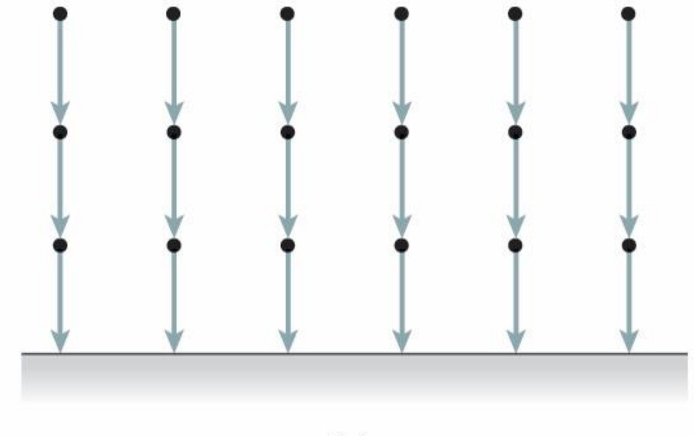
Course announcement

- Solution of Midterm 1 will be posted today.

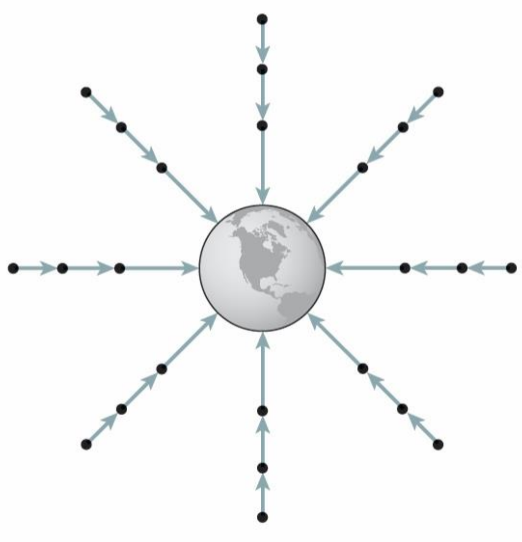
The Gravitational Field

- It's convenient to describe gravitation in terms of a **gravitational field** that results from the presence of mass and that exists at all points in space:
  - A massive object creates a gravitational field in its vicinity and other objects respond to the field **at their immediate locations**.
  - The gravitational field can be visualized with a set of vectors giving its strength (in N/k g; equivalently, m/s^2) and its direction.

$$\vec{g} = -\frac{GM}{r^2}\hat{r}$$



(a)



(b)

7	10/28(Fri.)	**Many Particles Motion and Rotation:** center of mass & liner momentum
8	11/1(Tue.)	**Many Particles Motion and Rotation:** rotation
8	11/4(Fri.)	**Many Particles Motion and Rotation:** torque & angular momentum
9	11/8(Tue.)	**Oscillation and Waves:** simple harmonic oscillation

GENERAL PHYSICS B1
MANY PARTICLES MOTION & ROTATION

Center of Mass and Linear Momentum

2022/10/28

Motion of objects in the real world

Up to now, we simplified the motion of an object to a single point. In real world, we have many particles or objects with finite volume.

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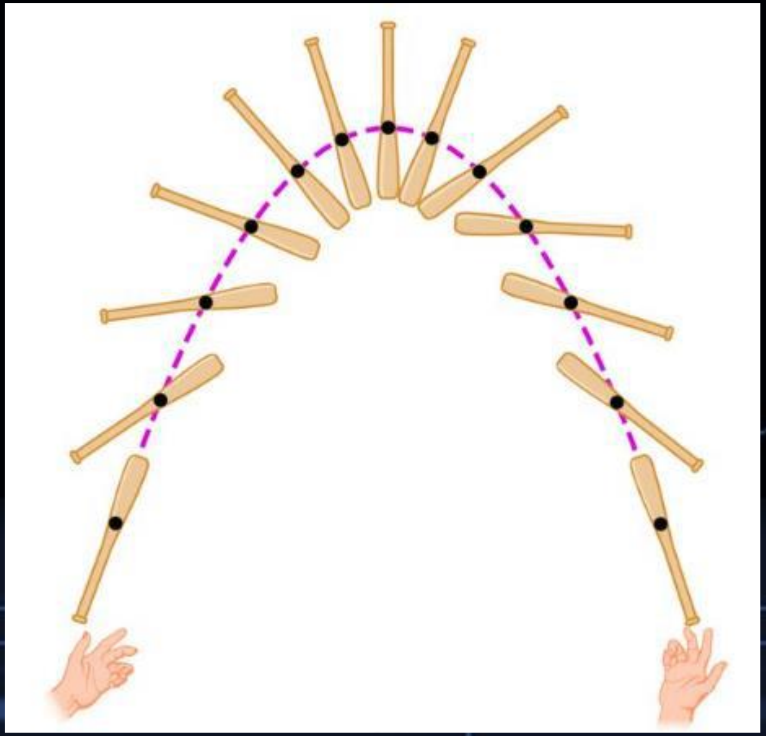


Today’s topic

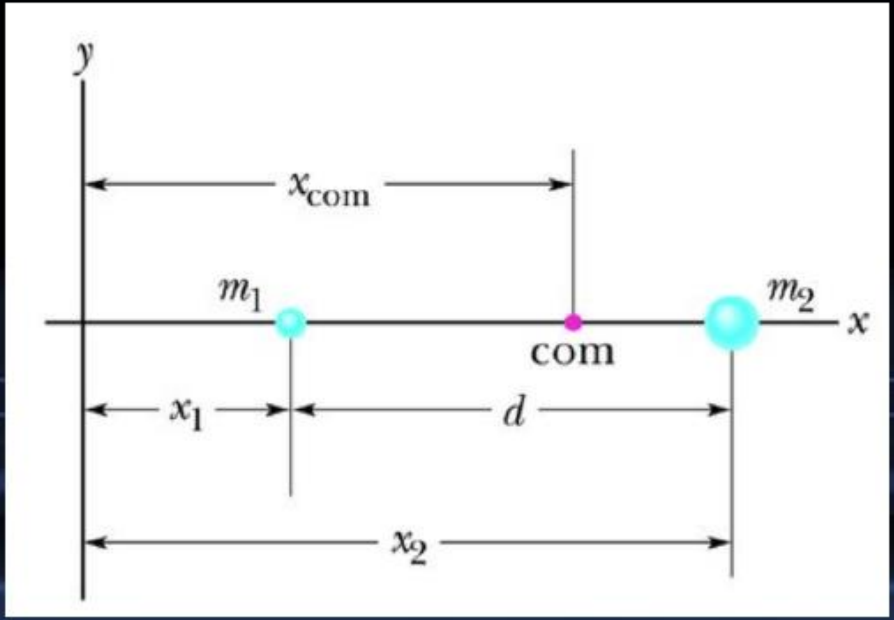
- Center of Mass
- Application of impulse and momentum: collision
- Momentum and energy in collision

The Center of Mass (COM)

- The center of mass of a system of particles is the point that represents the motion of the system.
- The center of mass moves as though (1) all of the system's mass were concentrated there (but it doesn't have to be inside the system) and (2) all external forces were applied there.

The center of mass in a two particles system

Two particles of masses m1 and m2 separated by distance d. We define the position of the center of mass (com) of this two-particle system to be $x_{com} = \frac{m_1x_1+m_2x_2}{m_1+m_2}$



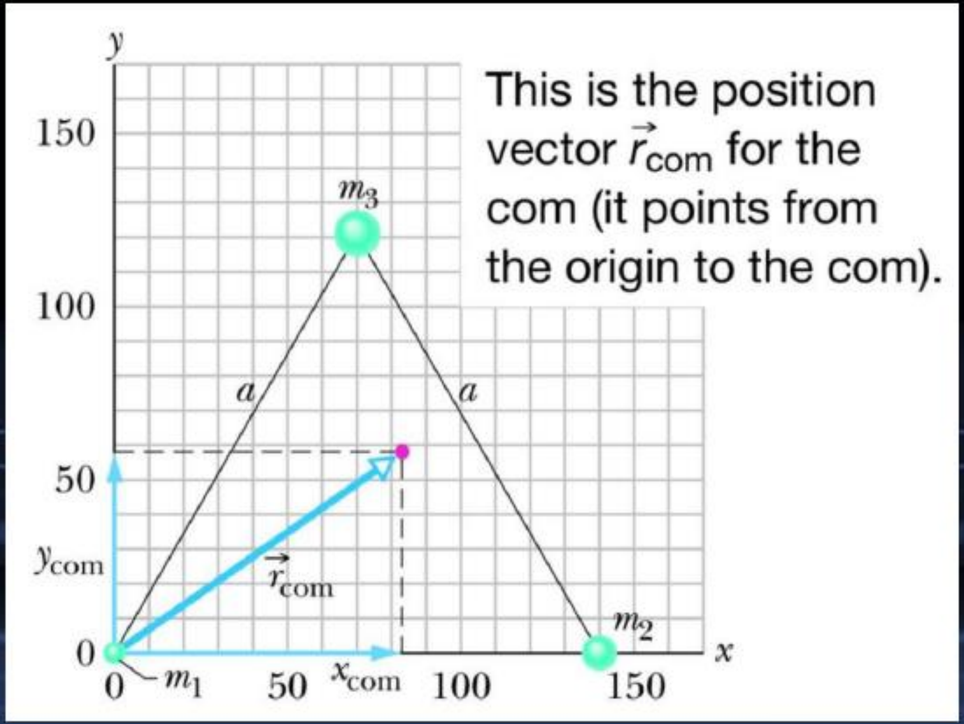
The center of mass in a many particles system

We can further extend this equation to a more general situation in which n particles in three dimension. Then the total mass is $M = m_1 + m_2 + \ldots + m_n$, and the location of the center of mass is

$$\overrightarrow{r_{com}} = \frac{1}{M}\sum_{i=1}^{n} m_i \overrightarrow{r_i}$$

Example: COM of three particles

Three particles of masses m_1 = 1.2 kg, m_2 = 2.5 kg, and m_3 = 3.4 kg form an equilateral triangle of edge length a = 140 cm. Where is the center of mass of this system?

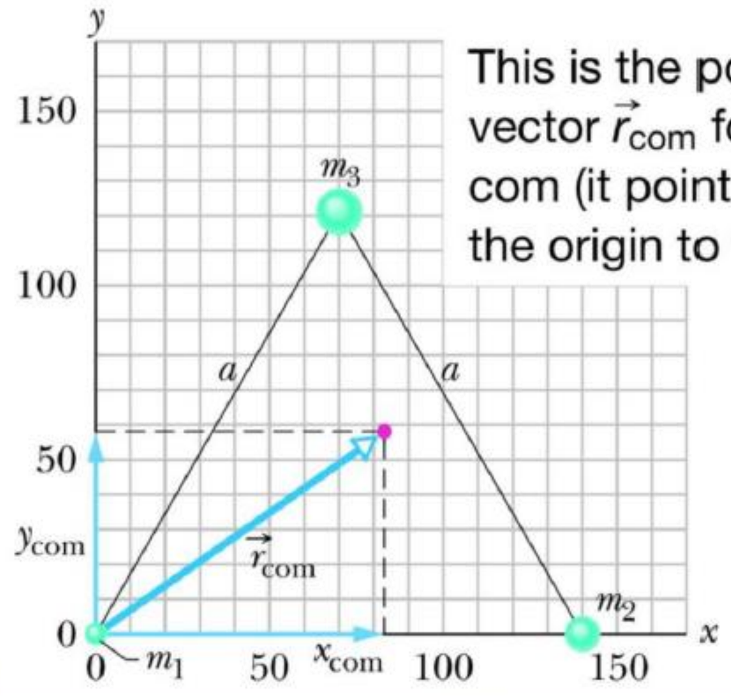


Example: COM of three particles

We can decompose $\overrightarrow{r_{com}}$ into x components and y components:

$$x_{com} = \frac{1}{M}\sum_{i=1}^{3} m_i x_i = \frac{m_1x_1+m_2x_2+m_3x_3}{M}$$
$$= \frac{(1.2\ \text{kg})(0)+(2.5\ \text{kg})(140\ \text{cm})+(3.4\ \text{kg})(70\ \text{m})}{7.1\ \text{kg}}$$
$$= 83\ \text{cm}$$

$$y_{com} = \frac{1}{M}\sum_{i=1}^{3} m_i y_i = \frac{m_1y_1+m_2y_2+m_3y_3}{M}$$
$$= \frac{(1.2\ \text{kg})(0)+(2.5\ \text{kg})(0)+(3.4\ \text{kg})(120\ \text{m})}{7.1\ \text{kg}}$$
$$= 58\ \text{cm}$$



This is the position vector $\vec{r}_{com}$ for the com (it points from the origin to the com).

The center of mass in a continuous system

An ordinary object contains so many particles (atoms) that we can best treat it as a continuous distribution of matter. The “particles” then become differential mass elements dm, the sums become integrals, and the coordinates of the center of mass are defined as

$$x_{com} = \frac{1}{M}\int x dm$$

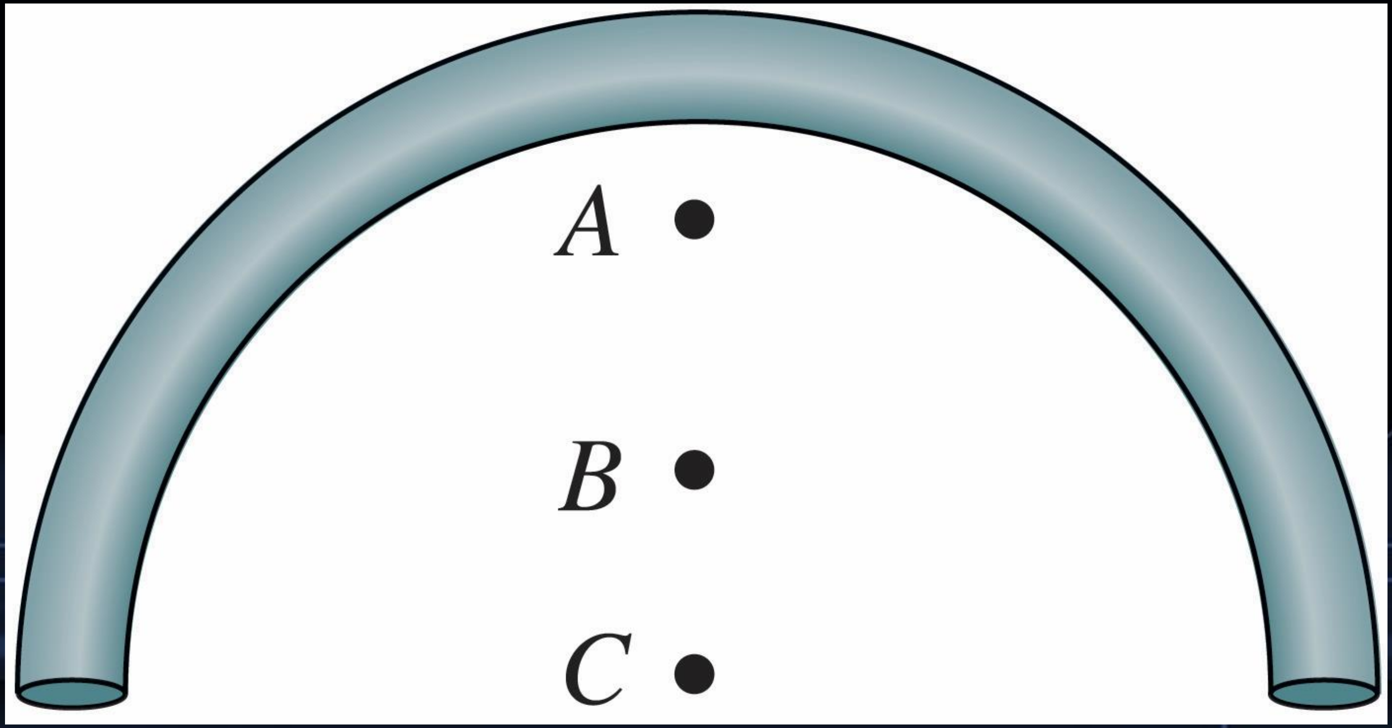
$$\overrightarrow{r_{com}} = \frac{1}{M}\int \vec{r}\, dm$$

$$y_{com} = \frac{1}{M}\int y dm$$

$$z_{com} = \frac{1}{M}\int z dm$$

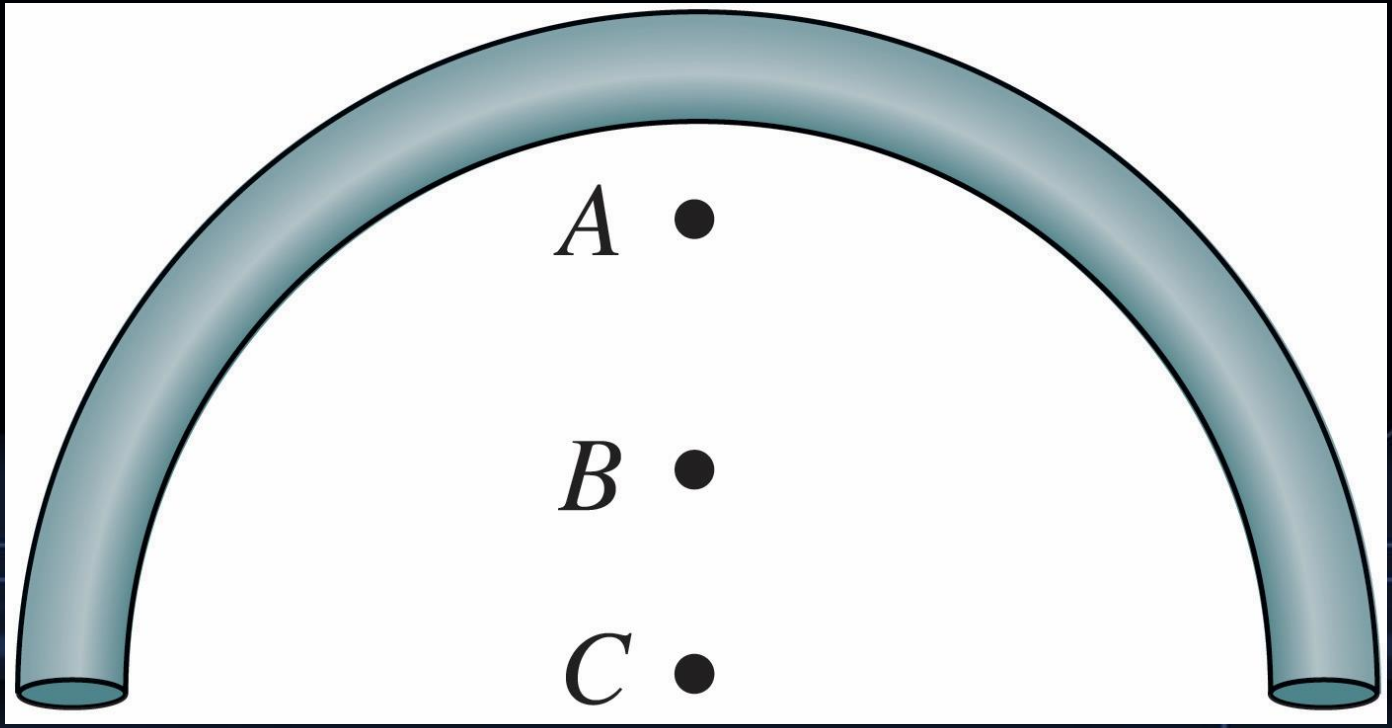
Think about it…

- The CM lies outside the semicircular wire but which point is it?



Think about it…

- The CM lies outside the semicircular wire but which point is it? A



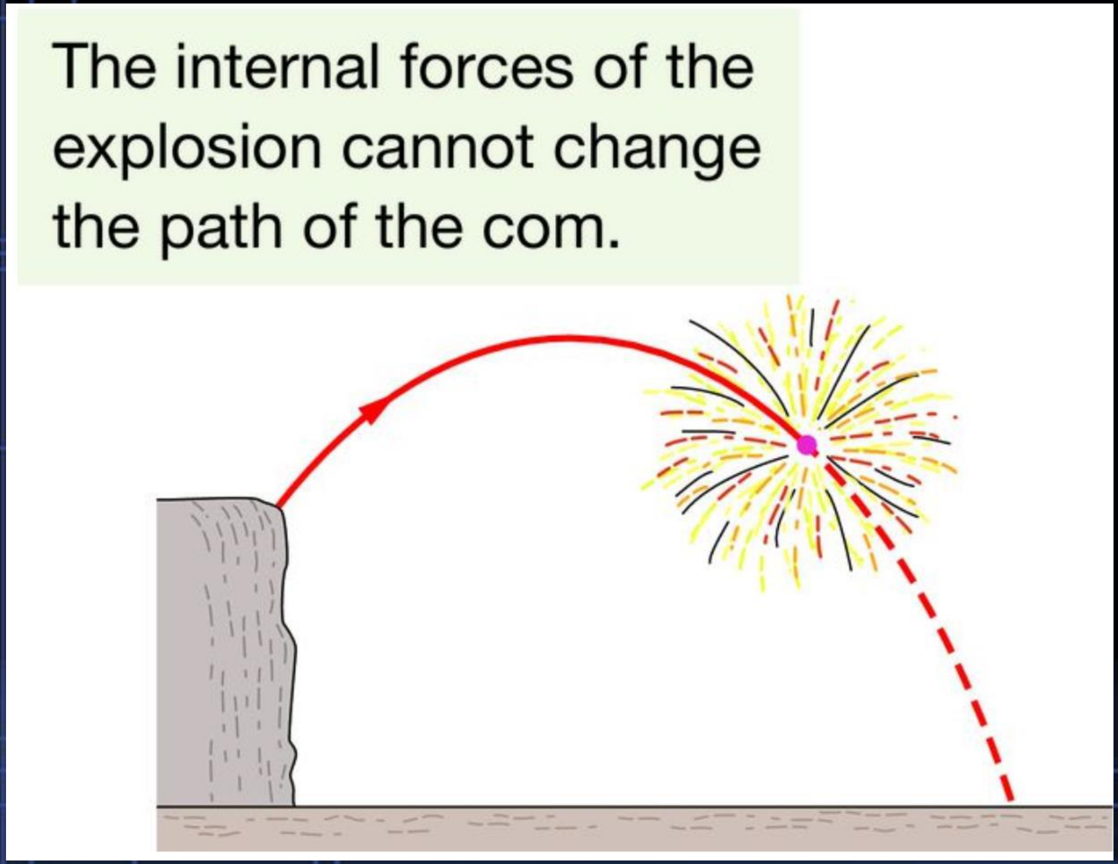
Newton's second law for COM

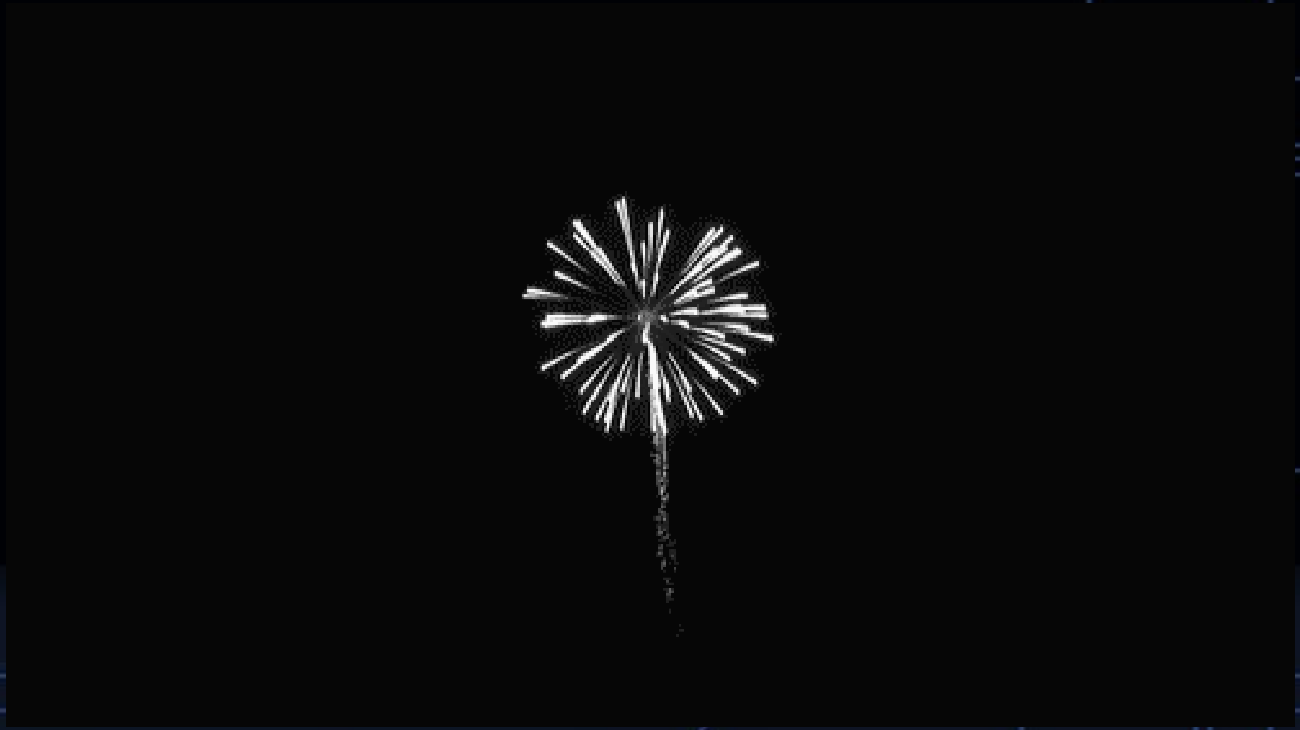
- The motion of the center of mass of any system of particles is governed by Newton's second law for a system of particles, which is

$$\overrightarrow{F_{NET}} = M\overrightarrow{a_{COM}}$$

- Here $\overrightarrow{F_{NET}}$ is the net force of all the external forces acting on the system, M is the total mass of the system, and $\overrightarrow{a_{COM}}$ is the acceleration of the system's center of mass.

Example: Explosion





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Linear momentum

- For a single particle, we define a quantity $\vec{p}$ called its linear momentum as

$$\vec{p} = m\vec{v}$$

which is a vector quantity that has the same direction as the particle's velocity. We can write Newton's second law in terms of this momentum:

$$\overrightarrow{F_{NET}} = \frac{d\vec{p}}{dt}$$

- For a system of particles these relations become

$$\vec{P} = m\overrightarrow{v_{COM}} \text{ and } \overrightarrow{F_{NET}} = \frac{d\vec{P}}{dt}$$

Impulse and Momentum

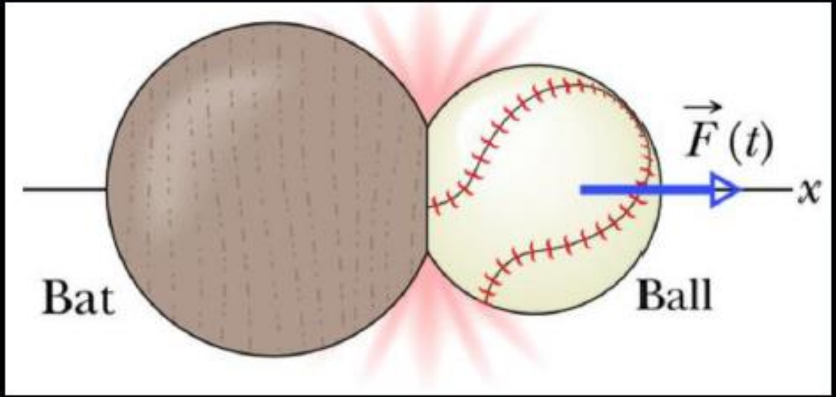
- Similar to our derivation to energy (integral EOM respect to displacement), we integral EOM respect to time:

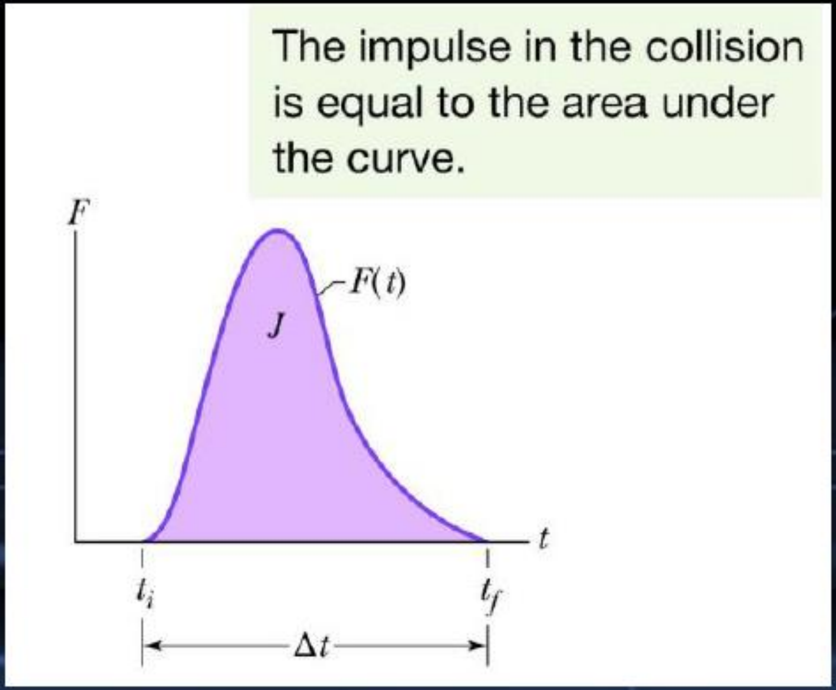
$$\int \overrightarrow{F_{NET}}\, dt = \vec{p}(t_{final}) - \vec{p}(t_{initial}) = \Delta\vec{p}$$

Where we define impulse as $\vec{J} = \int \overrightarrow{F_{NET}}\, dt$

and we have $\vec{J} = \Delta\vec{p}$

Example of Impulse





Conservation of linear momentum

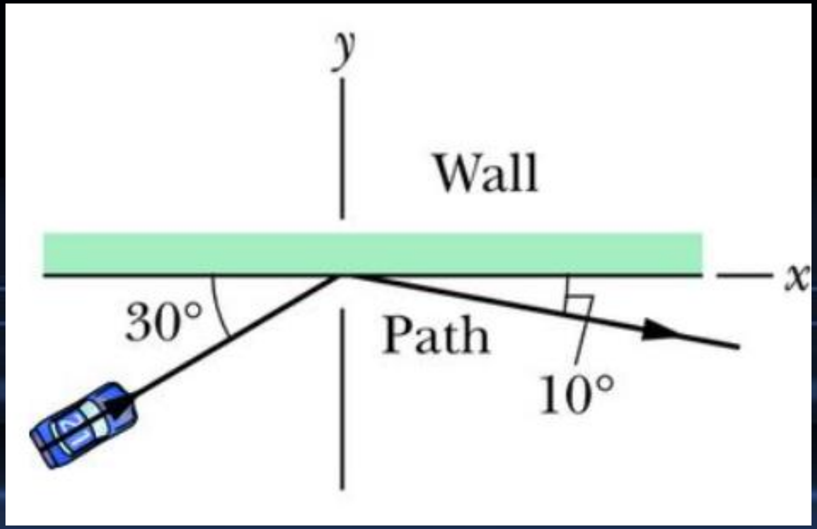
- Conservation law of momentum: If a system is closed and isolated so that no net external force acts on it, then the linear momentum $\vec{P}$ must be constant even if there are internal changes:

$$\vec{P} = constant \text{ (closed, isolated system)}$$

$$\overrightarrow{P_{initial}} = \overrightarrow{P_{final}}$$

Car-wall collision

A race car driver as his car collides with the racetrack wall. Just before the collision, he is traveling at speed v_i = 70 m/s along a straight line at 30° from the wall. Just after the collision, he is traveling at speed v_f = 50 m/s along a straight line at 10° from the wall. His mass m is 80 kg.

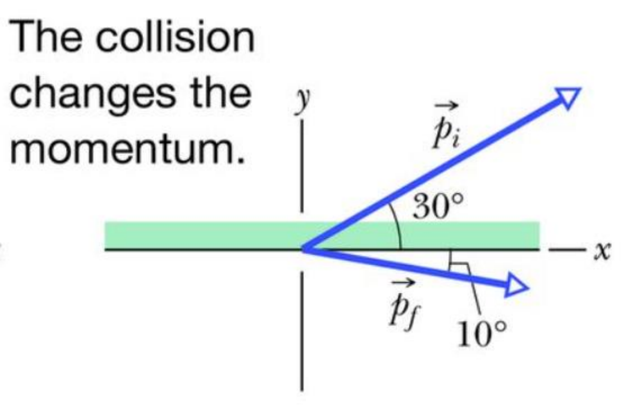


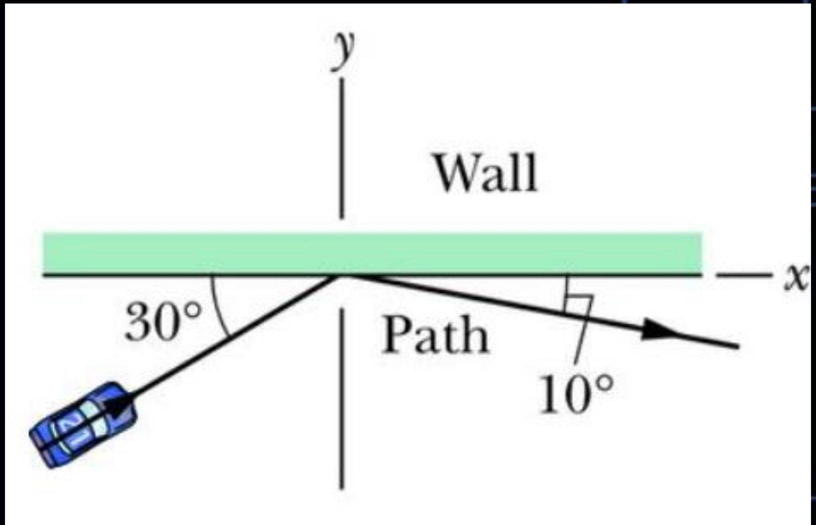
Car-wall collision

(a)What is the impulse on the driver due to the collision?

We know that:

$$\vec{J} = \overrightarrow{p_f} - \overrightarrow{p_i} = m(\overrightarrow{v_f} - \overrightarrow{v_i})$$





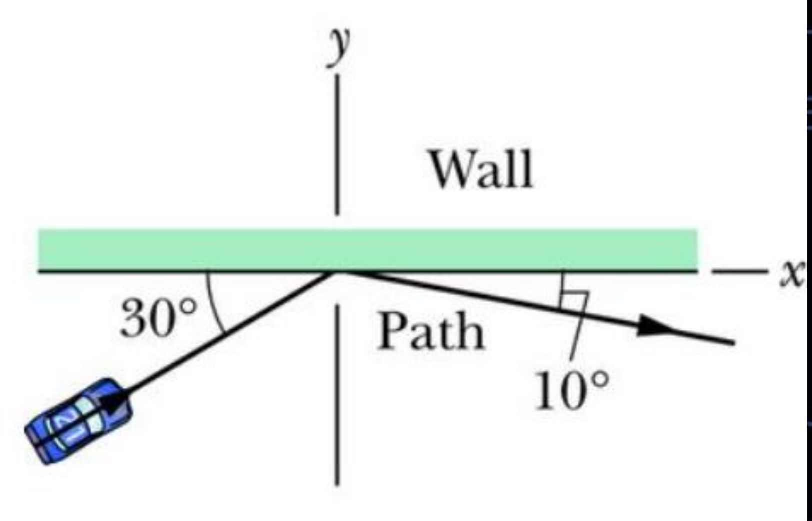
$$J_x = p_{fx} - p_{ix} = m\left(v_f cos30^{\circ} - v_i cos10^{\circ}\right) = -910(kg \cdot m/s)$$

$$J_y = p_{fy} - p_{iy} = m\left(v_f sin30^{\circ} - v_i sin10^{\circ}\right) = -3495(kg \cdot m/s)$$

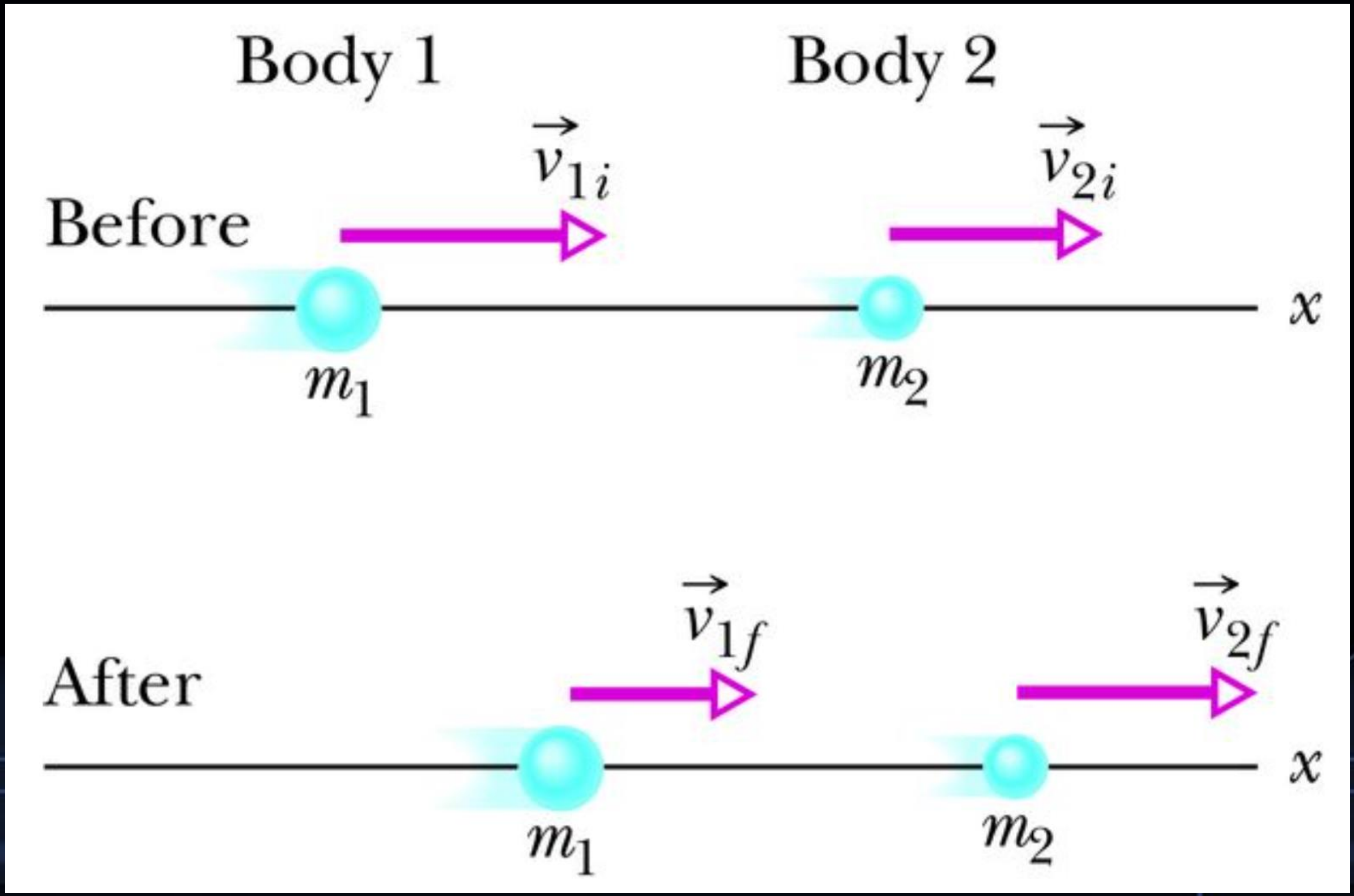
Car-wall collision

(b)The collision lasts for 14 ms. What is the magnitude of the average force on the driver during the collision?

$$F_{avg} = \frac{|\vec{J}|}{\Delta t} = \frac{\sqrt{J_x^2 + J_y^2}}{\Delta t} = 2.583 \times 10^5 N$$



Collision of two particles



Collision of two particles

- The total linear momentum of the system cannot change for a collision of two particles because there is no net external force to change the system.
- If that total happens to be unchanged by the collision, then the kinetic energy of the system is conserved (it is the same before and after the collision). Such a collision is called an elastic collision.
- If the kinetic energy of the system is not conserved, such a collision is called an inelastic collision.

Collision of two particles

- Conservation of linear momentum:

$$m_1\overrightarrow{v_{1i}} + m_2\overrightarrow{v_{2i}} = m_1\overrightarrow{v_{1f}} + m_2\overrightarrow{v_{2i}}$$

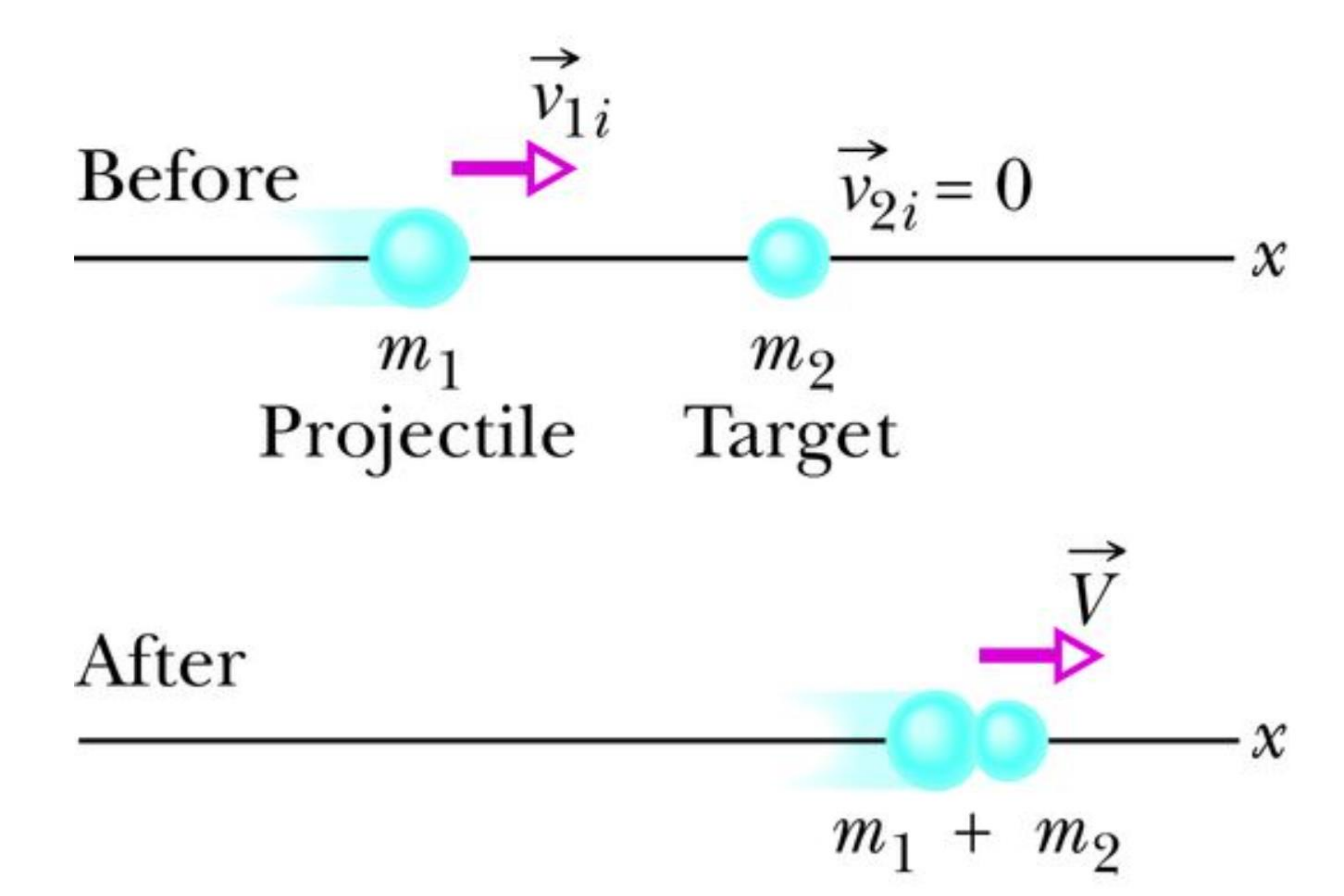
- Elastic collision(conservation of kinetic energy)

$$\frac{1}{2}m_1{v_{1i}}^2 + \frac{1}{2}m_2{v_{2i}}^2 = \frac{1}{2}m_1{v_{1f}}^2 + \frac{1}{2}m_2{v_{2f}}^2$$

- Inelastic collision

$$\frac{1}{2}m_1{v_{1i}}^2 + \frac{1}{2}m_2{v_{2i}}^2 \neq \frac{1}{2}m_1{v_{1f}}^2 + \frac{1}{2}m_2{v_{2f}}^2$$

Case 1: completely Inelastic collision



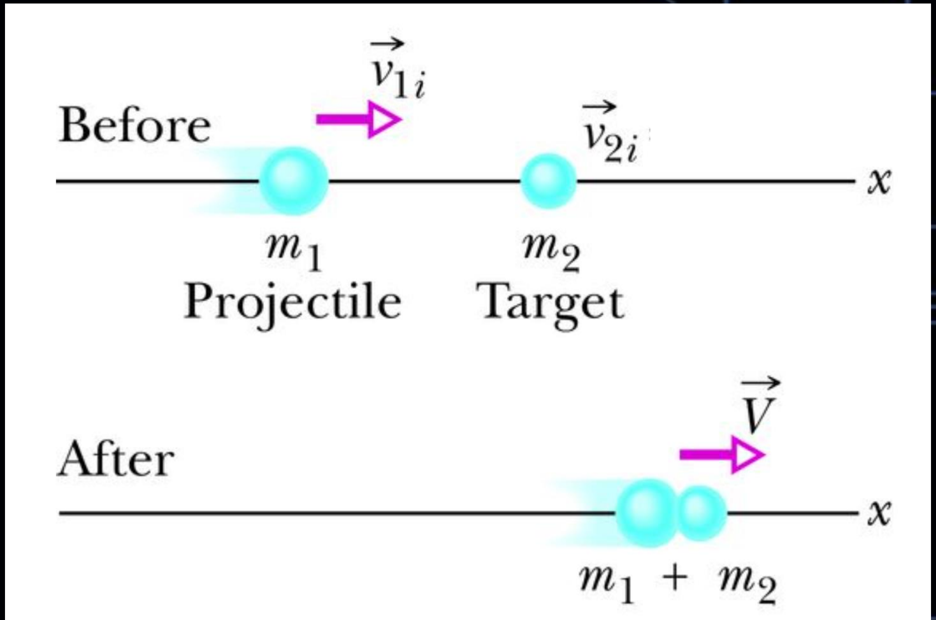
Case 1: completely Inelastic collision

- In a completely inelastic collision the two particles will stick together: the final velocities are the same
- Conservation of linear momentum:

$$m_1\overrightarrow{v_{1i}} + m_2\overrightarrow{v_{2i}} = (m_1+m_2)\vec{V}$$

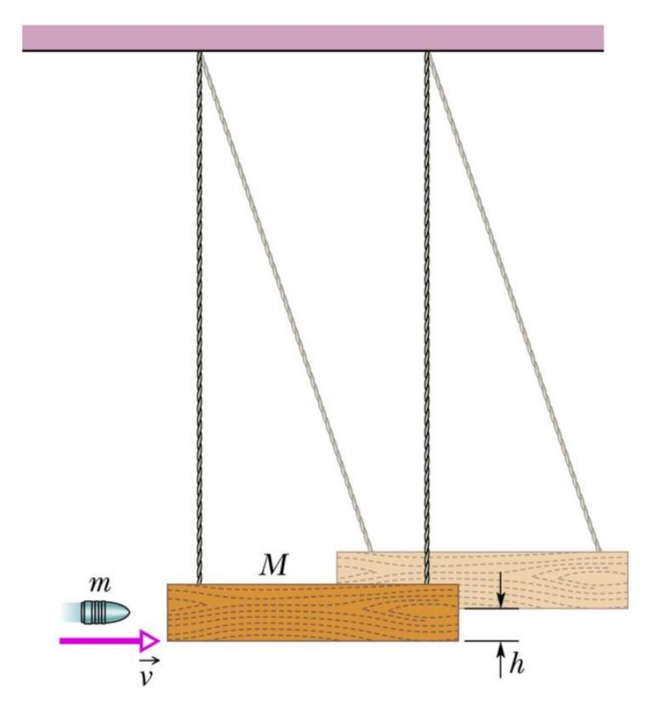
- If $\overrightarrow{v_{2i}} = 0$, we will have

$$\vec{V} = \frac{m_1}{m_1 + m_2}\overrightarrow{v_{1i}}$$



Example of completely inelastic collision

The ballistic pendulum was used to measure the speeds of bullets before electronic timing devices were developed. A large block of wood of mass M = 5.4 kg, hanging from two long cords. A bullet of mass m = 9.5 g is fired into the block, coming quickly to rest. The block + bullet then swing upward, their center of mass rising a vertical distance h = 6.3 cm What is the speed of the bullet just prior to the collision?



Example of completely inelastic collision

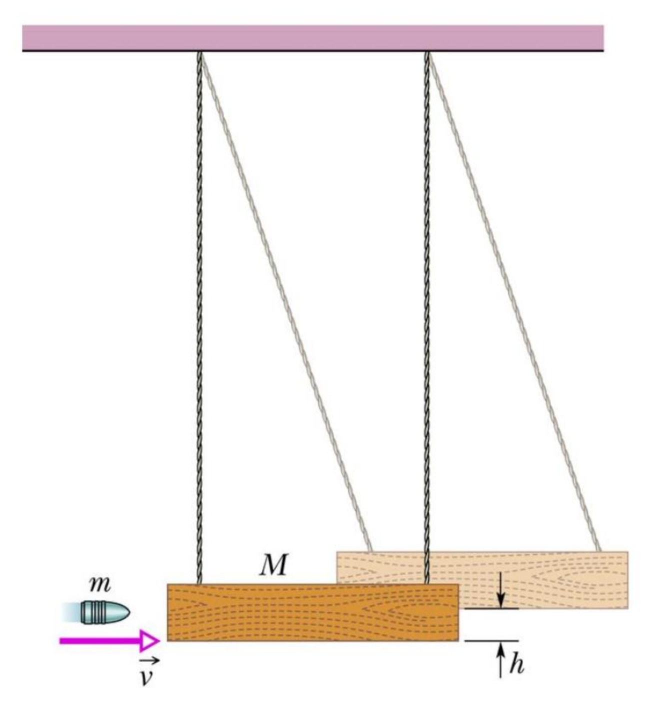
There are two events: 1. collision of bullet and block. 2. swinging of bullet and block.

- Collision of bullet and block: a completely inelastic collision:

$$\vec{V} = \frac{m_1}{m_1 + m_2}\overrightarrow{v_{1i}}$$

- Swing of bullet and block: conservation of energy

$$\frac{1}{2}(m_1 + m_2)\vec{V} = (m_1 + m_2)gh$$



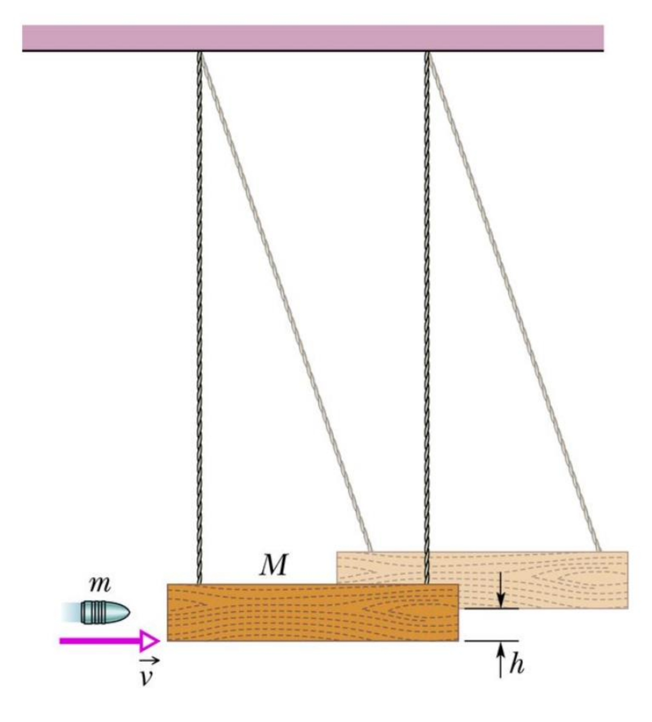
Example of completely inelastic collision

Therefore, we have:

$$\frac{1}{2}(m_1 + m_2)\left(\frac{m_1}{m_1 + m_2}\overrightarrow{v_{1i}}\right)^2 = (m_1 + m_2)gh$$

And we can get:

$$v_{1i} = 630m/s$$

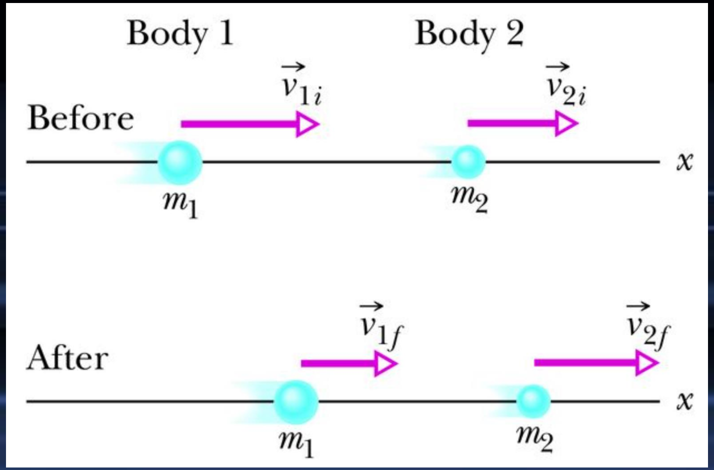


Case 2: Elastic collision

In an elastic collision, (1)total momentum is conserved (2) the kinetic energy of each colliding body may change, but the total kinetic energy of the system does not change.

$$m_1\overrightarrow{v_{1i}} + m_2\overrightarrow{v_{2i}} = m_1\overrightarrow{v_{1f}} + m_2\overrightarrow{v_{2i}}$$

$$\frac{1}{2}m_1{v_{1i}}^2 + \frac{1}{2}m_2{v_{2i}}^2 = \frac{1}{2}m_1{v_{1f}}^2 + \frac{1}{2}m_2{v_{2f}}^2$$

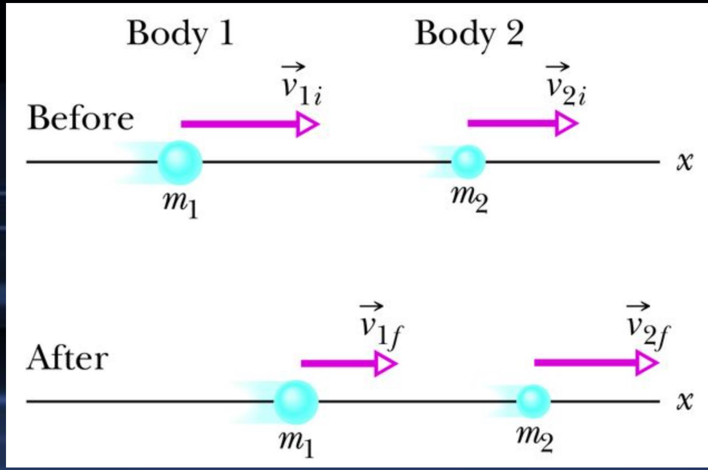


Case 2: Elastic collision

After rearrange, we can get final velocities of each particles in terms of initial velocities:

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} + \frac{2m_2}{m_1 + m_2} v_{2i}$$

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} + \frac{m_1 - m_2}{m_1 + m_2} v_{2i}$$

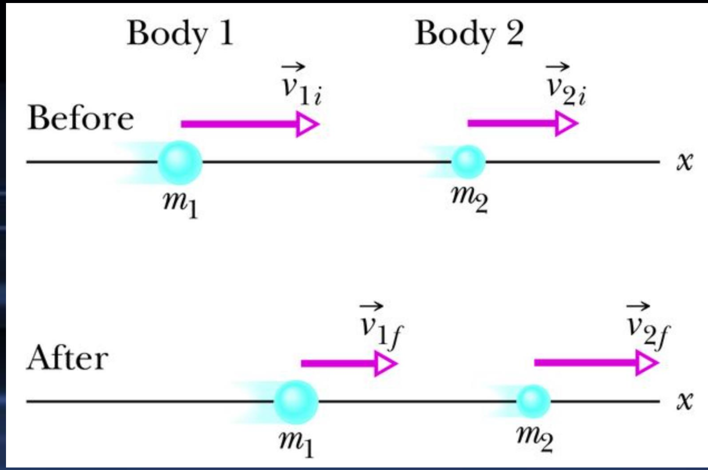


Case 2: Elastic collision

One special case: if $m_1=m_2$, we will have

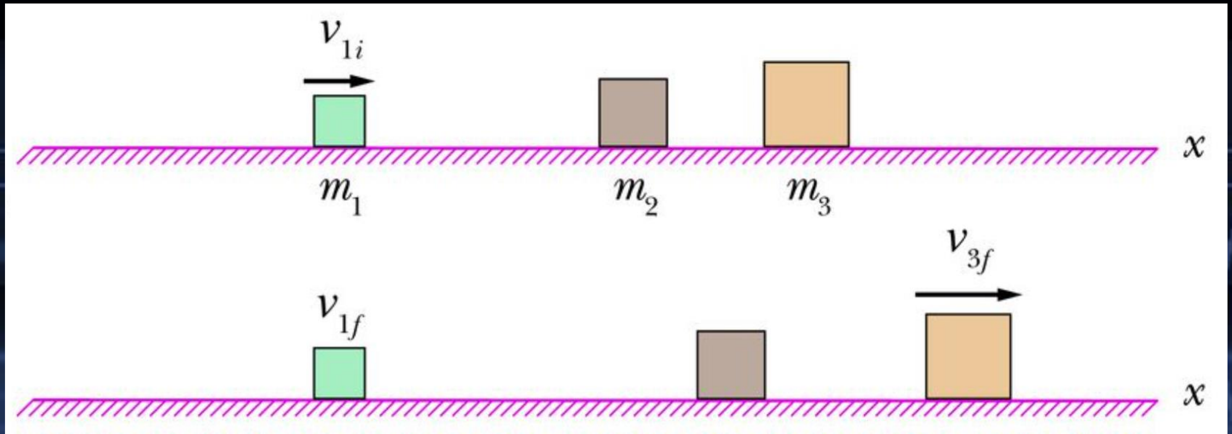
$$v_{1f} = v_{2i}$$
$$v_{2f} = v_{1i}$$

as if these two particles exchange their velocity.

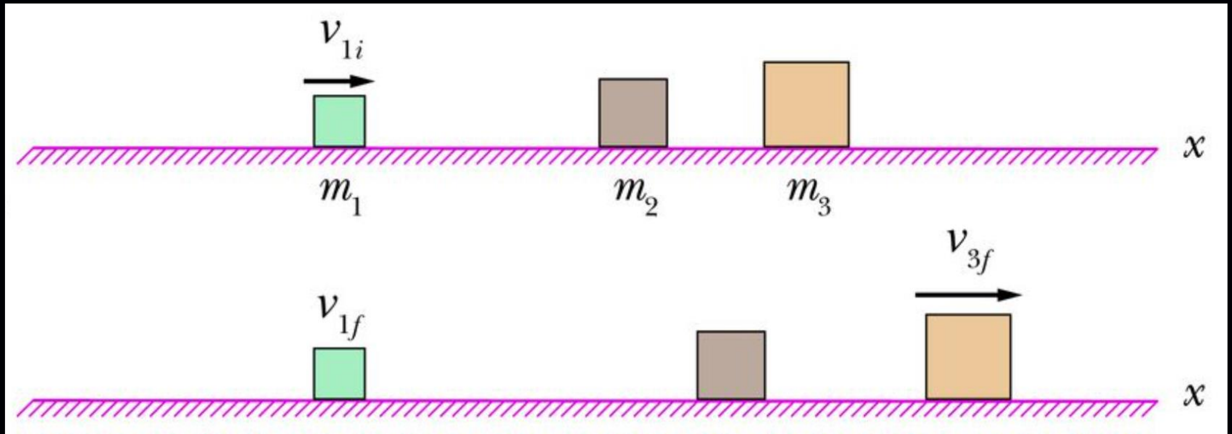


Example of elastic collision

A block 1 approaches a line of two stationary blocks with a velocity of v_{1i} = 10 m/s. It collides with block 2, which then collides with block 3, which has mass m_3 = 6.0 kg. After the second collision, block 2 is again stationary and block 3 has velocity v_{3f} = 5.0 m/s. Assume that the collisions are elastic. What are the masses of blocks 1 and 2? What is the final velocity v_{1f} of block 1?



Example of elastic collision



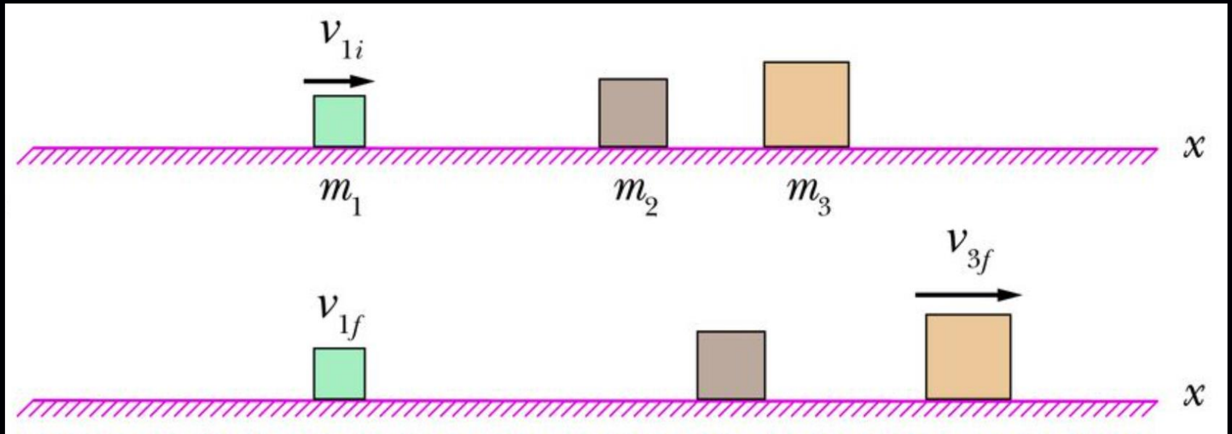
Let's think the second elastic collision first. Since initial velocity of m_3 is 0. we have

$$v_{2f} = \frac{m_2 - m_3}{m_2 + m_3} v_{2i}$$

Since the final velocity of m_2 is 0, the only way to have this is

$$m_2 = m_3 = 6kg$$

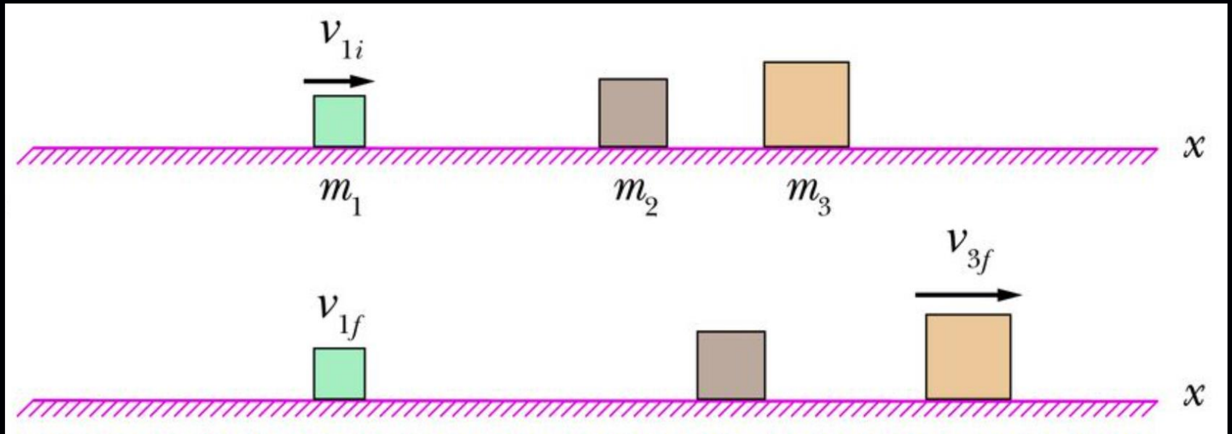
Example of elastic collision



We also have

$$v_{3f} = \frac{2m_2}{m_2+m_3} v_{2i} \text{ and we know } v_{3f} = v_{2i} = 5m/s$$

Example of elastic collision



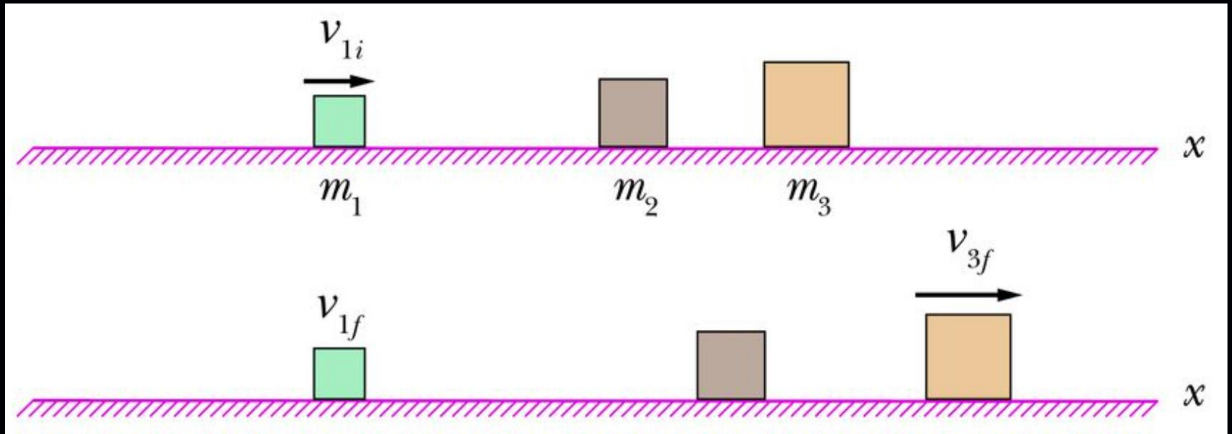
Now we come back in the first collision and we know that

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i}$$

Where $v_{2f} = 5m/s$, $v_{1i} = 10m/s$, and $m_2 = 6kg$

Thus we know $m_1 = 2kg$

Example of elastic collision



And finally we can find out that

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} = -5m/s$$

Summary

- A composite system behaves as though its mass is concentrated at the **C M:**

$$\overrightarrow{r_{com}} = \frac{1}{M}\sum_{i=1}^{n} m_i \overrightarrow{r_i}$$

- The C M obeys Newton's laws, so

$$\overrightarrow{F_{NET}} = M\overrightarrow{a_{COM}} = \frac{d\overrightarrow{P_{COM}}}{dt}$$

- In the absence of a net external force, a system's linear momentum is conserved, regardless of what happens internally to the system.
- Collisions are brief, intense, interactions that conserve momentum:
  - Elastic collisions also conserve kinetic energy.
  - Totally inelastic collisions occur when colliding objects join to make a single composite object.