Course announcement

- Midterm exam’s grade will be announced later this week.
- A midterm warning will be sent out according to homework sets and midterm exam.

8	11/1(Tue.)	**Many Particles Motion and Rotation:** rotation
8	11/4(Fri.)	**Many Particles Motion and Rotation:** torque & angular momentum
9	11/8(Tue.)	**Oscillation and Waves:** simple harmonic oscillation
9	11/11(Fri.)	**Oscillation and Waves:** damped and forced oscillation (Homework3)

GENERAL PHYSICS B1
MANY PARTICLES MOTION & ROTATION

Rotation
2022/11/01

Today’s topic

- Variables in rotation
- Relationship between linear motion and rotation
- Kinetic energy of rotational motion and rotational inertia

Final goal of this chapter

Pure Translation (Fixed Direction)		Pure Rotation (Fixed Axis)	
Position	x	Angular position	θ
Velocity	$v = dx/dt$	Angular velocity	$\omega = d\theta/dt$
Acceleration	$a = dv/dt$	Angular acceleration	$\alpha = d\omega/dt$
Mass	m	Rotational inertia	I
Newton's second law	$F_{net} = ma$	Newton's second law	$\tau_{net} = I\alpha$
Work	$W = \int F\,dx$	Work	$W = \int \tau\,d\theta$
Kinetic energy	$K = \frac{1}{2}mv^2$	Kinetic energy	$K = \frac{1}{2}I\omega^2$
Power (constant force)	$P = Fv$	Power (constant torque)	$P = \tau\omega$
Work–kinetic energy theorem	$W = \Delta K$	Work–kinetic energy theorem	$W = \Delta K$

Motion in different degrees of freedom

- so far we have examined the motion of translation, in which an object moves along a straight or curved line,
- The dynamics can be in different degrees of freedom.
- We now turn to the motion of rotation, in which an object turns about an axis

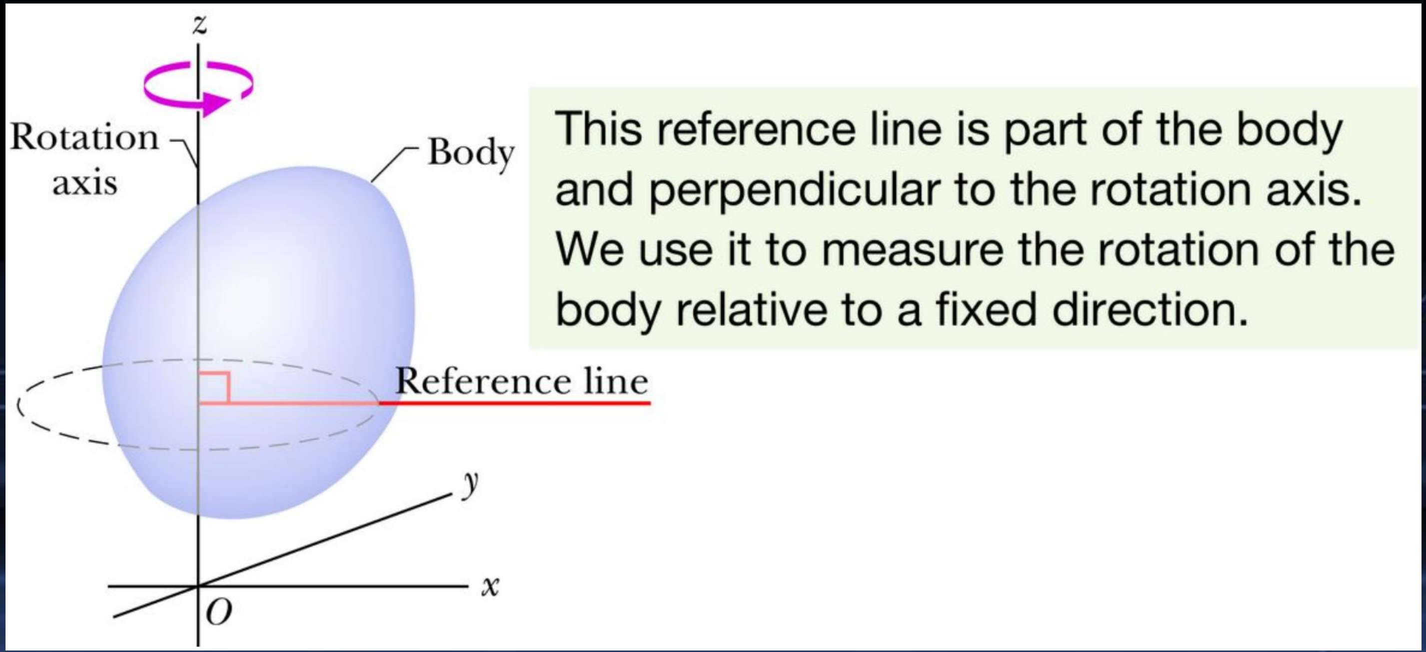
Translational motion vs rotational motion

https://tenor.com/view/racing-speed-skating-team-canada-olympics-racing-competition-gif-24961817

https://powerforwardblog.wordpress.com/2014/02/05/understanding-olympic-figure-skating-a-gif-guide-to-spins/

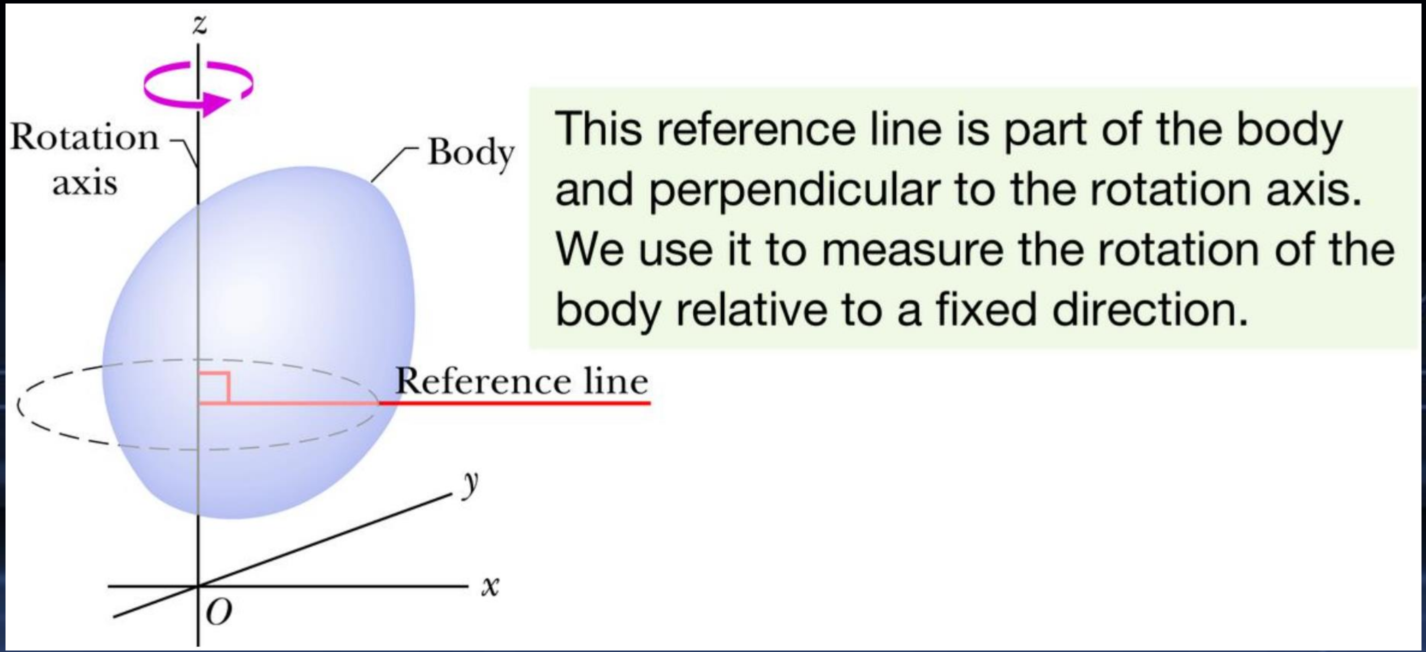
The axis of rotation

- In the following figure, a rigid body of arbitrary shape in rotation about a fixed axis, called the axis of rotation or the rotation axis.



The axis of rotation

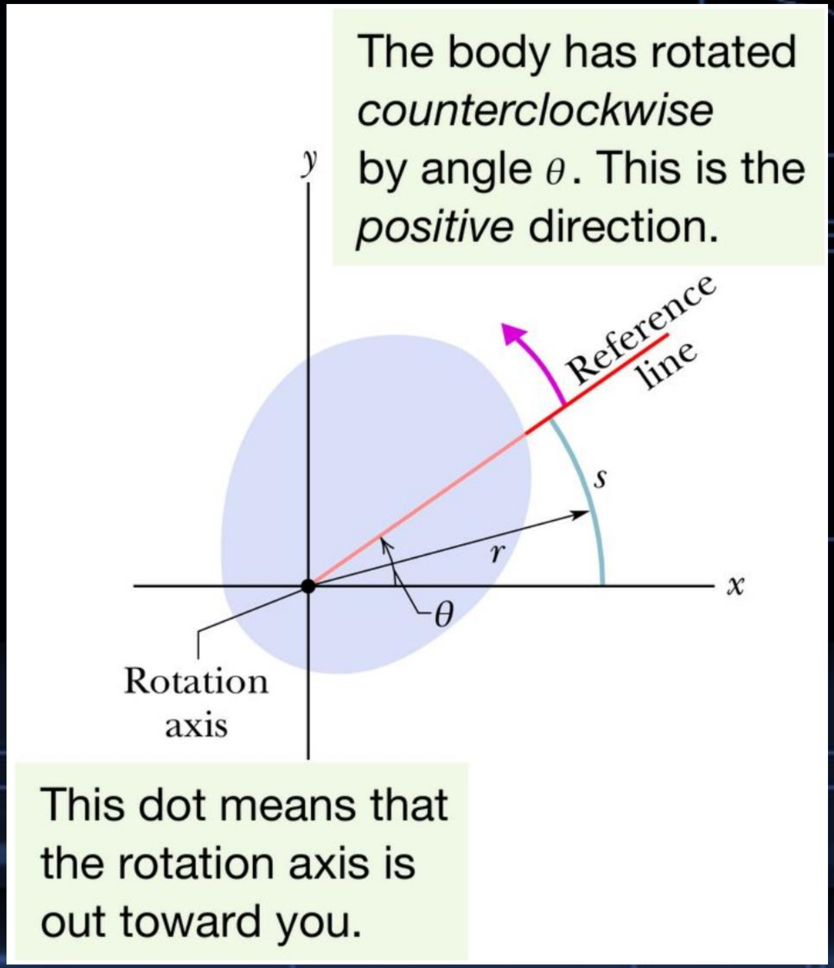
- In pure rotation (angular motion), every point of the body moves in a circle whose center lies on the axis of rotation, and every point moves through the same angle during a particular time interval.



Angular position

- In pure rotation (angular motion), every point of the body moves in a circle whose center lies on the axis of rotation, and every point moves through the same angle during a particular time interval.
- The angular position θ is measured relative to the positive direction of the x axis.

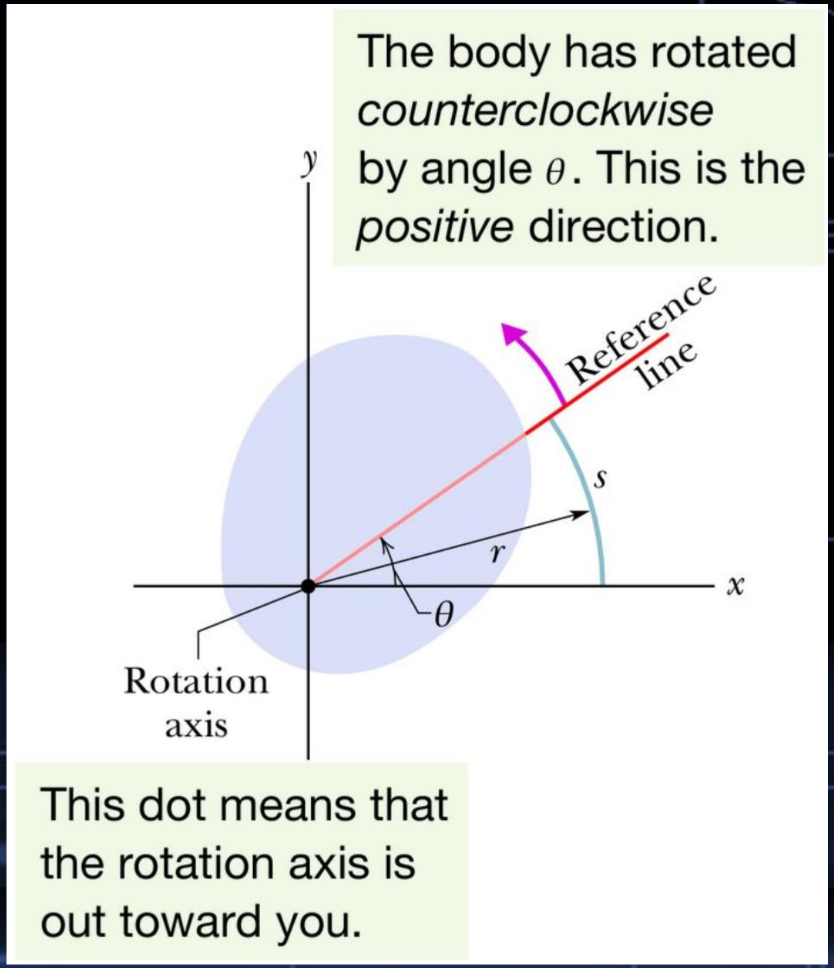
$$\theta = \frac{s}{r}$$



Angular position

- An angle defined in this way is measured in radians (rad).
- The radian, being the ratio of two lengths, is a pure number and thus has no dimension, and:

$$1\, rev = 360° = \frac{2\pi r}{r} = 2\pi\, rad$$

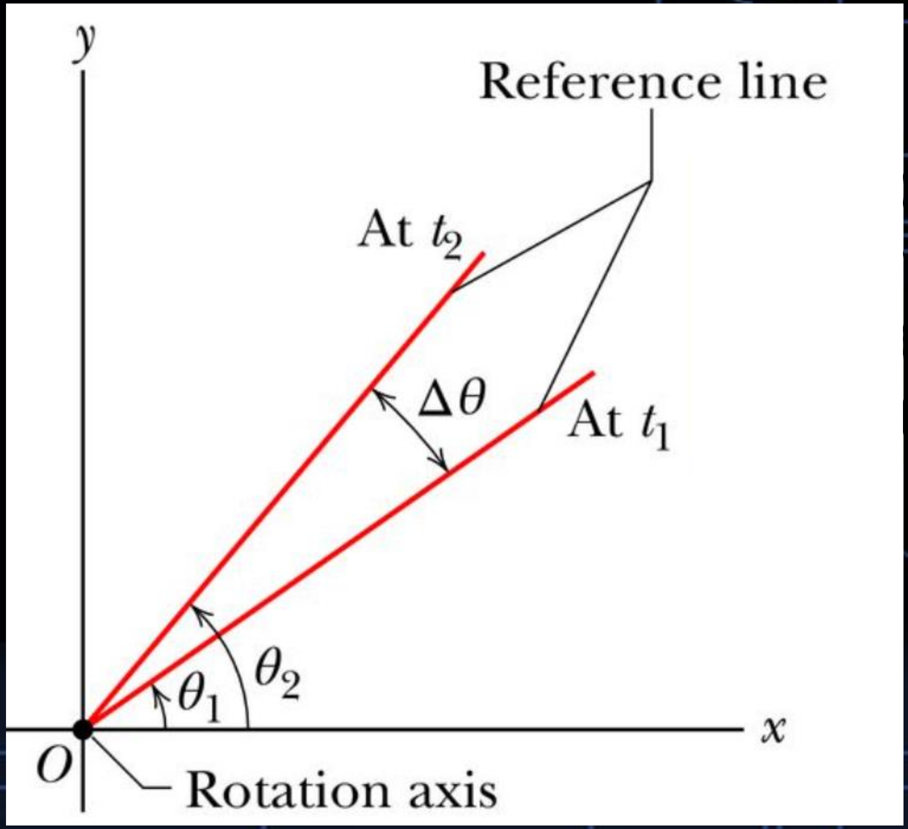


Angular displacement

If the body rotates about the rotation, changing the angular position of the reference line from θ1 to θ2, the body undergoes an angular displacement Δθ given by:

$$\Delta\theta = \theta_2 - \theta_1$$

This definition of angular displacement holds not only for the rigid body as a whole but also for every particle within that body.

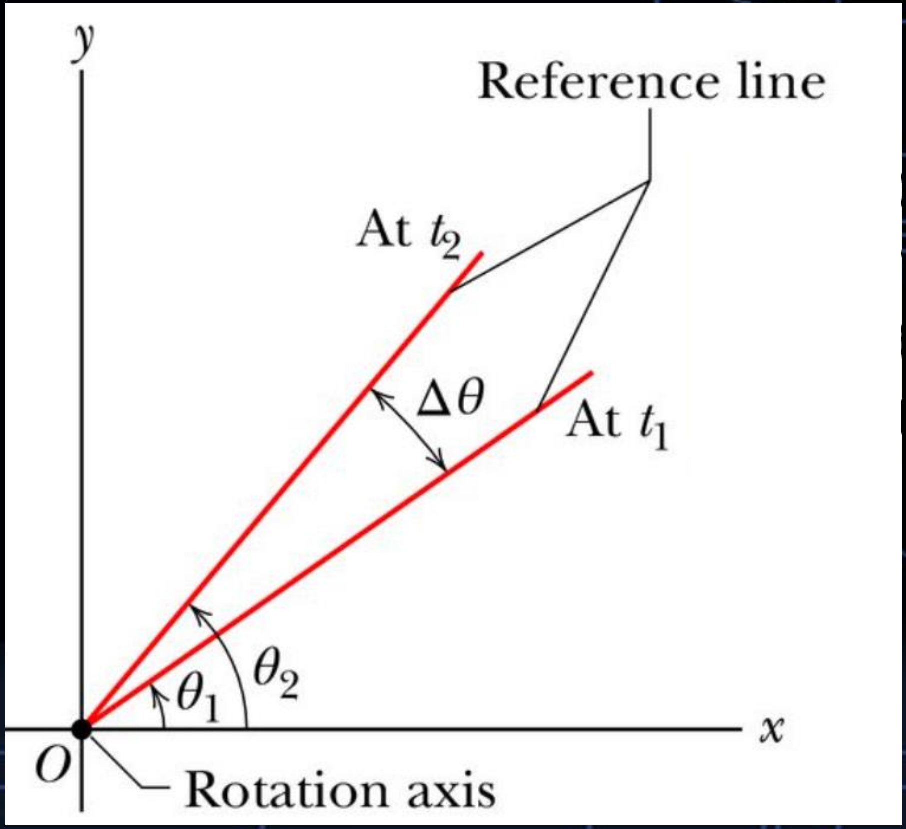


Angular displacement

Angular displacement Δθ given by:

$$\Delta\theta = \theta_2 - \theta_1$$

- An angular displacement in the counterclockwise direction is positive, and one in the clockwise direction is negative.



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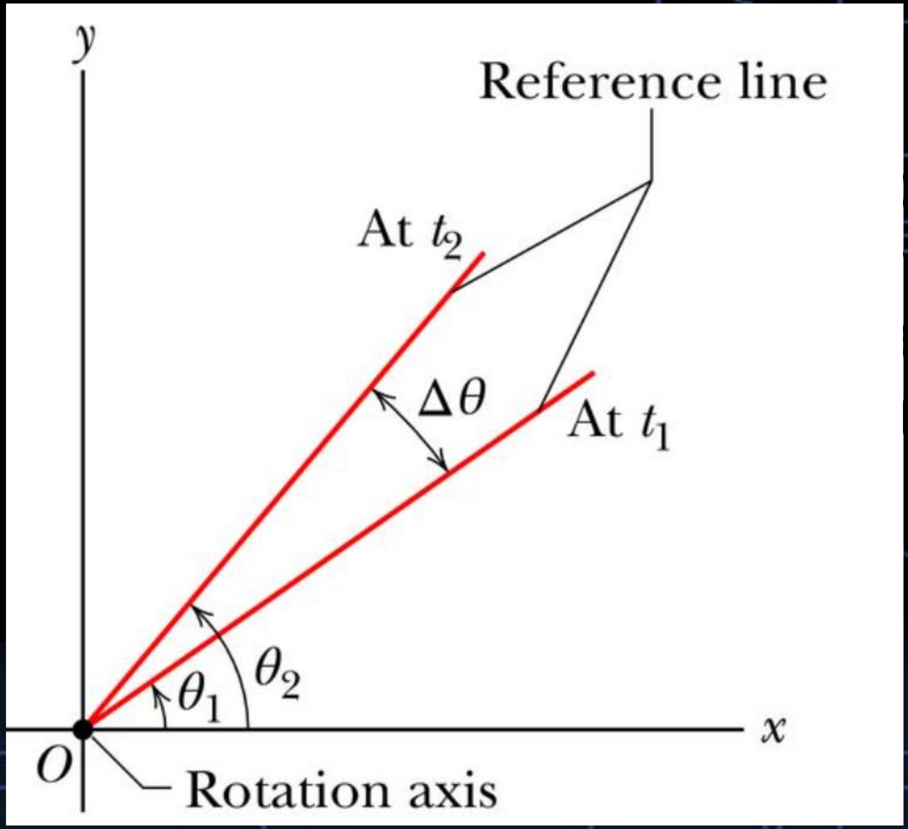
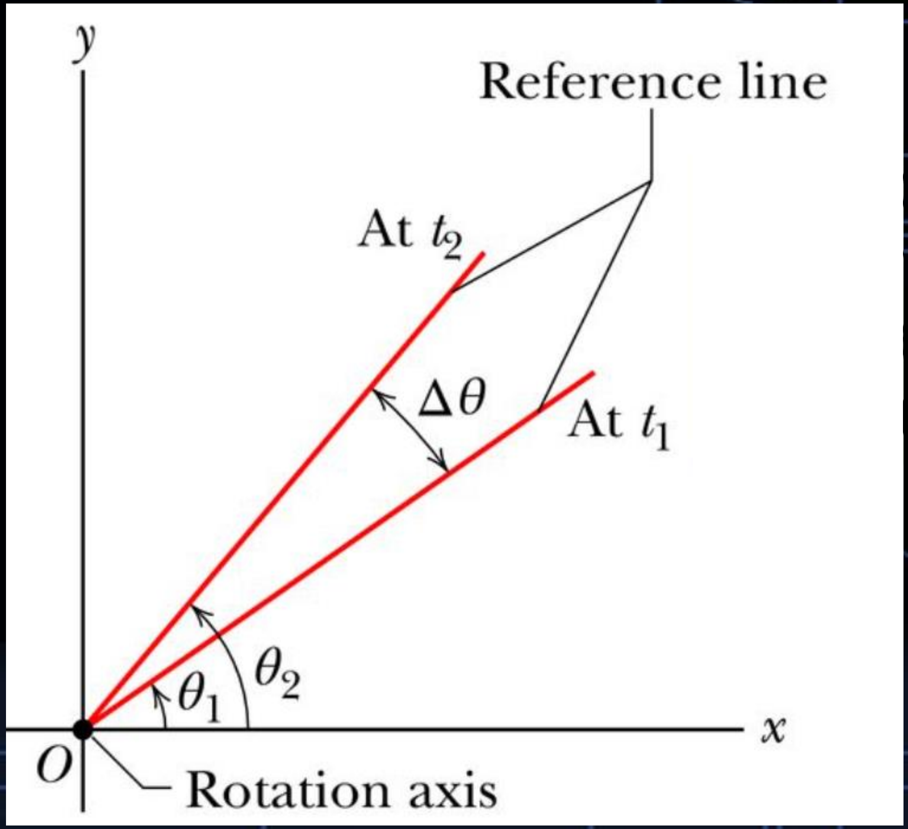


Angular velocity

Suppose that our rotating body is at angular position θ1 at time t1 and at angular position θ2 at time. We define the average angular velocity of the body in the time interval Δt from t1 to t2 to be:

$$\omega_{avg} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t}$$

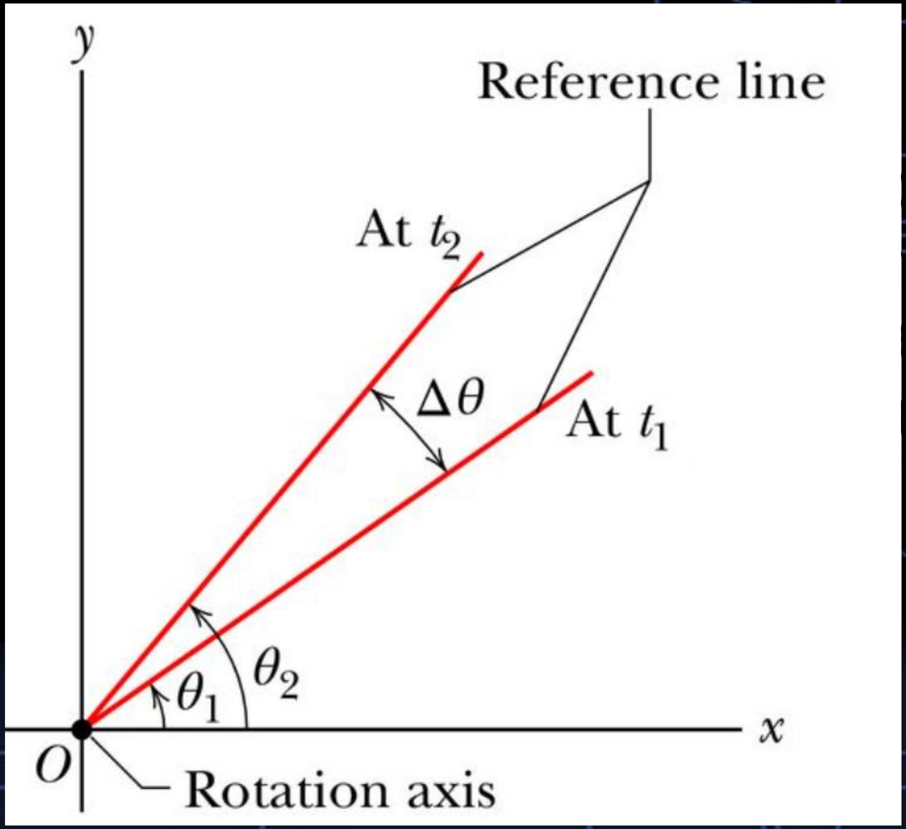


Angular velocity

The (instantaneous) angular velocity ω, with which we shall be most concerned, is the limit of the ratio as Δt approaches zero. Thus,:

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta \theta}{\Delta t} = \frac{d\theta}{dt}$$

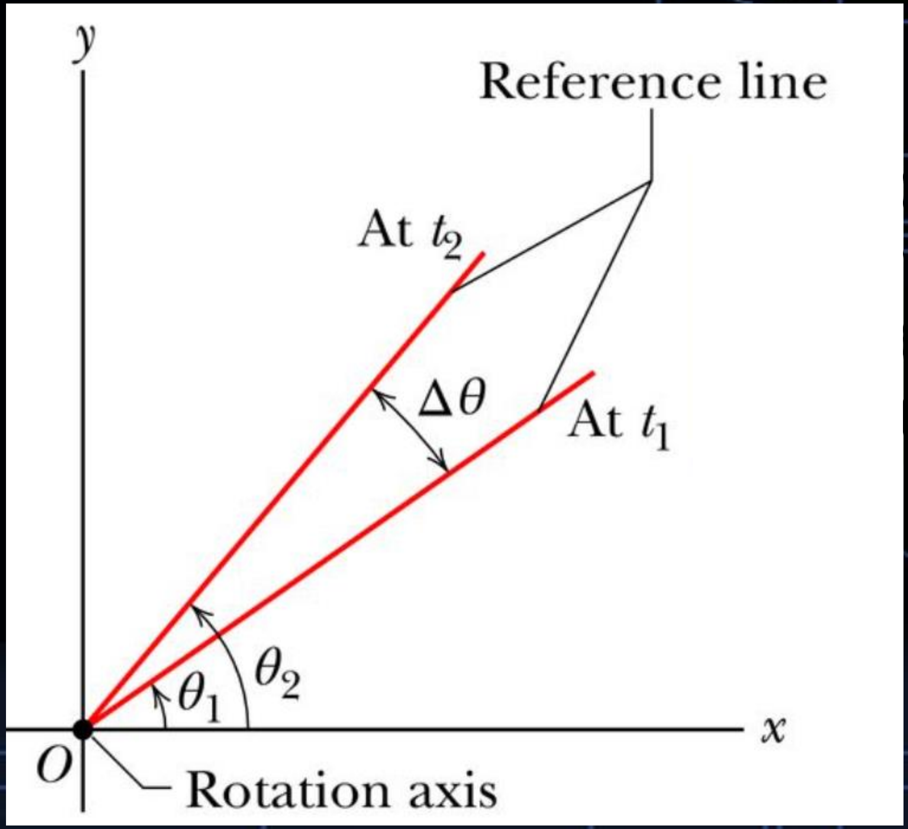
The angular velocity ω is positive if the body rotates counterclockwise.



Angular acceleration

If the angular velocity of a rotating body is not constant, then the body has an angular acceleration. Let ω2 and ω1 be its angular velocities at times t2 and t1, respectively. The average angular acceleration:

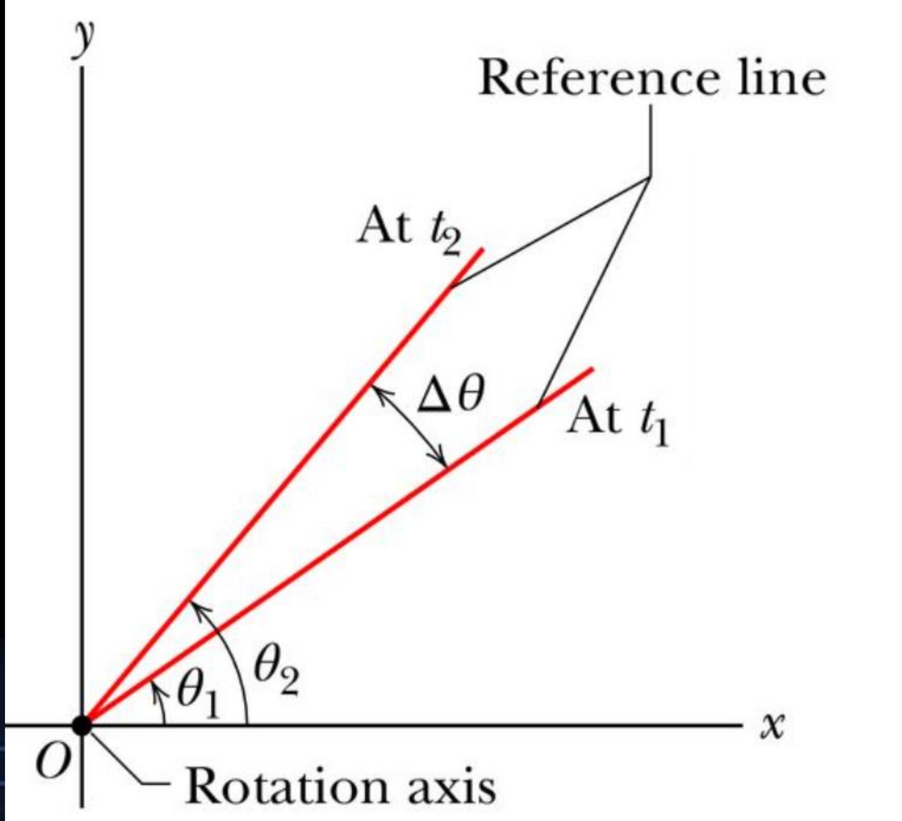
$$\alpha_{avg} = \frac{\omega_2 - \omega_1}{t_2 - t_1} = \frac{\Delta\omega}{\Delta t}$$



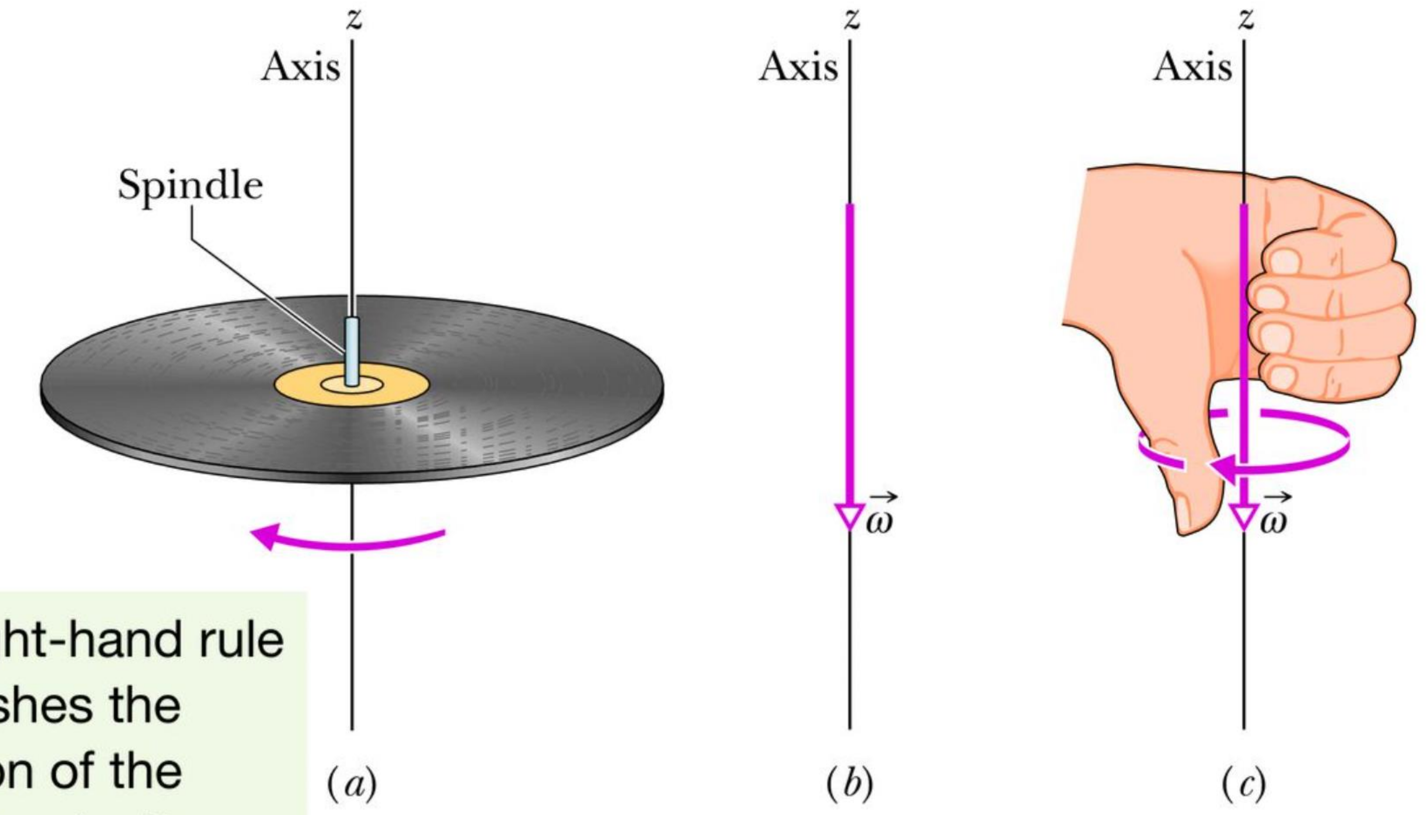
Angular acceleration

The (instantaneous) angular acceleration α, with which we shall be most concerned, is the limit of this quantity as Δt approaches zero. Thus,:

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt}$$



Angular velocity as a vector

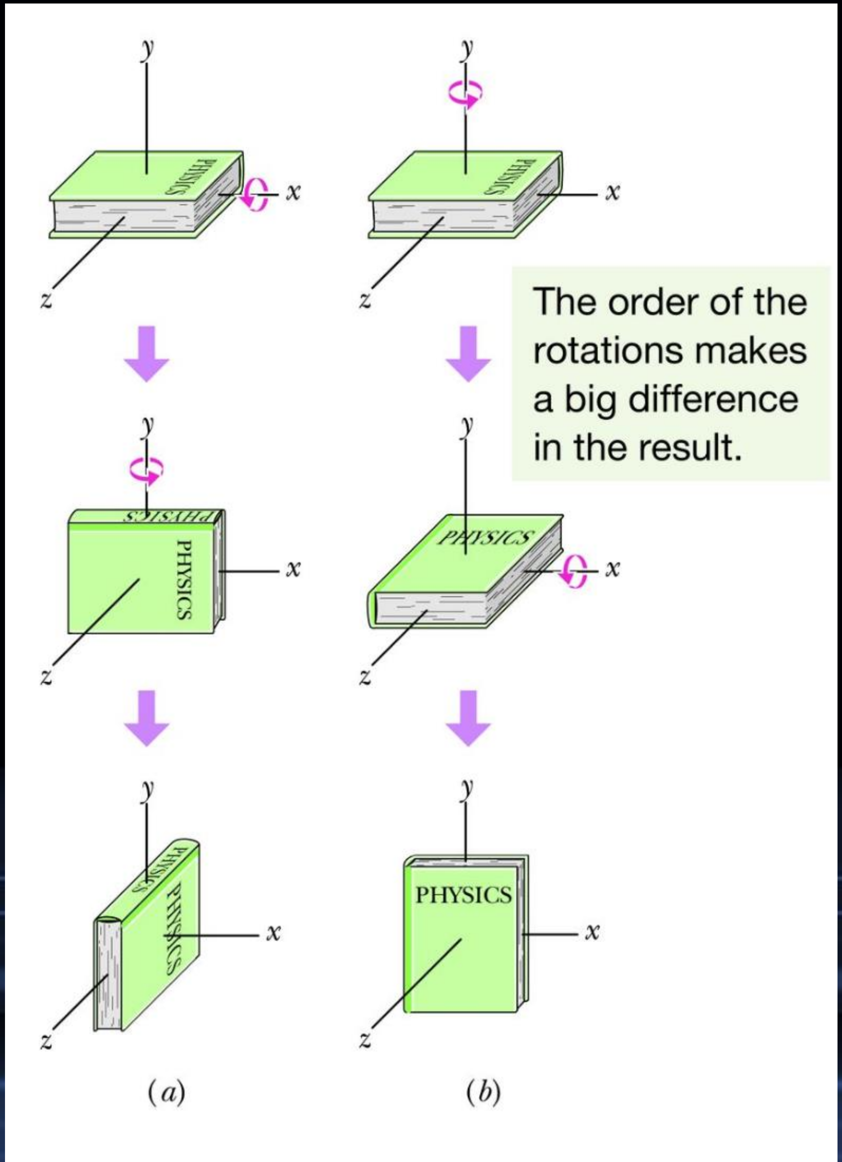


This right-hand rule establishes the direction of the angular velocity vector.

Angular displacement cannot be a vector

To be represented as a vector, a quantity must also obey the rules of vector addition, one of which says that if you add two vectors, the order in which you add them does not matter. Angular displacements fail this test.

Angular displacement cannot be a vector



Rotation with constant angular acceleration

In pure rotation, the case of constant angular acceleration is also important, and a parallel set of equations similar to linear motion holds.

Rotation with constant angular acceleration

Linear Equation	Missing Variable		Angular Equation
$v = v_0 + at$	$x - x_0$	$\theta - \theta_0$	$\omega = \omega_0 + \alpha t$
$x - x_0 = v_0 t + \frac{1}{2}at^2$	v	ω	$\theta - \theta_0 = \omega_0 t + \frac{1}{2}at^2$
$v^2 = v_0^2 + 2a\left(x - x_0\right)$	t	t	$\omega^2 = \omega_0^2 + 2a\left(\theta - \theta_0\right)$
$x - x_0 = \frac{1}{2}\left(v_0 + v\right)t$	a	α	$\theta - \theta_0 = \frac{1}{2}\left(\omega_0 + \omega\right)t$
$x - x_0 = vt - \frac{1}{2}at^2$	v_0	ω_0	$\theta - \theta_0 = \omega t - \frac{1}{2}\alpha t^2$

Relation between linear and angular variable

- A point in a rigid rotating body, at a perpendicular distance r from the rotation axis, moves in a circle with radius r. If the body rotates through an angle θ, the point moves along an arc with length s given by

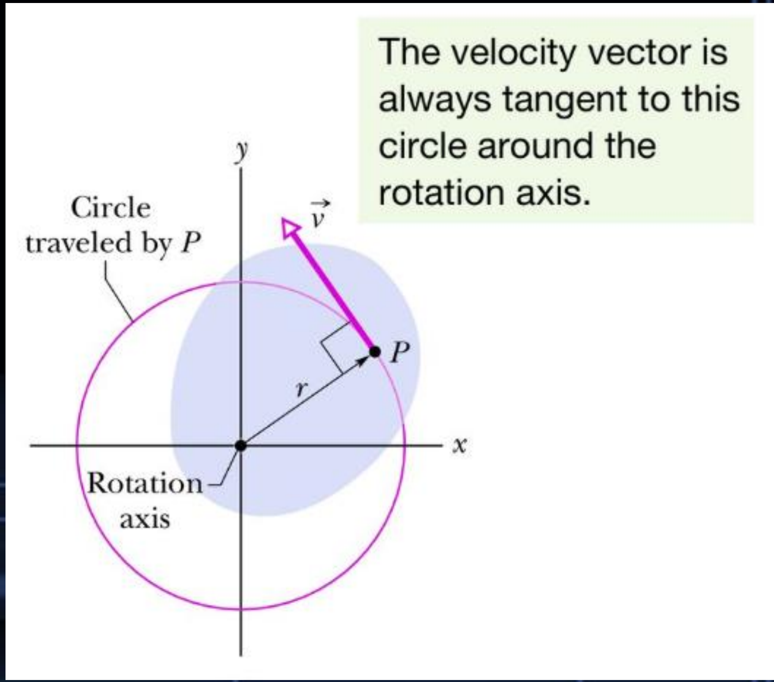
$$s = \theta r$$

where θ is in radians.

Relation between linear and angular variable

- Similarly, we can find other relationship between linear velocity and angular velocity:

$$v = \omega r$$



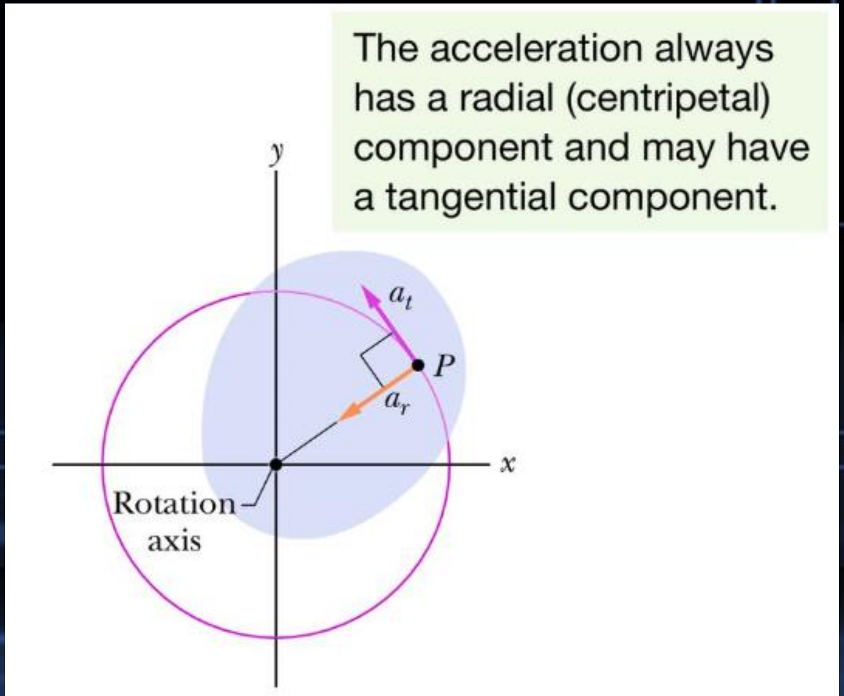
Relation between linear and angular variable

- For acceleration, the point has both tangential and radial components. The tangential component is

$$a_t = \alpha r$$

where α is the magnitude of the angular of the body. The radial component of is

$$a_r = \frac{v^2}{r} = \omega^2 r$$



Relation between linear and angular variable

- If the point moves in uniform circular motion, the period T of the motion for the point and the body is

$$T = \frac{2\pi r}{v} = \frac{2\pi}{\omega}$$

Kinetic energy of rotation

We shall treat the rotating rigid body as a collection of particles with different speeds. We can then add up the kinetic energies of all the particles to find the kinetic energy of the body as a whole:

$$K = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 + \cdots = \sum \frac{1}{2}m_i v_i^2$$

Rotational inertia (or moment of inertia)

In the rotating rigid body the rotational velocity is the same for every point. Thus we can rewrite:

$$K = \sum \frac{1}{2} m_i v_i^2 = \sum \frac{1}{2} m_i (r_i \omega)^2 = \frac{1}{2} \left(\sum m_i r_i^2 \right) \omega^2$$

We call that quantity the rotational inertia (or moment of inertia) I of the body with respect to the axis of rotation:

$$I = \sum m_i r_i^2$$

And thus the kinetic energy is $K = \frac{1}{2} I \omega^2$

Calculating the rotational inertia

- If a rigid body consists of a few particles, we can calculate its rotational inertia about a given rotation axis with $I = \sum m_i r_i^2$.
- If a rigid body consists of a great many adjacent particles, we replace the sum with an integral and define the rotational inertia of the body as

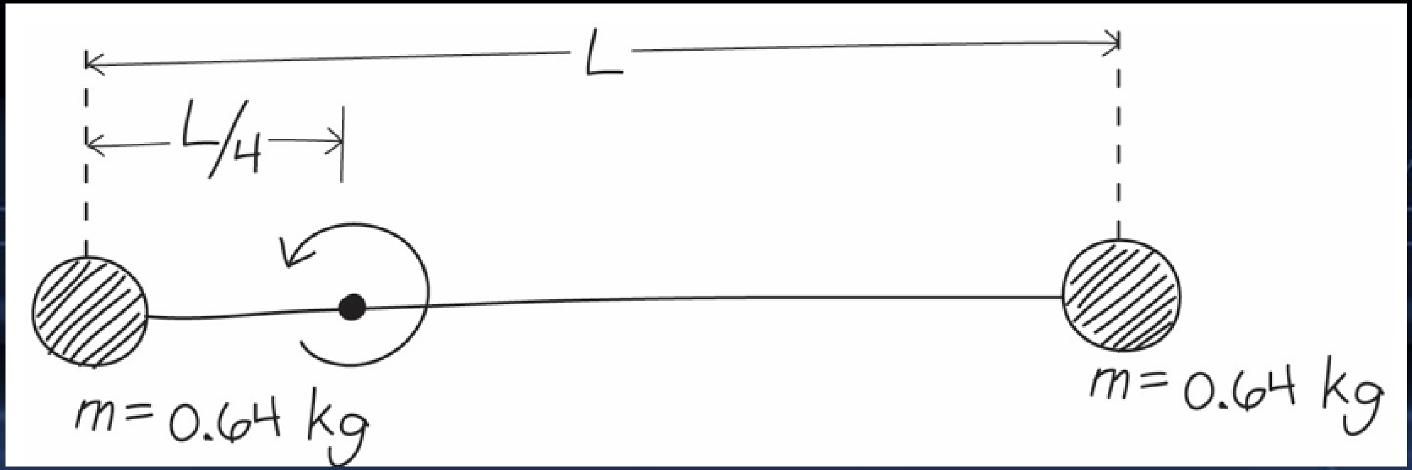
$$I = \int r^2 dm$$

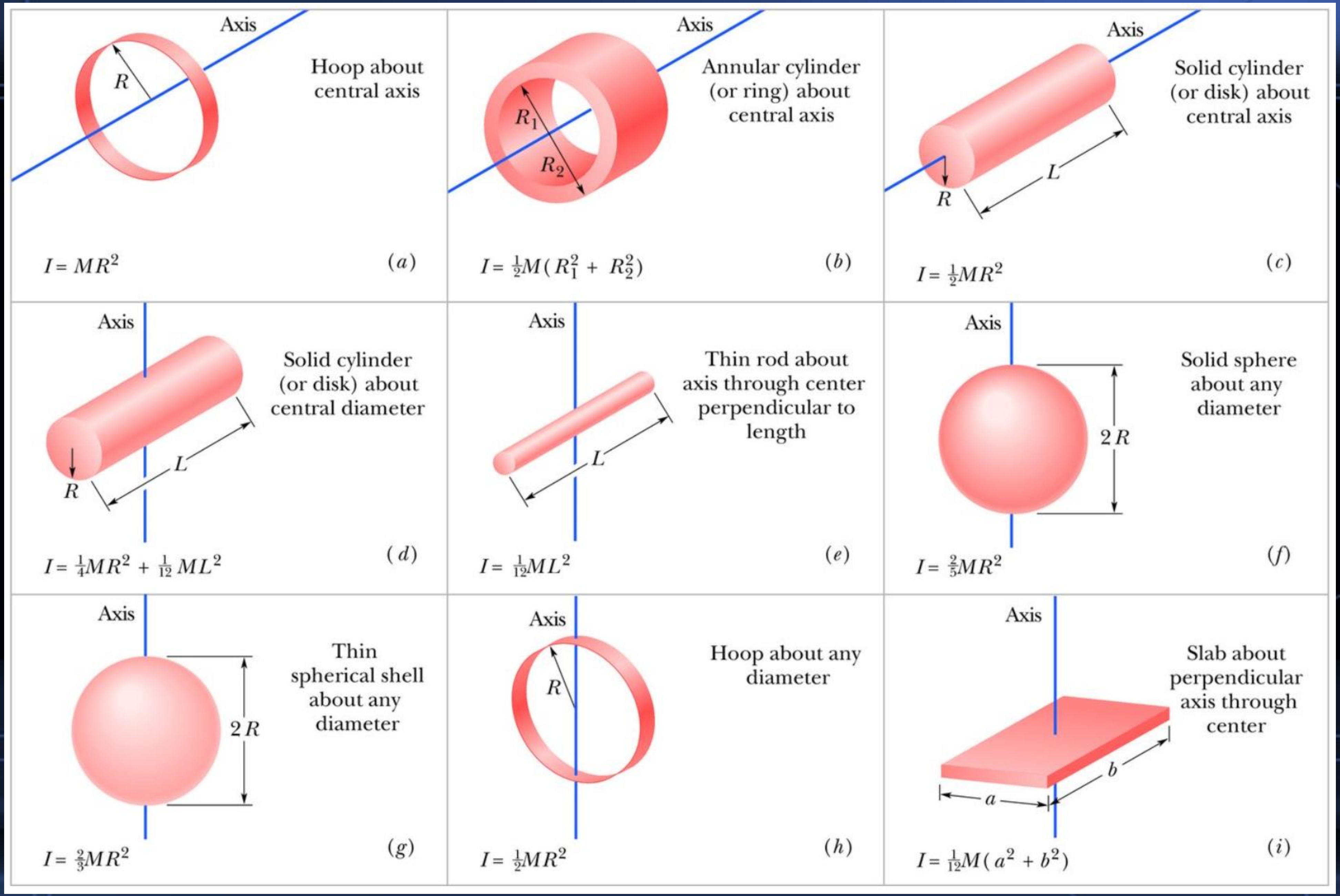
Think about it…

- What would happen to the rotational inertia of the two-mass dumbbell if its rotation axis were moved to the center of the rod? It would

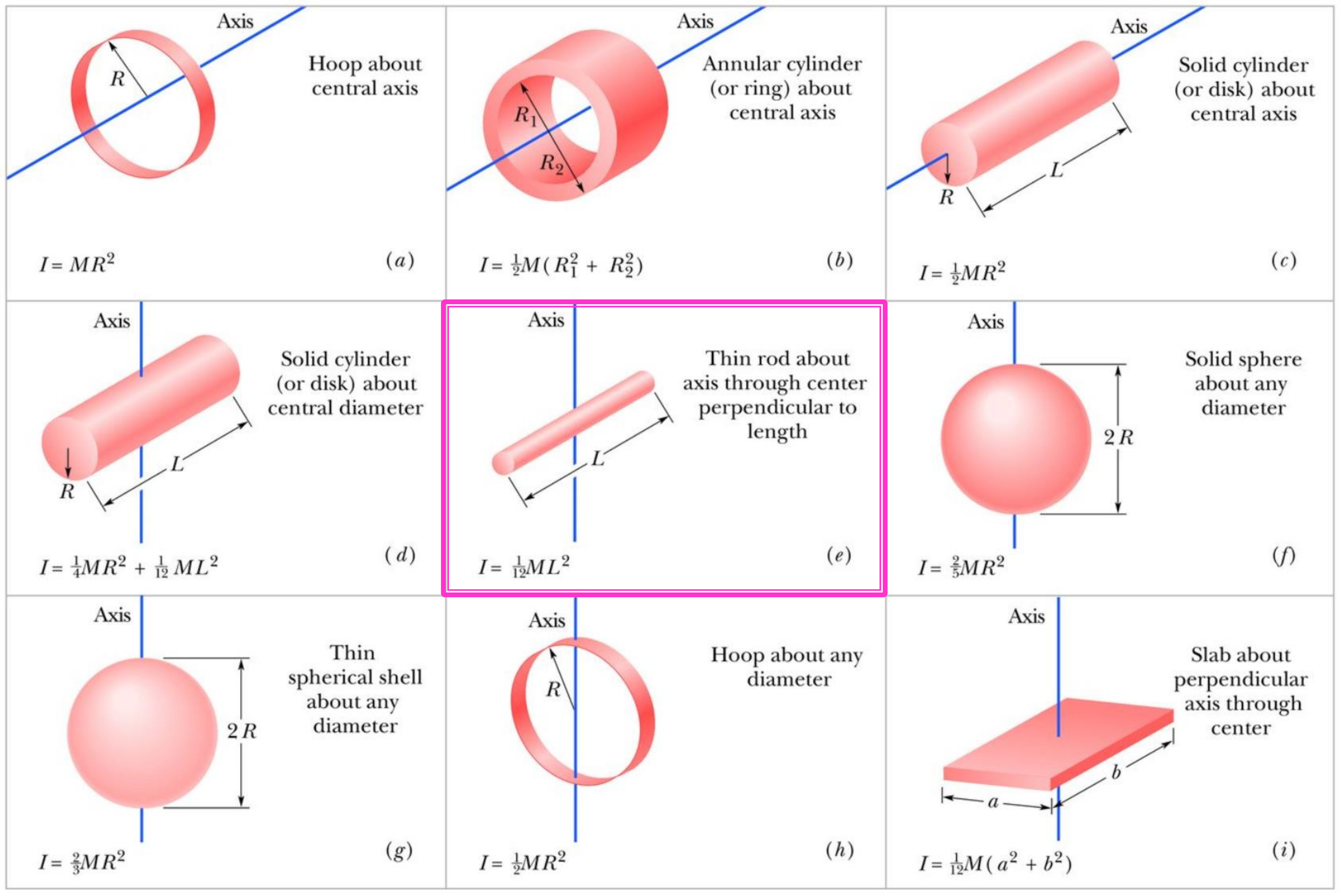
  (a) increase. (b) decrease. (c) stay the same.

- What would happen if the rotation axis were moved to one end of the rod?





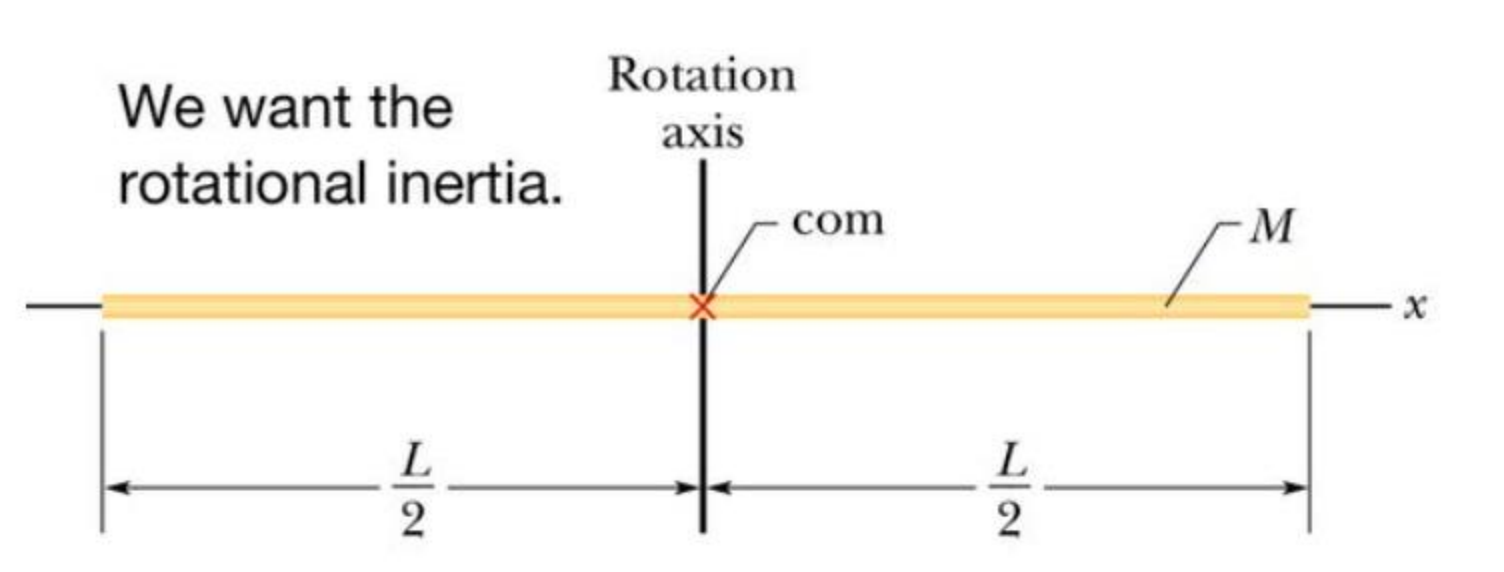
Axis
Hoop about central axis
R
$I = MR^2$
(a)
Axis
Annular cylinder (or ring) about central axis
R_1
R_2
$I = \frac{1}{2}M(R_1^2 + R_2^2)$
(b)
Axis
Solid cylinder (or disk) about central axis
L
R
$I = \frac{1}{2}MR^2$
(c)
Axis
Solid cylinder (or disk) about central diameter
L
R
$I = \frac{1}{4}MR^2 + \frac{1}{12}ML^2$
(d)
Axis
Thin rod about axis through center perpendicular to length
L
$I = \frac{1}{12}ML^2$
(e)
Axis
Solid sphere about any diameter
2R
$I = \frac{2}{5}MR^2$
(f)
Axis
Thin spherical shell about any diameter
2R
$I = \frac{2}{3}MR^2$
(g)
Axis
Hoop about any diameter
R
$I = \frac{1}{2}MR^2$
(h)
Axis
Slab about perpendicular axis through center
a
b
$I = \frac{1}{12}M(a^2 + b^2)$
(i)



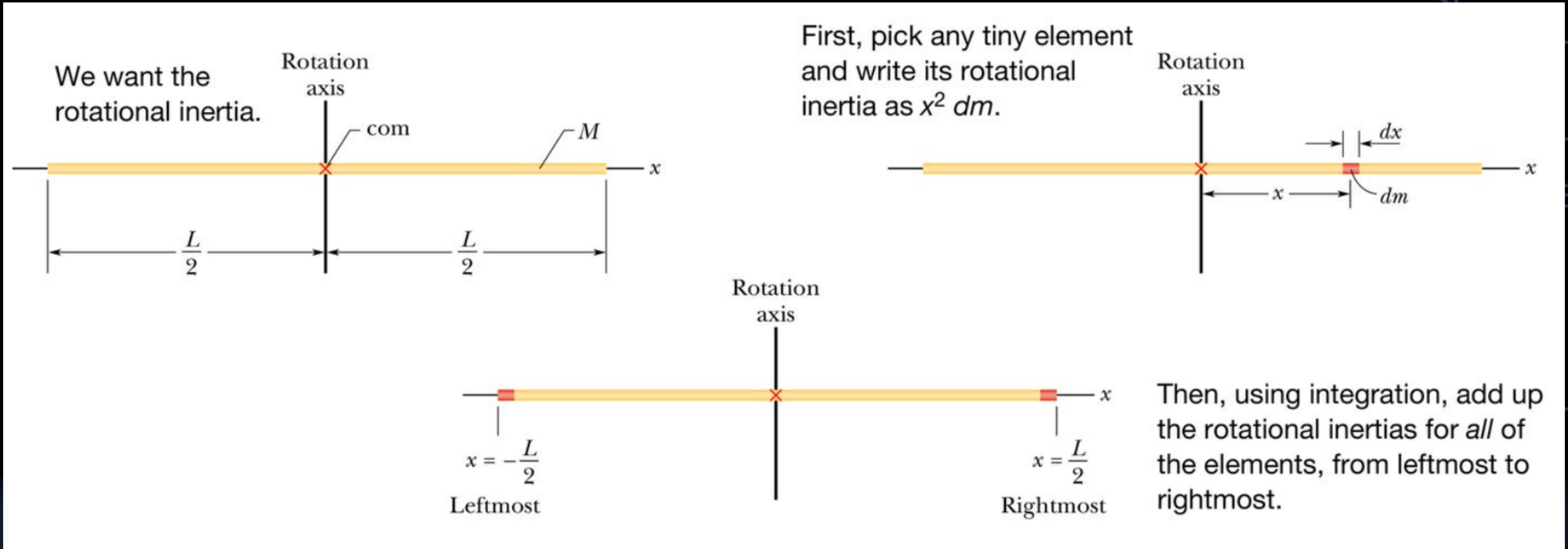
Axis
Hoop about central axis
R
$I = MR^2$
(a)
Axis
Annular cylinder (or ring) about central axis
R_1
R_2
$I = \frac{1}{2}M(R_1^2 + R_2^2)$
(b)
Axis
Solid cylinder (or disk) about central axis
L
R
$I = \frac{1}{2}MR^2$
(c)
Axis
Solid cylinder (or disk) about central diameter
L
R
$I = \frac{1}{4}MR^2 + \frac{1}{12}ML^2$
(d)
Axis
Thin rod about axis through center perpendicular to length
L
$I = \frac{1}{12}ML^2$
(e)
Axis
Solid sphere about any diameter
2R
$I = \frac{2}{5}MR^2$
(f)
Axis
Thin spherical shell about any diameter
2R
$I = \frac{2}{3}MR^2$
(g)
Axis
Hoop about any diameter
R
$I = \frac{1}{2}MR^2$
(h)
Axis
Slab about perpendicular axis through center
a
b
$I = \frac{1}{12}M(a^2 + b^2)$
(i)

Example of calculating rotational inertia

A thin, uniform rod of mass M and length L, on an x axis with the origin at the rod's center. What is the rotational inertia of the rod about the perpendicular rotation axis through the center?



Example of calculating rotational inertia



Example of calculating rotational inertia

$$
\begin{aligned}
I &= \int_{x=-L/2}^{x=+L/2} x^2 \left(\frac{M}{L}\right) dx \\
&= \frac{M}{3L}\left[x^3\right]_{-L/2}^{+L/2} = \frac{M}{3L}\left[\left(\frac{L}{2}\right)^3 - \left(-\frac{L}{2}\right)^3\right] \\
&= \frac{1}{12} M L^2. \qquad \text{(Answer)}
\end{aligned}
$$