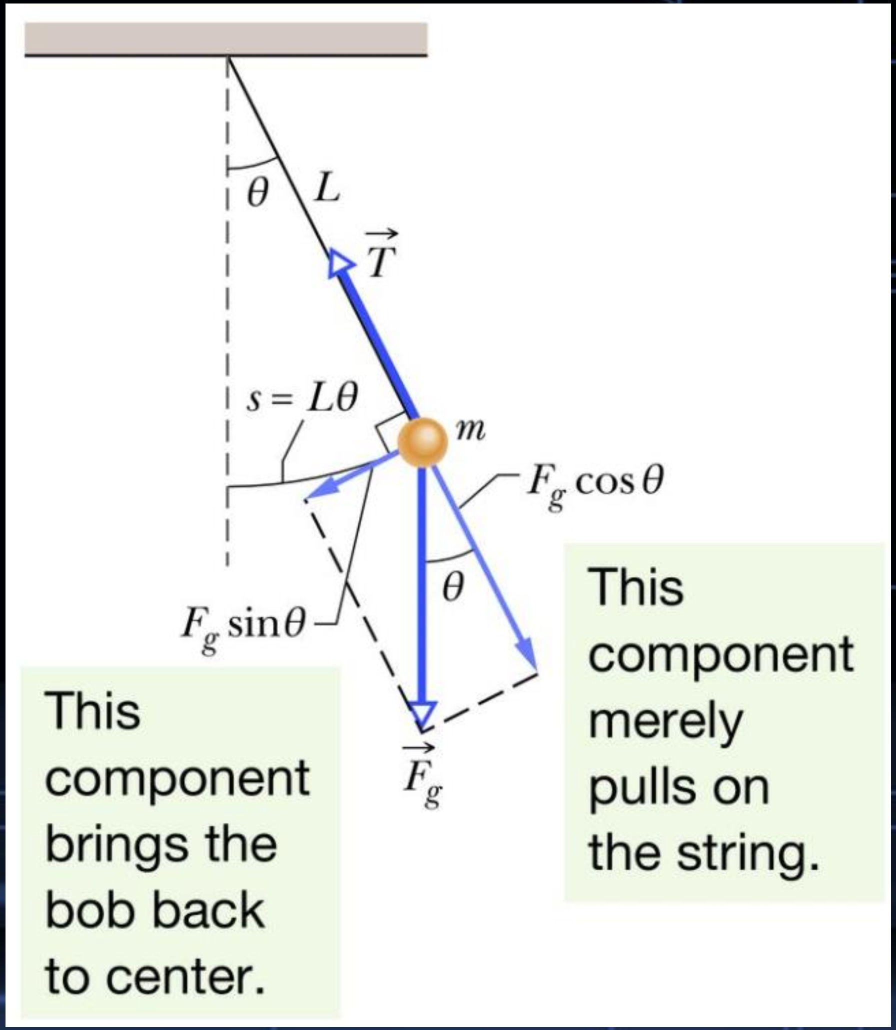
- This leads to:

$$-L\left(mg \sin \theta\right) = I\alpha$$

- If the angle is small:

$$\alpha = -\frac{mgL}{I}\theta$$

This is also SHM!



Simple pendulums

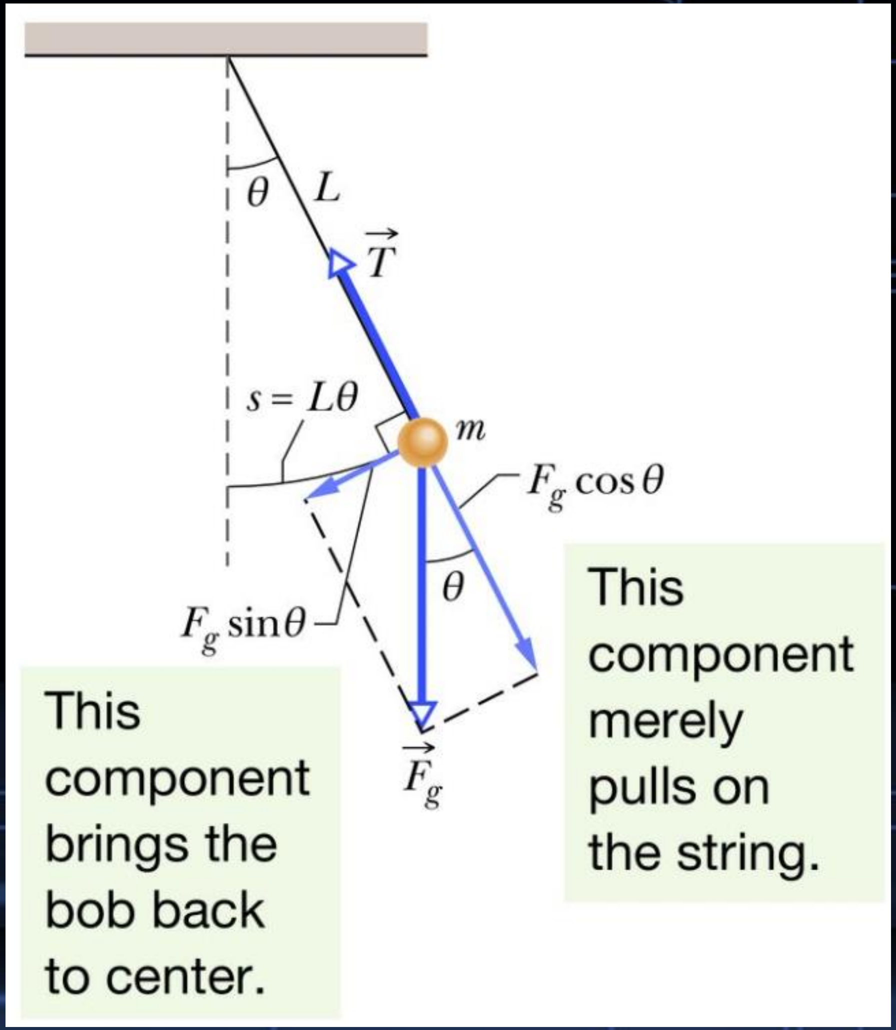
- We can find that:

$$\omega = \sqrt{\frac{mgL}{I}}$$

$$T = 2\pi\sqrt{\frac{I}{mgL}}$$

- With $I = mL^2$, we have

$$T = 2\pi\sqrt{\frac{L}{g}}$$



The Torsional Oscillator

- A disk suspended from a wire forms a torsional oscillator. The twisted wire (with torsional constant κ) exerts a restoring torque on the disk and is given by:

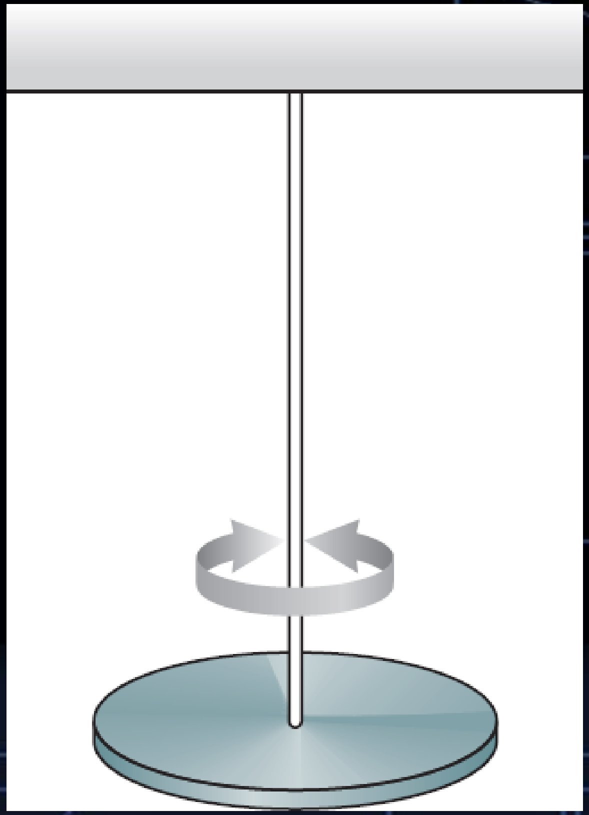
$$\tau = -\kappa\theta$$

- If the disk has a rotational inertia *I*, we have:

$$I\frac{d^2\theta}{dt^2} = -\kappa\theta$$

- The disk oscillates in SHM with angular frequency:

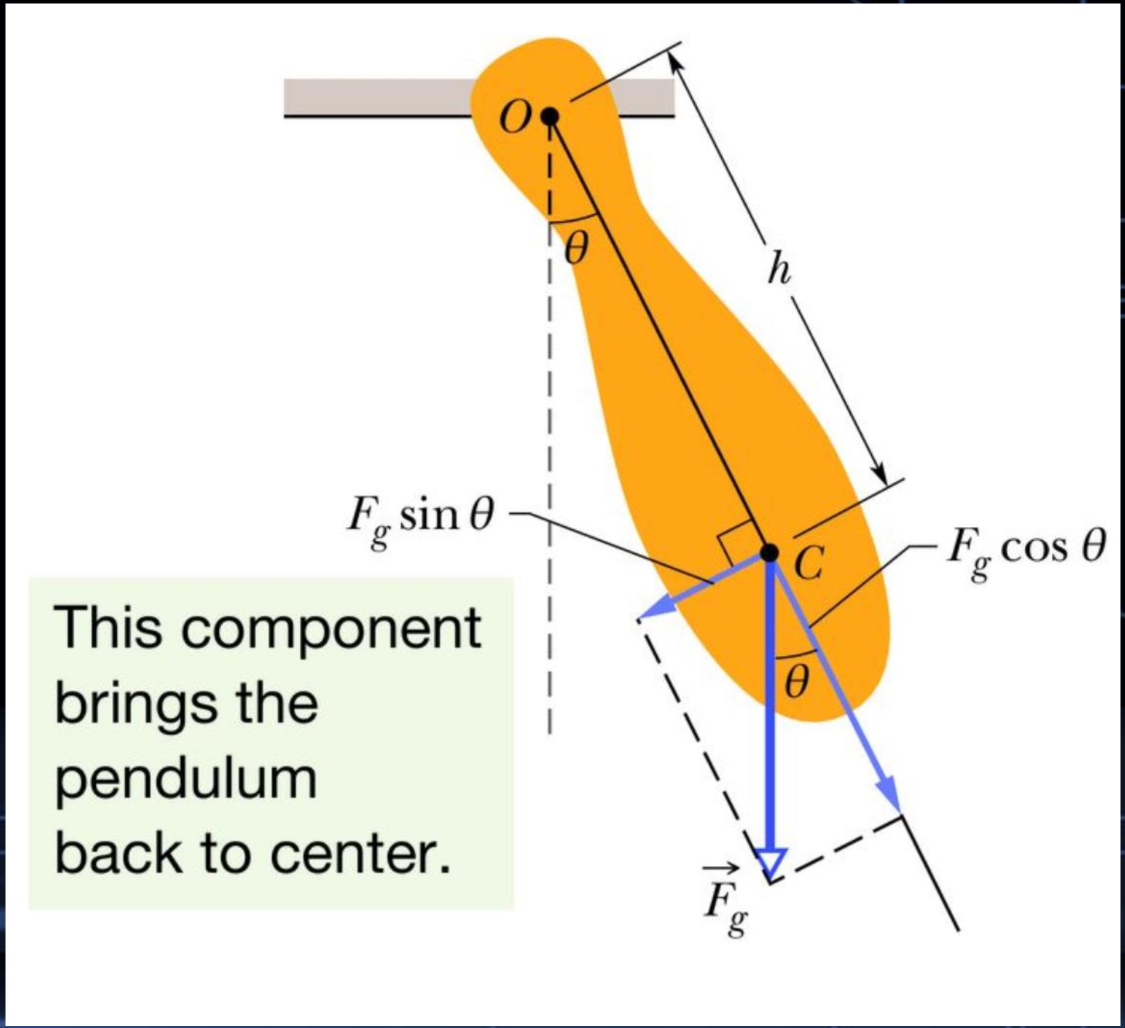
$$\omega = \sqrt{\kappa/I}$$



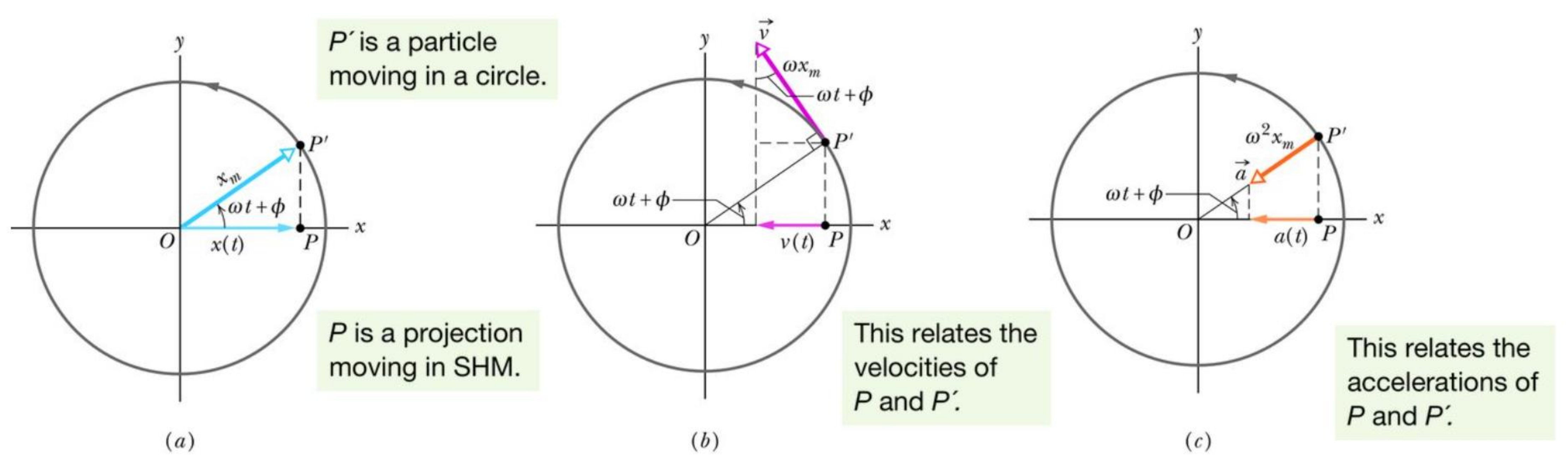
Physical Pendulums

- The analysis is the same to the COM even if the pendulum have a complicated distribution of mass(physical pendulum).

$$T = 2\pi \sqrt{\frac{I}{mgh}}$$



SHM and Uniform Circular Motion



- Simple harmonic motion is the projection of uniform circular motion on a diameter of the circle in which the circular motion occurs.

Summary

- Oscillatory motion is periodic motion that results from a force or torque that tends to restore a system to a point of stable equilibrium.
- In SHM, the restoring force or torque is directly proportional to displacement:
  - The mass-spring system is the paradigm simple harmonic oscillator.

$$\omega = \sqrt{\frac{k}{m}} \quad \text{(angular frequency)}$$

$$T = 2\pi\sqrt{\frac{m}{k}} \quad \text{(period)}$$