Course announcement

- The homework set 4 has been posted on eLearn. Please submit your homework via eLearn by 5PM, 12/02. No late homework will be accepted. Solution will be posted tonight.
- The first midterm will on 12/06 (Tuesday). Exam will be started 8:00AM.

11	11/22(Tue.)	**Oscillation and Waves:** propagation of waves
11	11/25(Fri.)	**Fluid Motion:** Density, Pressure, and Hydrostatic Equilibrium (Homework4)
12	11/29(Tue.)	**Fluid Motion:** Fluid Dynamics and Application
12	12/2(Fri.)	**Review II**
13	12/6(Tue.)	Mid Term 2

Midterm Exam 2

- Exams will be started at 8:00AM and ends at 9:50AM.
- Please bring student ID and calculator.
- You can bring one A4 information sheet for the exam.
- Cheating will result in 0 points for the whole exam and will be reported to university.
- No Exams Corrections for Midterm 2

Policy for COVID-19

- We follow university guideline about course under COVID-19.
- Please have facial mask with you
- For students who cannot attend exam due to COVID-19, they can have test remotely with monitor of web camera.

(https://teams.live.com/meet/9570955571789). The problem will be posted on eLearn and can be handed via eLearn. Only students inform me in advance can have test via eLearn.

Problem about Exam

- There will be 6 problem sets. Range: 9~15 chapter (skip chapter 12) of Essential University Physics by Richard Wolfson.
- All the problems will be related to materials covered in class, problem discussed in class, and homework problems.

GENERAL PHYSICS B1
REVIEW II

2022/12/02

The Center of Mass (COM)

- The center of mass of a system of particles is the point that represents the motion of the system.
- The center of mass moves as though (1) all of the system's mass were concentrated there (but it doesn't have to be inside the system) and (2) all external forces were applied there.

The center of mass in a many particles system

We can further extend this equation to a more general situation in which n particles in three dimension. Then the total mass is $M = m_1 + m_2 + \ldots + m_n$, and the location of the center of mass is

$$\overrightarrow{r_{com}} = \frac{1}{M}\sum_{i=1}^{n} m_i \overrightarrow{r_i}$$

The center of mass in a continuous system

An ordinary object contains so many particles (atoms) that we can best treat it as a continuous distribution of matter. The “particles” then become differential mass elements dm, the sums become integrals, and the coordinates of the center of mass are defined as

$$x_{com} = \frac{1}{M}\int x dm$$

$$\overrightarrow{r_{com}} = \frac{1}{M}\int \vec{r}\, dm$$

$$y_{com} = \frac{1}{M}\int y dm$$

$$z_{com} = \frac{1}{M}\int z dm$$

Linear momentum

- For a single particle, we define a quantity $\vec{p}$ called its linear momentum as

$$\vec{p} = m\vec{v}$$

which is a vector quantity that has the same direction as the particle's velocity. We can write Newton's second law in terms of this momentum:

$$\overrightarrow{F_{NET}} = \frac{d\vec{p}}{dt}$$

- For a system of particles these relations become

$$\vec{P} = m\overrightarrow{v_{COM}} \text{ and } \overrightarrow{F_{NET}} = \frac{d\vec{P}}{dt}$$

Impulse and Momentum

- Similar to our derivation to energy (integral EOM respect to displacement), we integral EOM respect to time:

$$\int \overrightarrow{F_{NET}}\, dt = \vec{p}(t_{final}) - \vec{p}(t_{initial}) = \Delta\vec{p}$$

Where we define impulse as $\vec{J} = \int \overrightarrow{F_{NET}}\, dt$

and we have $\vec{J} = \Delta\vec{p}$

Collision of two particles

- The total linear momentum of the system cannot change for a collision of two particles because there is no net external force to change the system.
- If that total happens to be unchanged by the collision, then the kinetic energy of the system is conserved (it is the same before and after the collision). Such a collision is called an elastic collision.
- If the kinetic energy of the system is not conserved, such a collision is called an inelastic collision.

Collision of two particles

- Conservation of linear momentum:

$$m_1\overrightarrow{v_{1i}} + m_2\overrightarrow{v_{2i}} = m_1\overrightarrow{v_{1f}} + m_2\overrightarrow{v_{2f}}$$

- Elastic collision(conservation of kinetic energy)

$$\frac{1}{2}m_1{v_{1i}}^2 + \frac{1}{2}m_2{v_{2i}}^2 = \frac{1}{2}m_1{v_{1f}}^2 + \frac{1}{2}m_2{v_{2f}}^2$$

- Inelastic collision

$$\frac{1}{2}m_1{v_{1i}}^2 + \frac{1}{2}m_2{v_{2i}}^2 \neq \frac{1}{2}m_1{v_{1f}}^2 + \frac{1}{2}m_2{v_{2f}}^2$$

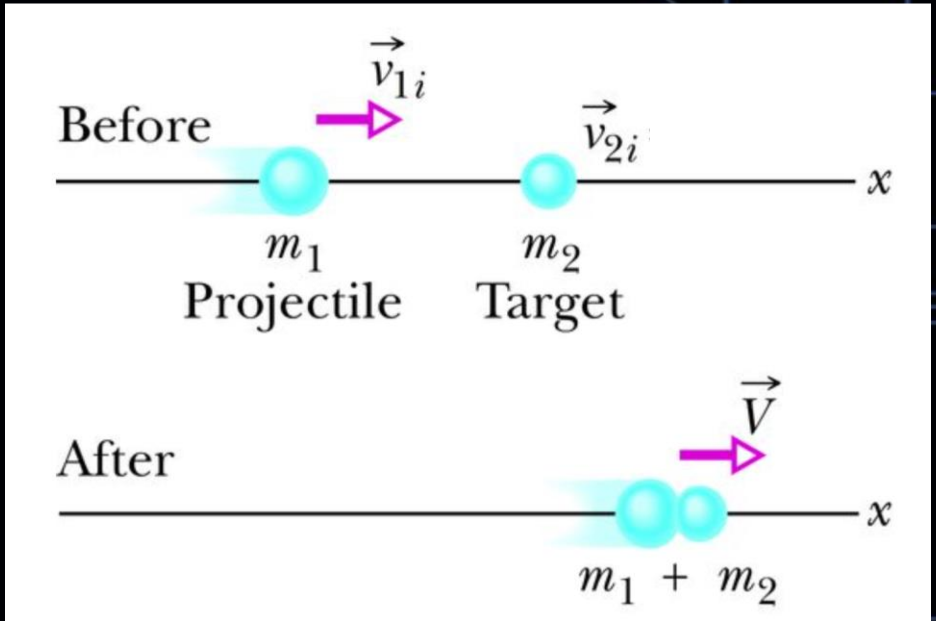
Case 1: completely Inelastic collision

- In a completely inelastic collision the two particles will stick together: the final velocities are the same
- Conservation of linear momentum:

$$m_1\overrightarrow{v_{1i}} + m_2\overrightarrow{v_{2i}} = (m_1+m_2)\vec{V}$$

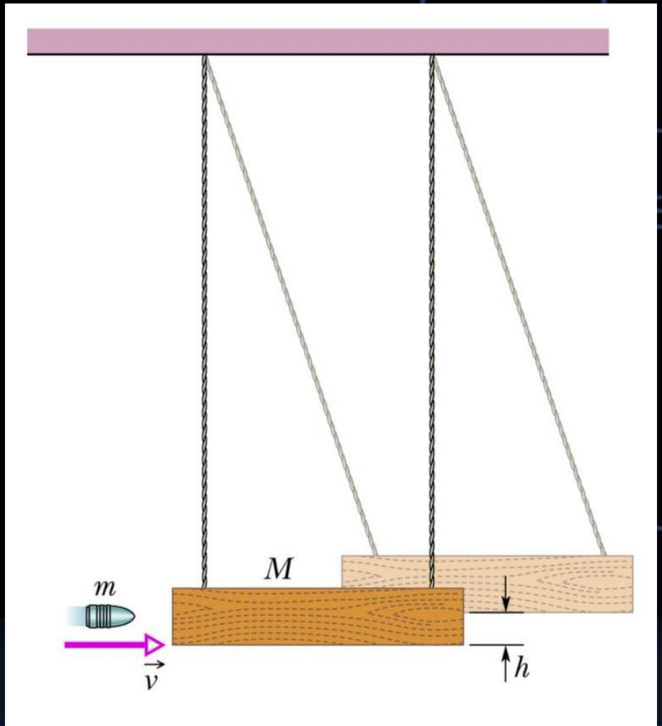
- If $\overrightarrow{v_{2i}} = 0$, we will have

$$\vec{V} = \frac{m_1}{m_1+m_2}\overrightarrow{v_{1i}}$$



Example of completely inelastic collision

The ballistic pendulum was used to measure the speeds of bullets before electronic timing devices were developed. A large block of wood of mass M = 5.4 kg, hanging from two long cords. A bullet of mass m = 9.5 g is fired into the block, coming quickly to rest. The block + bullet then swing upward, their center of mass rising a vertical distance h = 6.3 cm What is the speed of the bullet just prior to the collision?



Example of completely inelastic collision

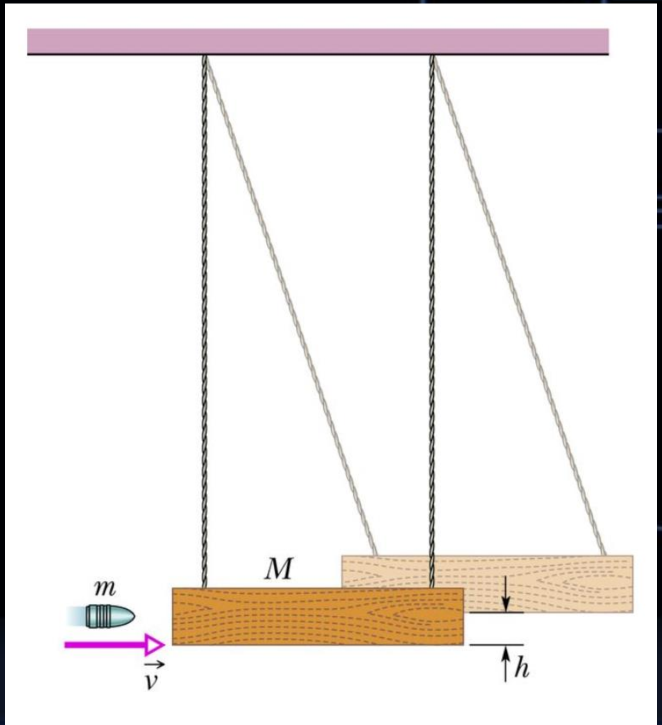
There are two events: 1. collision of bullet and block. 2. swinging of bullet and block.

- Collision of bullet and block: a completely inelastic collision:

$$\vec{V} = \frac{m_1}{m_1 + m_2}\overrightarrow{v_{1i}}$$

- Swing of bullet and block: conservation of energy

$$\frac{1}{2}(m_1 + m_2)(\vec{V})^2 = (m_1 + m_2)gh$$



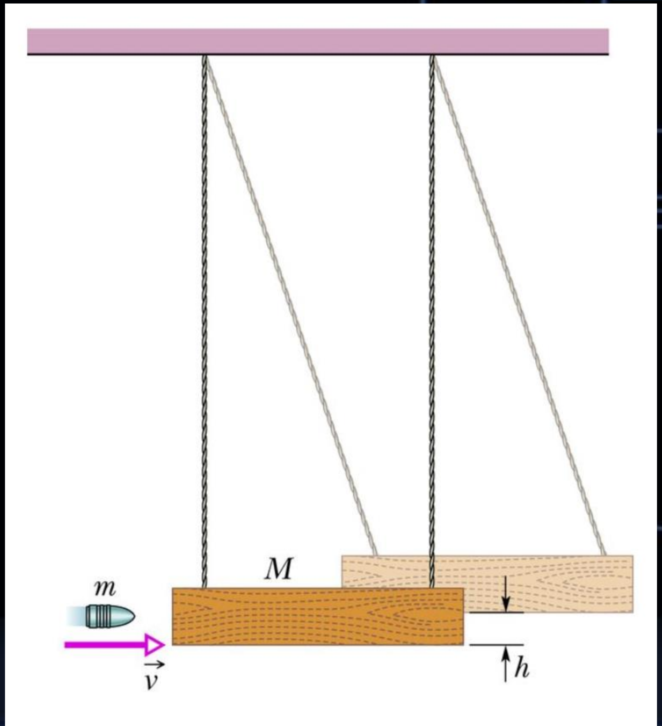
Example of completely inelastic collision

Therefore, we have:

$$\frac{1}{2}(m_1+m_2)\left(\frac{m_1}{m_1+m_2}\overrightarrow{v_{1i}}\right)^2=(m_1+m_2)gh$$

And we can get:

$$v_{1i}=630m/s$$

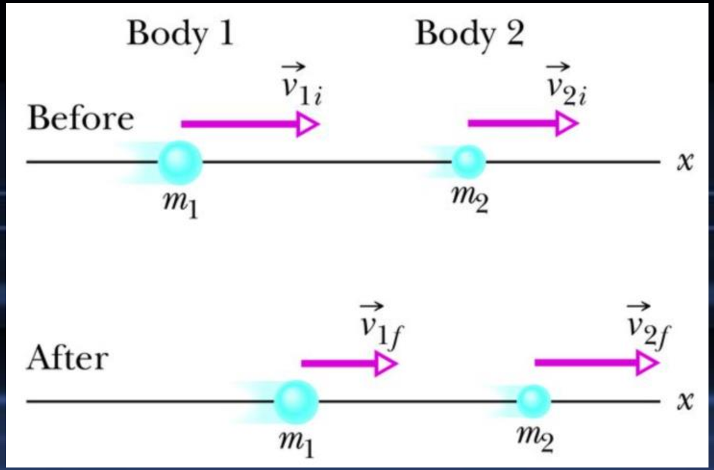


Case 2: Elastic collision

In an elastic collision, (1)total momentum is conserved (2) the kinetic energy of each colliding body may change, but the total kinetic energy of the system does not change.

$$m_1\overrightarrow{v_{1i}} + m_2\overrightarrow{v_{2i}} = m_1\overrightarrow{v_{1f}} + m_2\overrightarrow{v_{2f}}$$

$$\frac{1}{2}m_1{v_{1i}}^2 + \frac{1}{2}m_2{v_{2i}}^2 = \frac{1}{2}m_1{v_{1f}}^2 + \frac{1}{2}m_2{v_{2f}}^2$$

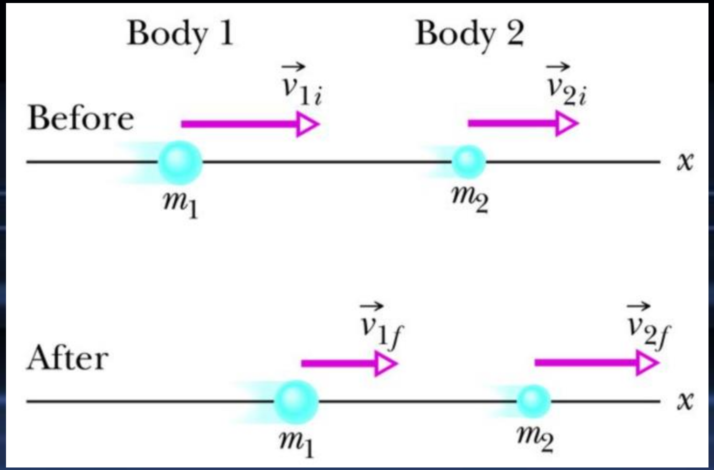


Case 2: Elastic collision

After rearrange, we can get final velocities of each particles in terms of initial velocities:

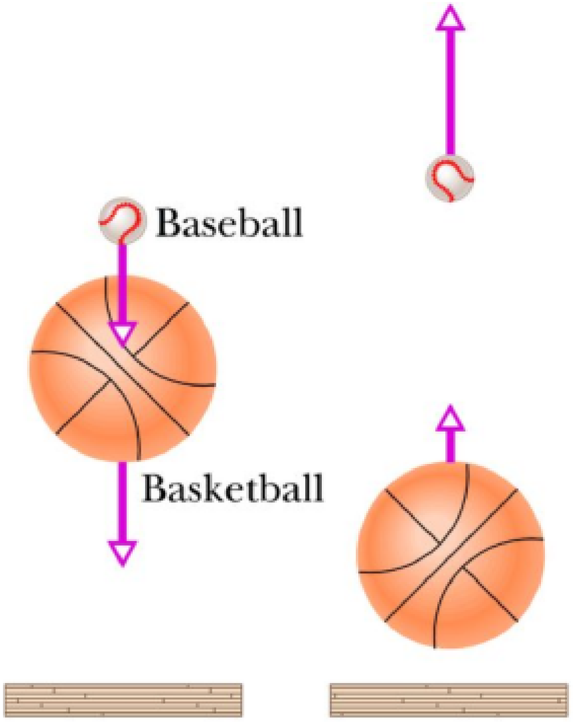
$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} + \frac{2m_2}{m_1 + m_2} v_{2i}$$

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} + \frac{m_1 - m_2}{m_1 + m_2} v_{2i}$$



2.Elastic Collision

A small ball of mass m is aligned above a larger ball of mass $M = 0.63$ kg (with a slight separation, as with the baseball and basketball of Fig.(a), and the two are dropped simultaneously from a height of $h = 1.8$ m. (Assume the radius of each ball is negligible relative to h.) (a) If the larger ball rebounds elastically from the floor and then the small ball rebounds elastically from the larger ball, what value of m results in the larger ball stopping when it collides with the small ball? (0.5points) (b) What height does the small ball then reach Fig.(b)? (0.5points)



(a) Before (b) After

Solution:

(a) Right before the basketball hit the floor, both basket ball and baseball have a velocity with magnitude $v_0 = \sqrt{2gh} = -5.94m/s$ and the negative sign means the direction of momention is pointing down. After the basketball hit the floor, the basketball will have a velocity with the same magnitude but the direction is upward because it rebounds elastically. Next, the collision process between basketball and baseball is elastic and the final velocity of basketball is zero. Therefore, we will have conservation of energy and conservation of momentum and the expression for final velocity of ellastic collision is valid.

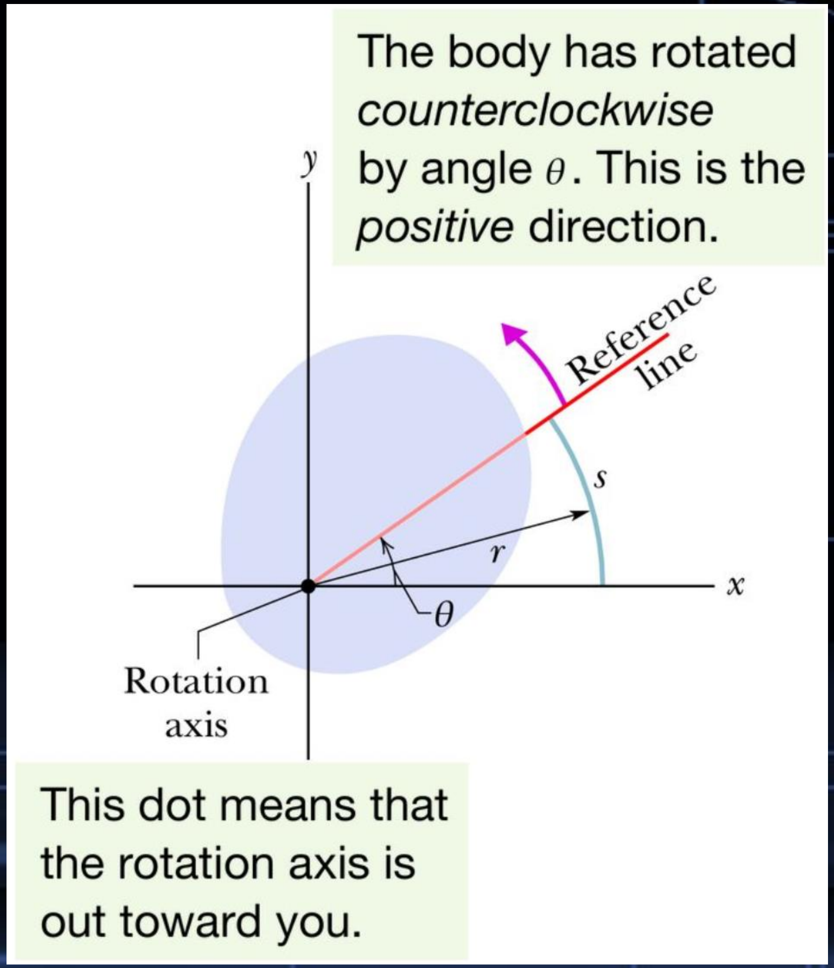
Therefore final velocity of basketball $v_{basketball-f} = \frac{M-m}{M+m}v_0 - \frac{2m}{M+m}v_0 = \frac{M-3m}{M+m}v_0$. Thus, if the final velocity of basketball is zero, the requirement is $m = \frac{M}{3} = 0.21kg$.

(b)The final velocity of baseball $v_{baseball-f} = \frac{2M}{M+m}v_0 + \frac{M-m}{M+m}v_0 = \frac{3M-m}{M+m}v_0 = \frac{3M-\frac{M}{3}}{M+\frac{M}{3}}v_0 = 2v_0 = 11.88m/s$. Thus, the final height is $h = \frac{v_{baseball-f}^2}{2g} = 7.2m$

Angular position

- In pure rotation (angular motion), every point of the body moves in a circle whose center lies on the axis of rotation, and every point moves through the same angle during a particular time interval.
- The angular position θ is measured relative to the positive direction of the x axis.

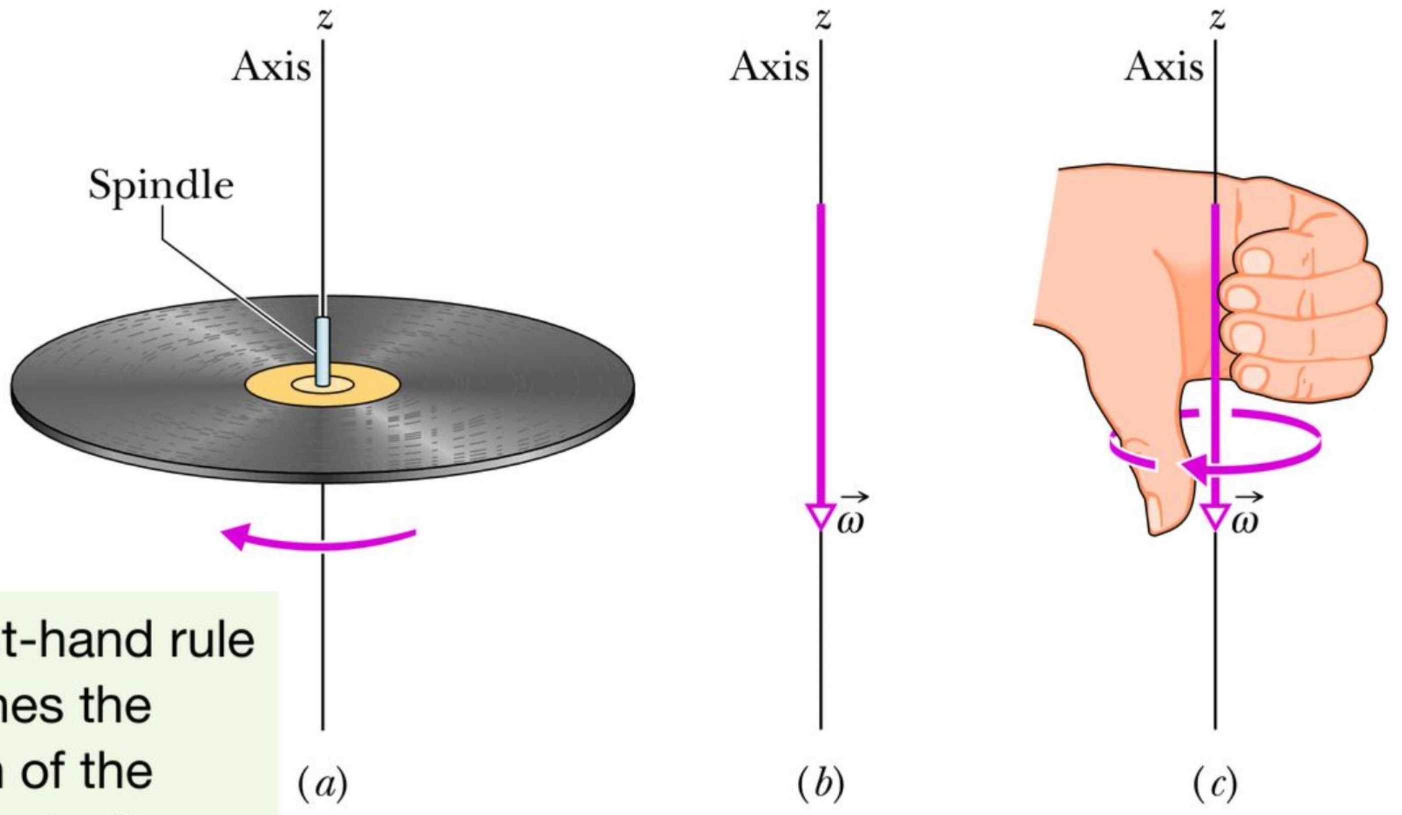
$$\theta = \frac{s}{r}$$



Rotation with constant angular acceleration

Linear Equation	**Missing Variable**		**Angular Equation**
$v = v_0 + at$	$x - x_0$	$\theta - \theta_0$	$\omega = \omega_0 + \alpha t$
$x - x_0 = v_0 t + \frac{1}{2} a t^2$	v	ω	$\theta - \theta_0 = \omega_0 t + \frac{1}{2} a t^2$
$v^2 = v_0^2 + 2a\left(x - x_0\right)$	t	t	$\omega^2 = \omega_0^2 + 2a\left(\theta - \theta_0\right)$
$x - x_0 = \frac{1}{2}\left(v_0 + v\right) t$	a	α	$\theta - \theta_0 = \frac{1}{2}\left(\omega_0 + \omega\right) t$
$x - x_0 = vt - \frac{1}{2} a t^2$	v_0	ω_0	$\theta - \theta_0 = \omega t - \frac{1}{2} \alpha t^2$

Angular velocity as a vector



This right-hand rule establishes the direction of the angular velocity vector.

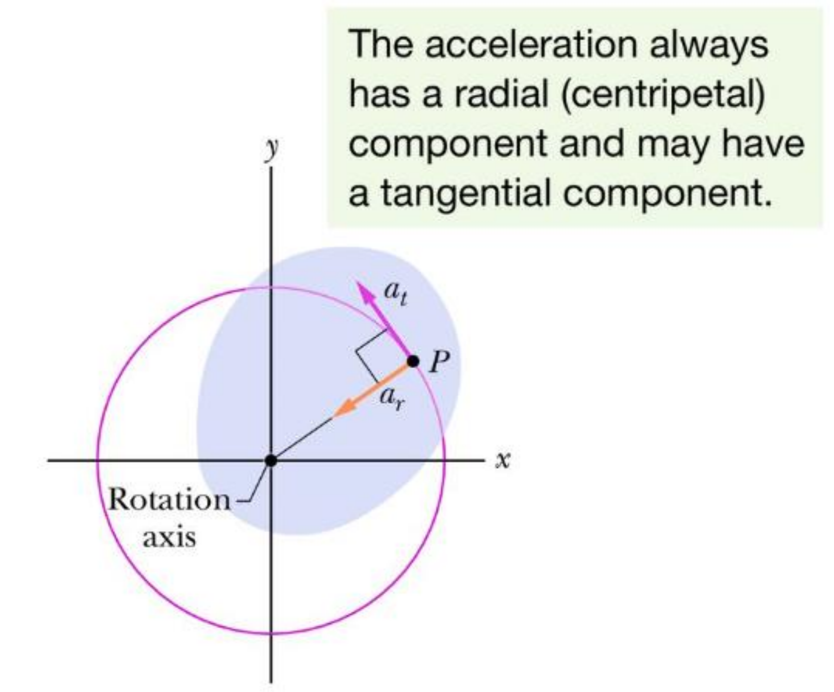
Relation between linear and angular variable

- For acceleration, the point has both tangential and radial components. The tangential component is

$$a_t = \alpha r$$

where α is the magnitude of the angular of the body. The radial component of is

$$a_r = \frac{v^2}{r} = \omega^2 r$$



Rotational inertia (or moment of inertia)

In the rotating rigid body the rotational velocity is the same for every point. Thus we can rewrite:

$$K = \sum \frac{1}{2} m_i v_i^2 = \sum \frac{1}{2} m_i (r_i \omega)^2 = \frac{1}{2} \left(\sum m_i r_i^2 \right) \omega^2$$

We call that quantity the rotational inertia (or moment of inertia) I of the body with respect to the axis of rotation:

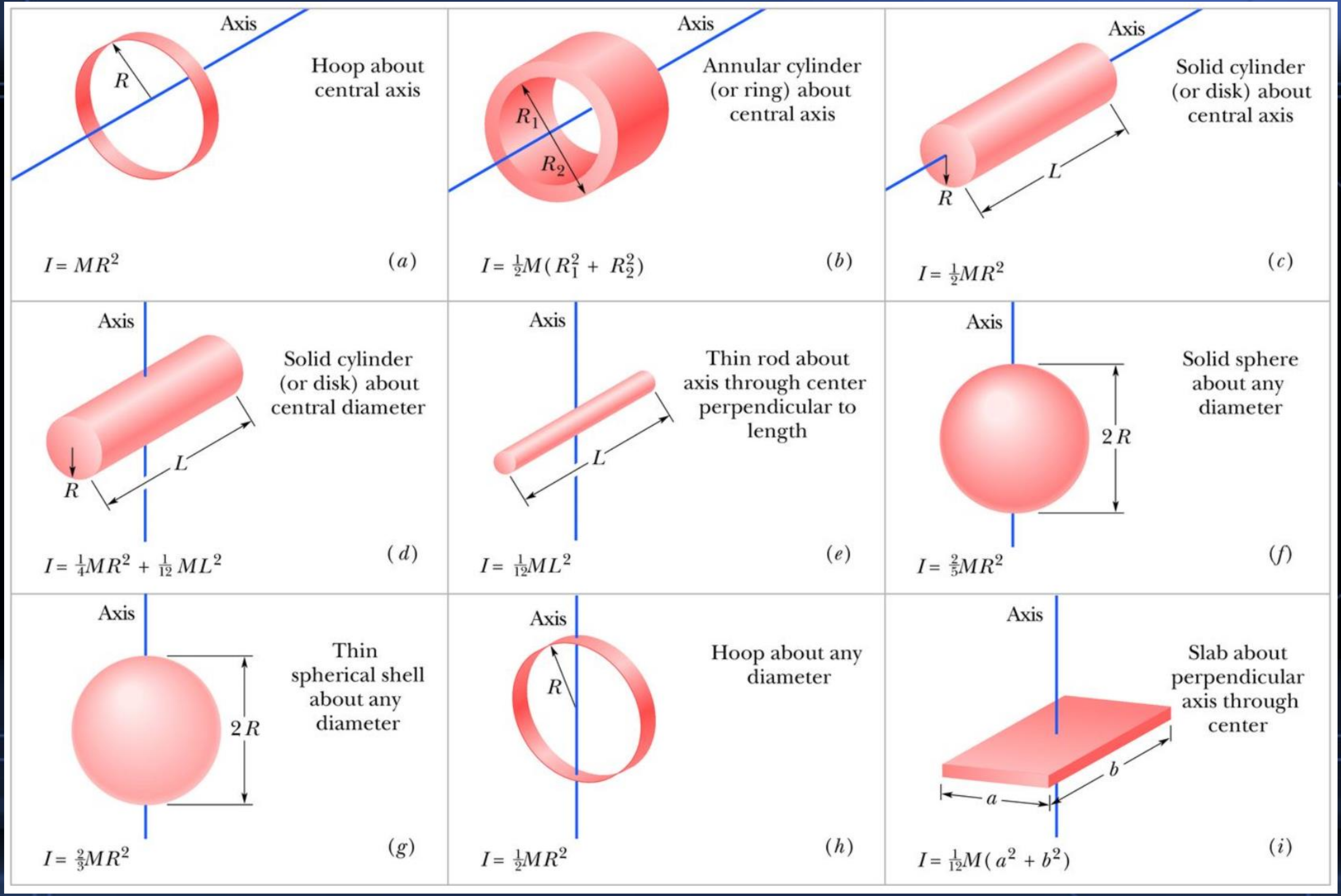
$$I = \sum m_i r_i^2$$

And thus the kinetic energy is $K = \frac{1}{2} I \omega^2$

Calculating the rotational inertia

- If a rigid body consists of a few particles, we can calculate its rotational inertia about a given rotation axis with $I = \sum m_i r_i^2$.
- If a rigid body consists of a great many adjacent particles, we replace the sum with an integral and define the rotational inertia of the body as

$$I = \int r^2 dm$$


Axis
Hoop about central axis
R
$I = MR^2$
(a)
Axis
Annular cylinder (or ring) about central axis
R_1
R_2
$I = \frac{1}{2}M(R_1^2 + R_2^2)$
(b)
Axis
Solid cylinder (or disk) about central axis
L
R
$I = \frac{1}{2}MR^2$
(c)
Axis
Solid cylinder (or disk) about central diameter
L
R
$I = \frac{1}{4}MR^2 + \frac{1}{12}ML^2$
(d)
Axis
Thin rod about axis through center perpendicular to length
L
$I = \frac{1}{12}ML^2$
(e)
Axis
Solid sphere about any diameter
2R
$I = \frac{2}{5}MR^2$
(f)
Axis
Thin spherical shell about any diameter
2R
$I = \frac{2}{3}MR^2$
(g)
Axis
Hoop about any diameter
R
$I = \frac{1}{2}MR^2$
(h)
Axis
Slab about perpendicular axis through center
b
a
$I = \frac{1}{12}M(a^2 + b^2)$
(i)

Parallel-Axis Theorem

- For calculating rotational inertia, directly calculation through $I = \int r^2 dm$ can work.
- Assuming we know the rotational inertia I_{com}, where the rotational axis is through the body's center of mass. Then, the rotational inertia I about a rotational axis parallel to the rotational axis through COM is:

$$I = I_{COM} + Mh^2$$

where M is the mass of the body and h is the distance that we have shifted the rotation axis from being through the com.

Momentum: translational vs. rotational

Translational		Rotational	
Force	$\vec{F}$	Torque	$\vec{\tau}\left(=\vec{r}\times\vec{F}\right)$
Linear momentum	$\vec{p}$	Angular momentum	$\vec{\ell}\left(=\vec{r}\times\vec{p}\right)$
Linear momentum[b]	$\vec{P}\left(=\Sigma\vec{p}_i\right)$	Angular momentum[b]	$\vec{L}\left(=\Sigma\vec{\ell}_i\right)$
Linear momentum[b]	$\vec{P}=M\vec{v}_{\text{com}}$	Angular momentum[c]	$L=I\omega$
Newton's second law[b]	$\vec{F}_{\text{net}}=\frac{d\vec{P}}{dt}$	Newton's second law[b]	$\vec{\tau}_{\text{net}}=\frac{d\vec{L}}{dt}$
Conservation law[d]	$\vec{P}=\text{a constant}$	Conservation law[d]	$\vec{L}=\text{a constant}$

Summary I

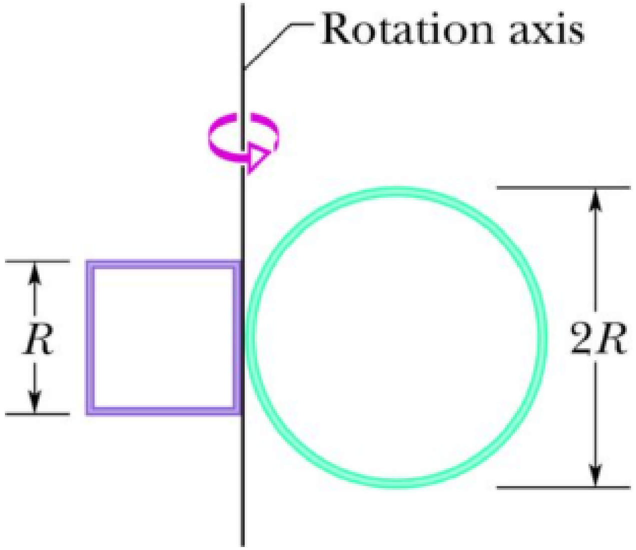
Pure Translation (Fixed Direction)		**Pure Rotation (Fixed Axis)**	
Position	x	Angular position	θ
Velocity	$v = dx/dt$	Angular velocity	$\omega = d\theta/dt$
Acceleration	$a = dv/dt$	Angular acceleration	$\alpha = d\omega/dt$
Mass	m	Rotational inertia	I
Newton's second law	$F_{\text{net}} = ma$	Newton's second law	$\tau_{\text{net}} = I\alpha$
Work	$W = \int F\,dx$	Work	$W = \int \tau\,d\theta$
Kinetic energy	$K = \frac{1}{2}mv^2$	Kinetic energy	$K = \frac{1}{2}I\omega^2$
Power (constant force)	$P = Fv$	Power (constant torque)	$P = \tau\omega$
Work–kinetic energy theorem	$W = \Delta K$	Work–kinetic energy theorem	$W = \Delta K$

Summary II

Translational		Rotational	
Force	$\vec{F}$	Torque	$\vec{\tau}\left(=\vec{r}\times\vec{F}\right)$
Linear momentum	$\vec{p}$	Angular momentum	$\vec{\ell}\left(=\vec{r}\times\vec{p}\right)$
Linear momentum[b]	$\vec{P}\left(=\Sigma\vec{p}_i\right)$	Angular momentum[b]	$\vec{L}\left(=\Sigma\vec{\ell}_i\right)$
Linear momentum[b]	$\vec{P}=M\vec{v}_{\text{com}}$	Angular momentum[c]	$L=I\omega$
Newton's second law[b]	$\vec{F}_{\text{net}}=\frac{d\vec{P}}{dt}$	Newton's second law[b]	$\vec{\tau}_{\text{net}}=\frac{d\vec{L}}{dt}$
Conservation law[d]	$\vec{P}=$ a constant	Conservation law[d]	$\vec{L}=$ a constant

3. Rotational Inertia and Angular Momentum

As shown in the following figure, a rigid structure consisting of a circular hoop of radius R and mass m, and a square made of four thin bars, each of length R and mass m. The rigid structure rotates at a constant speed about a vertical axis, with a period of rotation of 2.5 s. Assuming R = 0.50 m and m = 2.0 kg, calculate (a) the structure's rotational inertia about the axis of rotation and (0.5points) (b) its angular momentum about that axis. (0.5points)



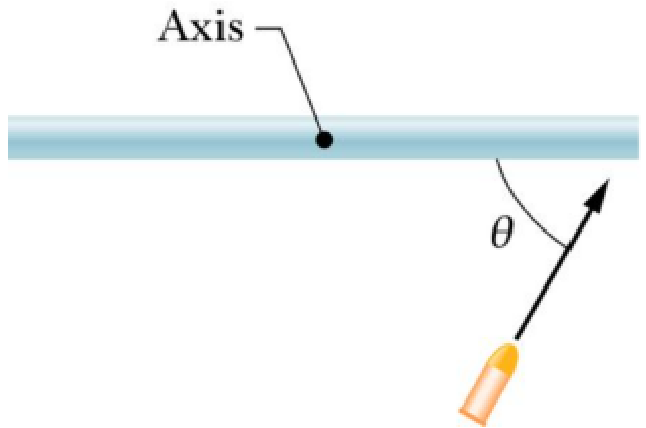
Solution:

(a) For the square, the rotational inertia $I_{square} = MR^2 + 2 \times [\frac{1}{12}MR^2 + M(\frac{R}{2})^2] + 0 = \frac{5}{3}MR^2$. For the circle loop, we can start with rotaional inertia $= \frac{1}{2}MR^2$ when the rotation is panetrating the circle center. (You can find this in the course slides.) Then use parallel axis theorem, we have $I_{circle} = \frac{1}{2}MR^2 + MR^2 = \frac{3}{2}MR^2$. Therefore, the total rotaional intertia $I_{total} = I_{square} + I_{circle} = \frac{5}{3}MR^2 + \frac{3}{2}MR^2 = \frac{19}{6}MR^2 = 1.58kgm^2$

(b) The angular moment $L = I_{total}\omega = 3.97kgm/s^2$

4.Conservation of Angular Momentum

In the following fig. (an overhead view), a uniform thin rod of length 0.500 m and mass 4.00 kg can rotate in a horizontal plane about a vertical axis through its center. The rod is at rest when a 3.00 g bullet traveling in the rotation plane is fired into one end of the rod. In the view from above, the bullet's path makes angle j = 60.0° with the rod. If the bullet lodges in the rod and the angular velocity of the rod is 10 rad/s immediately after the collision, what is the bullet's speed just before impact?



Solution:

During the process, there is no external torque exerting on the system. Therefore, the angular momentum is conserved:

$$L_{initial} = rpsin\theta = (\frac{L}{2})mvsin\theta$$

$$L_{final} = I\omega = [\frac{1}{12}ML^2 + m(\frac{L}{2})^2]\omega$$

Thus the velocity of bullet: $v = \frac{[\frac{1}{12}ML^2+m(\frac{L}{2})^2]\omega}{\frac{L}{2}msin\theta} = 1285m/s$

Simple Harmonic Motion (SHM)

- A particle that is oscillating about the origin of an x axis, repeatedly going left and right by identical amounts. A simple harmonic motion (SHM) is that the displacement can be described as a sinusoidal function of time t:

$$x(t) = x_m \cos(\omega t + \phi)$$

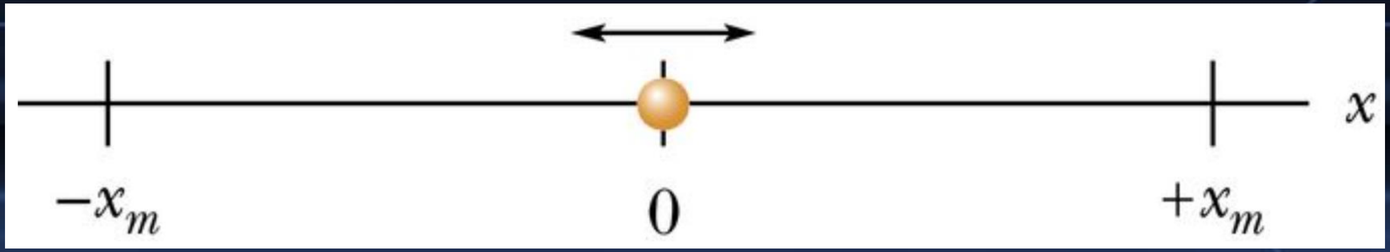
Phase

Displacement as a function of time t

Amplitude

Angular frequency

Phase constant

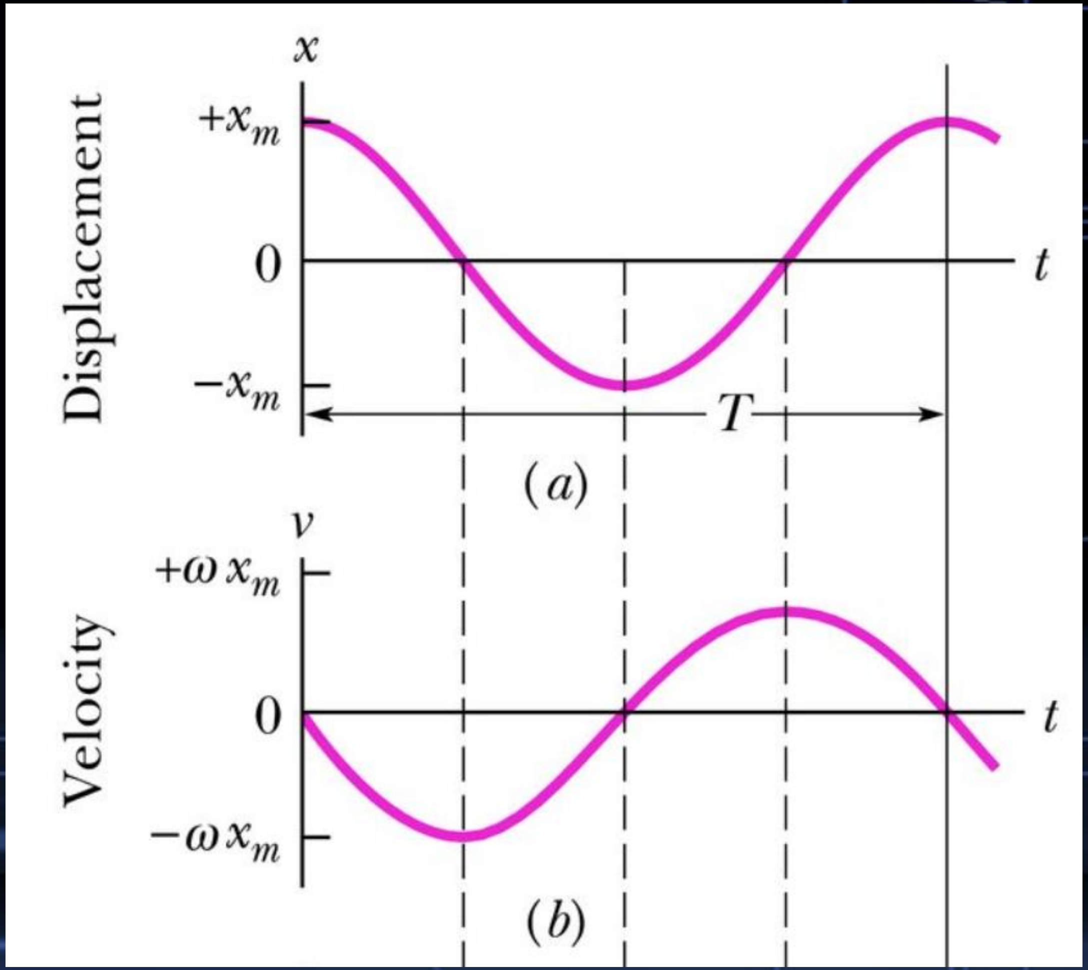


The Velocity of SHM

- By taking derivative of displacement respect to time, one can get velocity of SHM:

$$v(t) = \frac{dx(t)}{dt} = \frac{d}{dt}\left[x_m \cos(\omega t + \phi)\right]$$

$$v(t) = -\omega x_m \sin(\omega t + \phi) \quad \text{(velocity)}$$



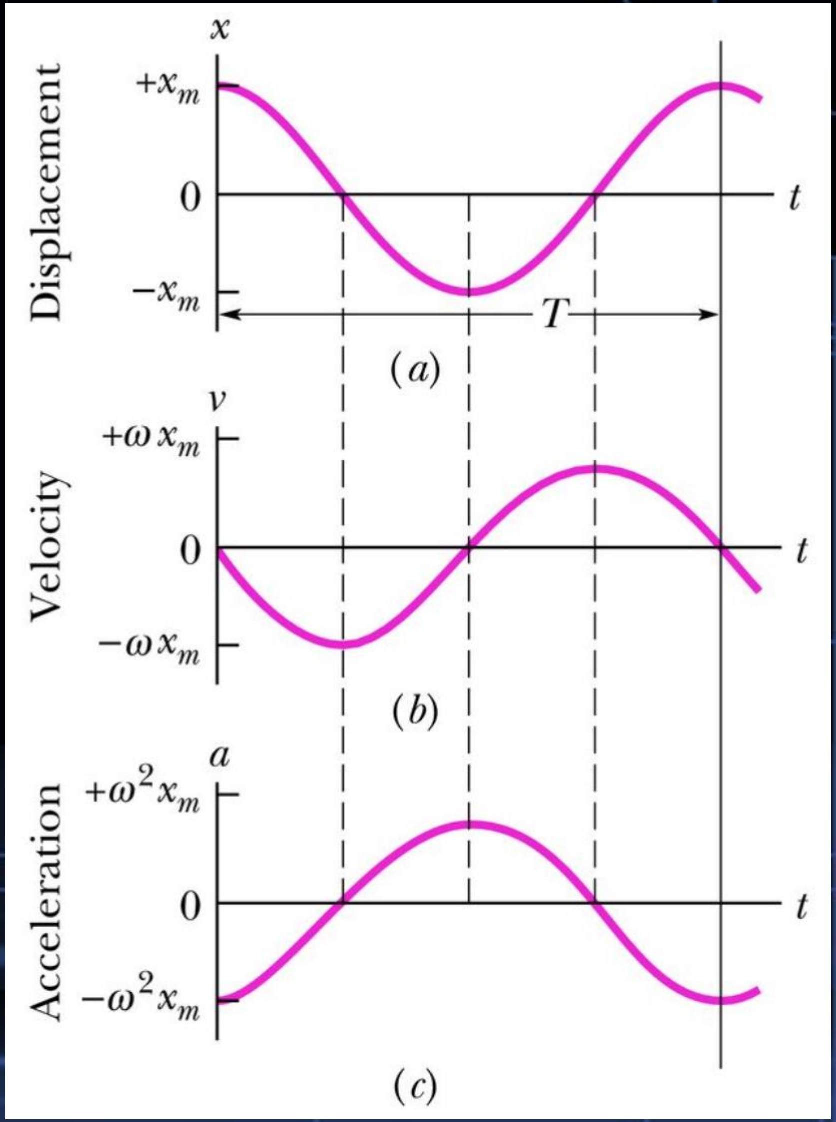
The acceleration of SHM

- By taking derivative of velocity respect to time, one can get velocity of SHM:

$$a(t) = -\omega^2 x_m \cos(\omega t + \phi) \quad \text{(acceleration)}$$

- We can also find that:

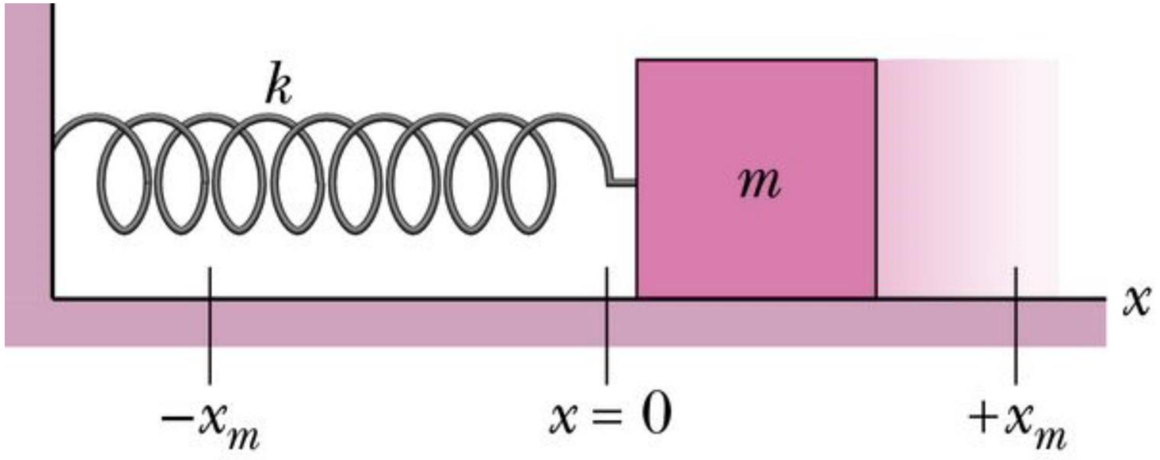
$$a(t) = -\omega^2 x(t)$$



The force law of simple harmonic oscillation

- From Newton's second law we can know that the force in the SHM should have the form of:

$$F = ma = m\left(-\omega^2 x\right) = -\left(m\omega^2\right) x$$

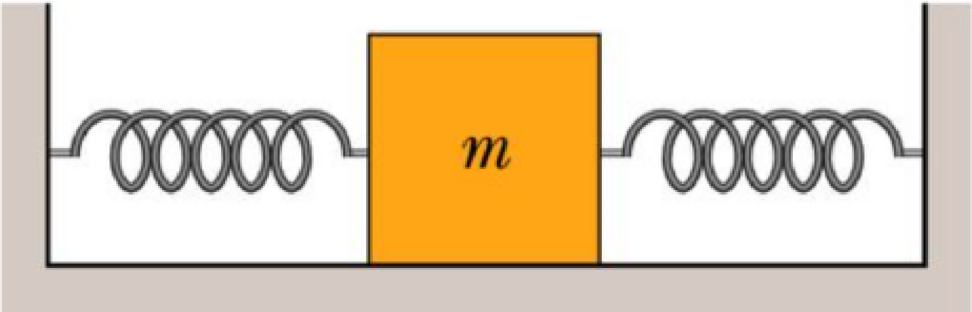


- Thus:

$$\omega = \sqrt{\frac{k}{m}} \quad \text{(angular frequency)} \qquad T = 2\pi\sqrt{\frac{m}{k}} \quad \text{(period)}$$

5.Oscillation with two springs

In the following figure, two springs are attached to a block that can oscillate over a frictionless floor. If the left spring is removed, the block oscillates at a frequency of 30 Hz. If, instead, the spring on the right is removed, the block oscillates at a frequency of 45 Hz. At what frequency does the block oscillate with both springs attached? (1point)



olution:

The oscillation frequency of a mass m attached to one spring with spring constant k is $f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

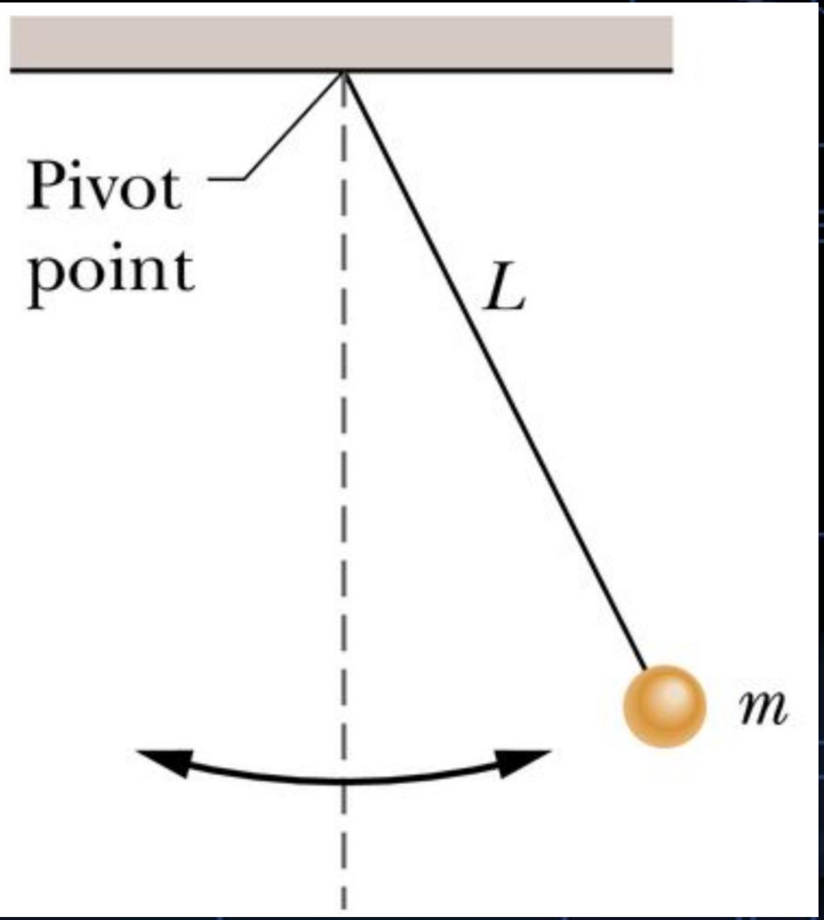
For the setup in the figure, the two springs exert forces that have the same direction and sum up at all time. Therefore, we can conclude that $f_{tot} = \frac{1}{2\pi}\sqrt{\frac{k_1+k_2}{m}}$.

Thus, $f_{tot}^2 = (\frac{1}{2\pi}\sqrt{\frac{k_1+k_2}{m}})^2 = (\frac{1}{2\pi})^2\frac{k_1+k_2}{m} = f_1^2 + f_2^2$.

And $f_{tot} = \sqrt{f_1^2 + f_2^2} = 54Hz$.

Simple pendulums

Consider a simple pendulum, which consists of a particle of mass m suspended from one end of an unstretchable, massless string of length L that is fixed at the other end. The mass is free to swing back and forth in the plane of the page, to the left and right of a vertical line through the pendulum's pivot point



Simple pendulums

- We can find the torque on m:

$$\tau = -L\,(F_g\ \sin\ \theta)$$

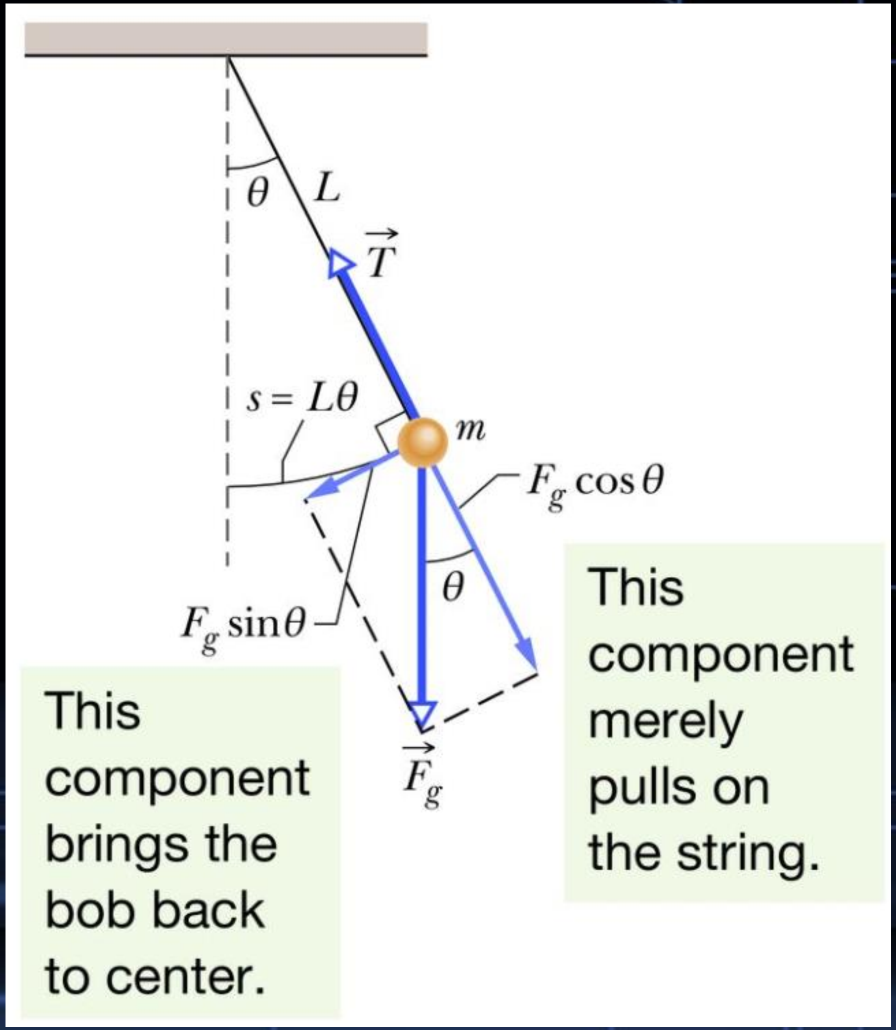
- This leads to:

$$-L\,(mg\ \sin\ \theta) = I\alpha$$

- If the angle is small:

$$\alpha = -\frac{mgL}{I}\,\theta$$

This is also SHM!



Simple pendulums

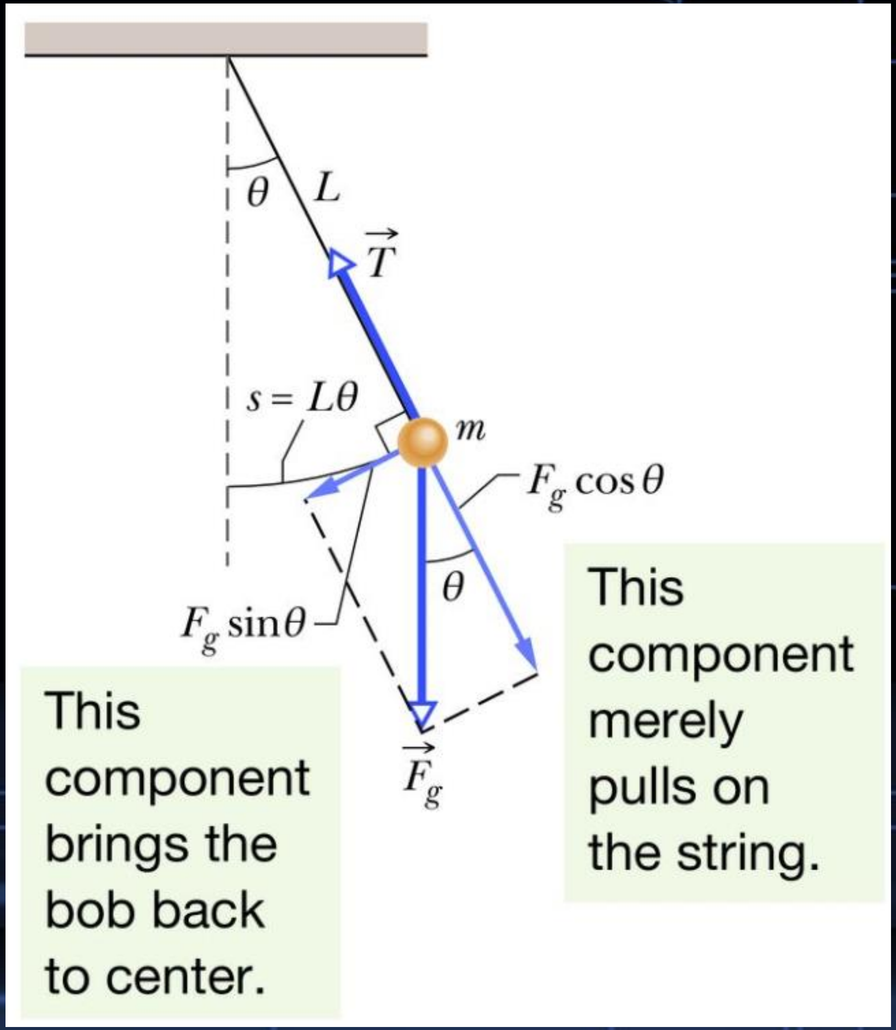
- We can find that:

$$\omega = \sqrt{\frac{mgL}{I}}$$

$$T = 2\pi \sqrt{\frac{I}{mgL}}$$

- With $I = mL^2$, we have

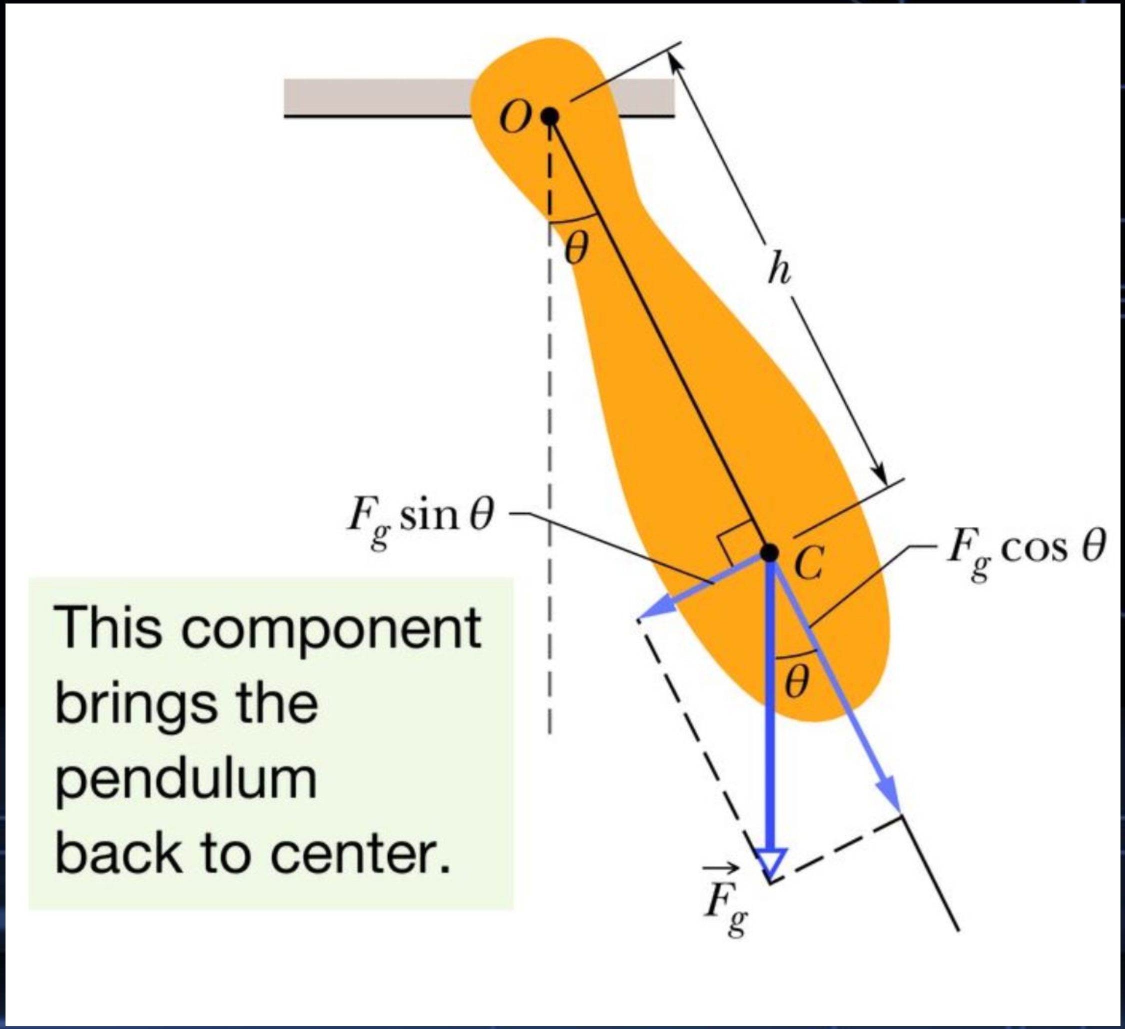
$$T = 2\pi \sqrt{\frac{L}{g}}$$



Physical Pendulums

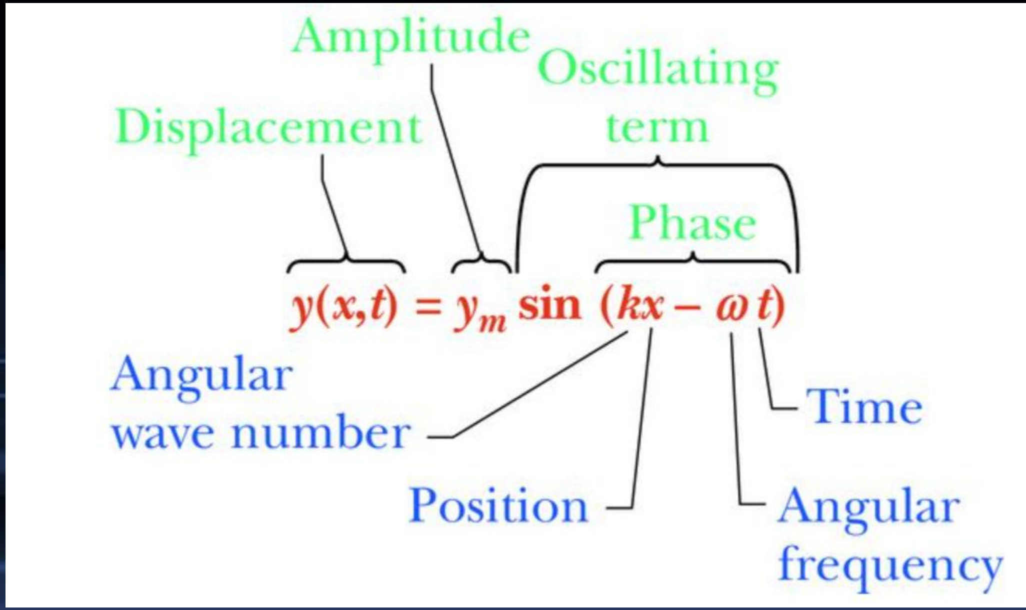
- The analysis is the same to the COM even if the pendulum have a complicated distribution of mass(physical pendulum).

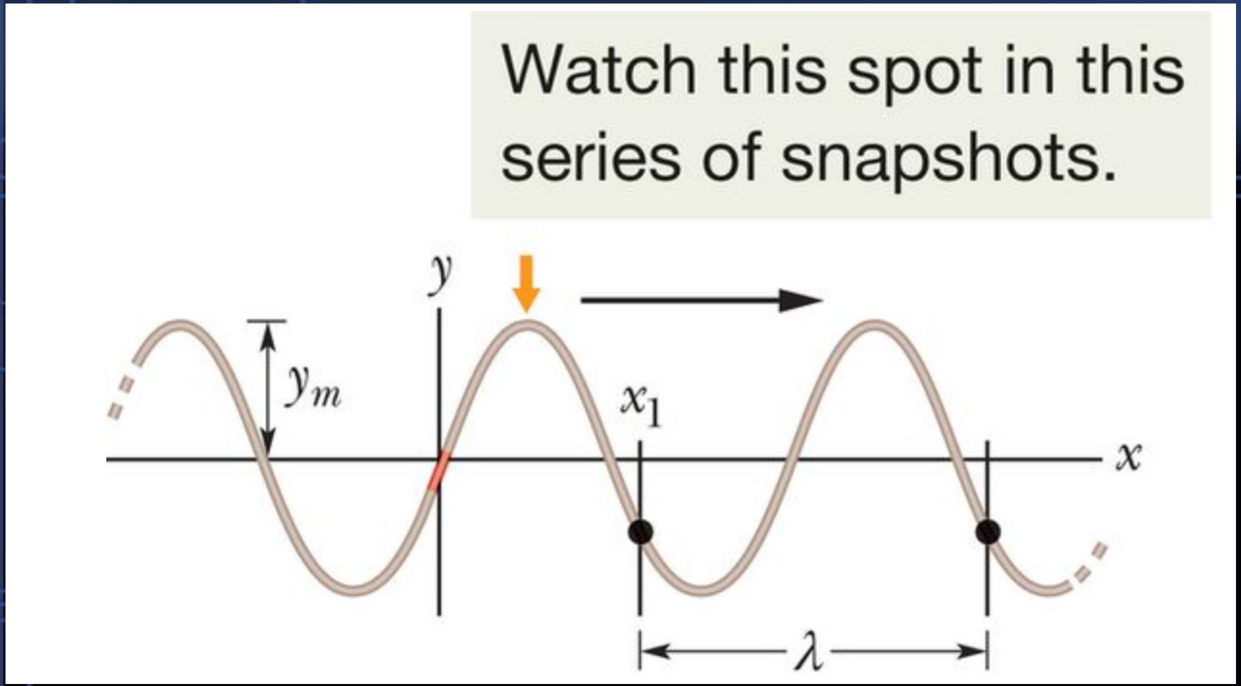
$$T = 2\pi \sqrt{\frac{I}{mgh}}$$



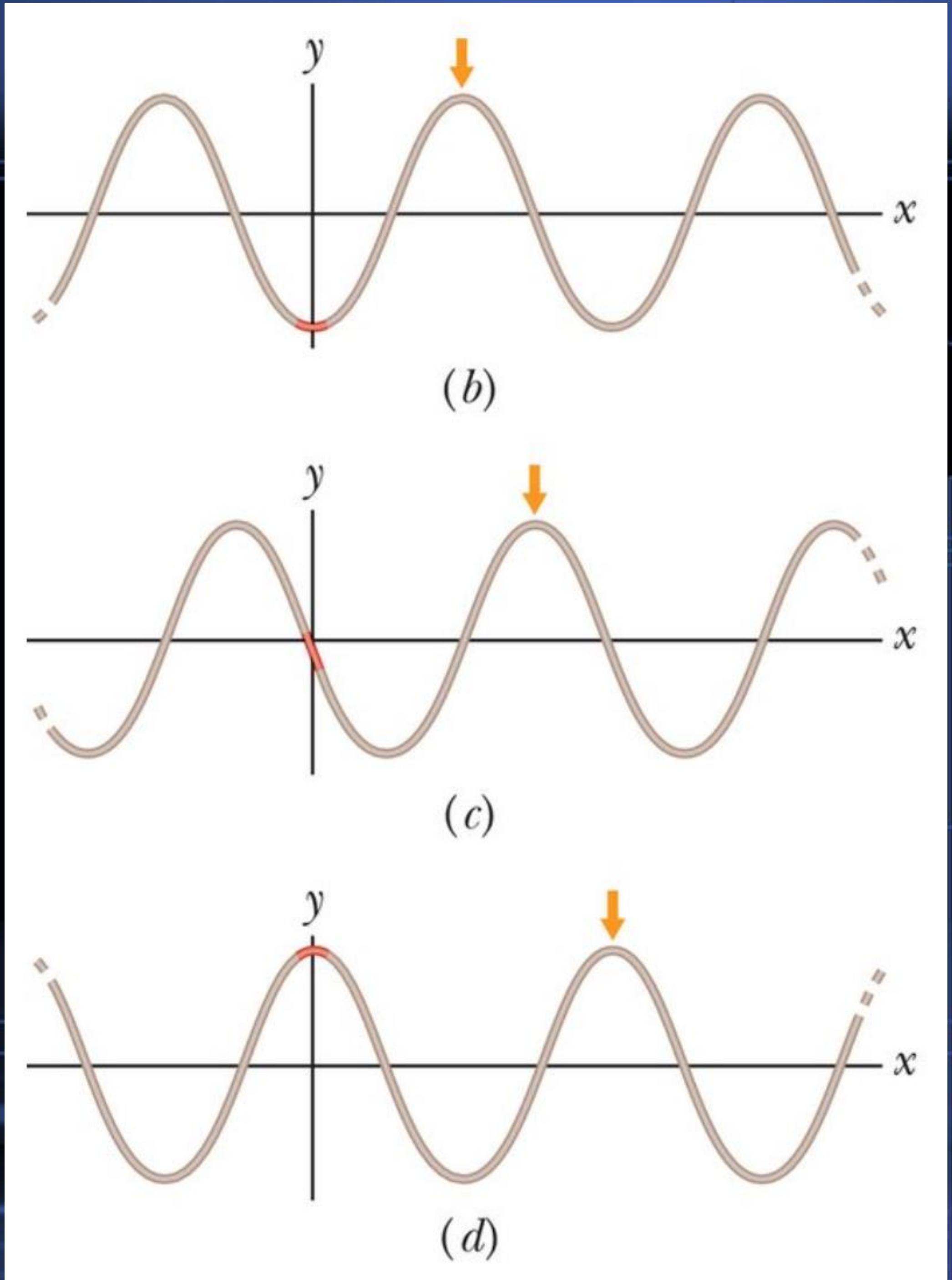
Description of wave

- Use wave on a string as an example. Imagine a sinusoidal wave traveling in the positive direction of an x axis. The elements oscillate parallel to the y axis. At time t, the displacement y(x,t) of the element located at position x is given by

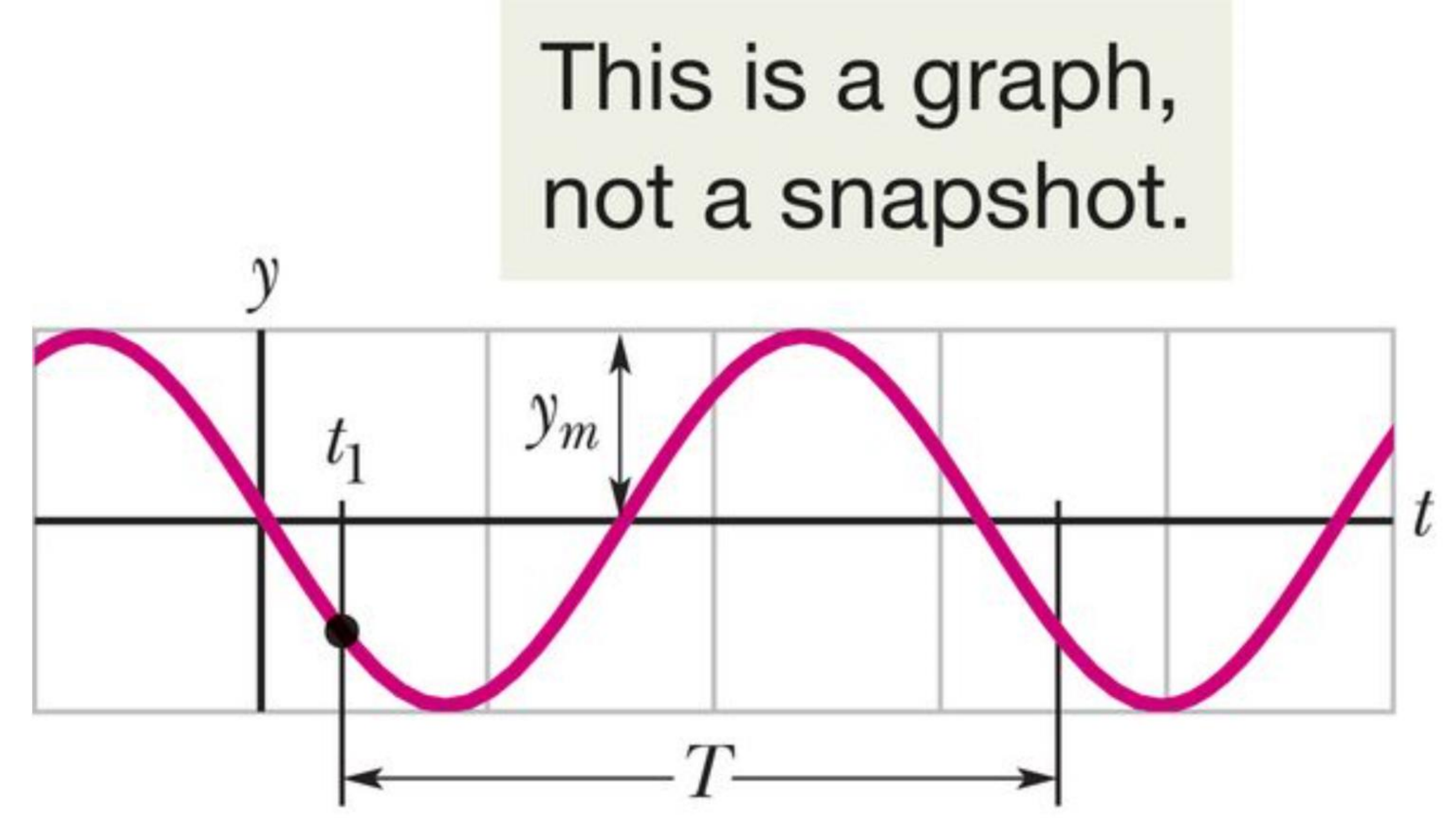




- $k = \frac{2\pi}{\lambda}$ (angular wave number)
- phase of the wave is the *argument* $(kx - \omega t)$ of the sine: a wave traveling to positive x direction



Angular frequency and frequency of wave



- $\omega = \frac{2\pi}{T}$ (angular wave frequency)
- $f = \frac{1}{T} = \frac{\omega}{2\pi}$ (frequency)

Direction of wave propagation

- With the concept of wave speed, one can find that:

$$y(x,t) = y_m \sin(kx - \omega t + \phi)$$

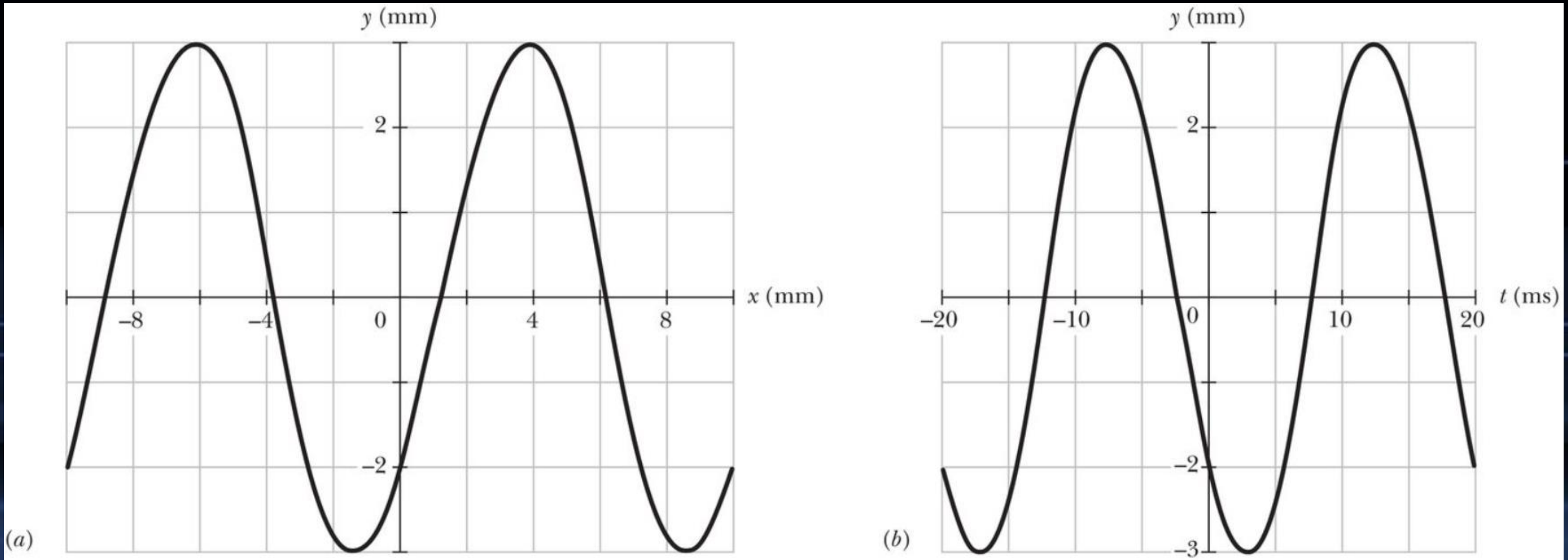
is a wave traveling to positive x direction with positive wave speed $v = \frac{\omega}{k}$.

$$y(x,t) = y_m \sin(kx + \omega t + \phi)$$

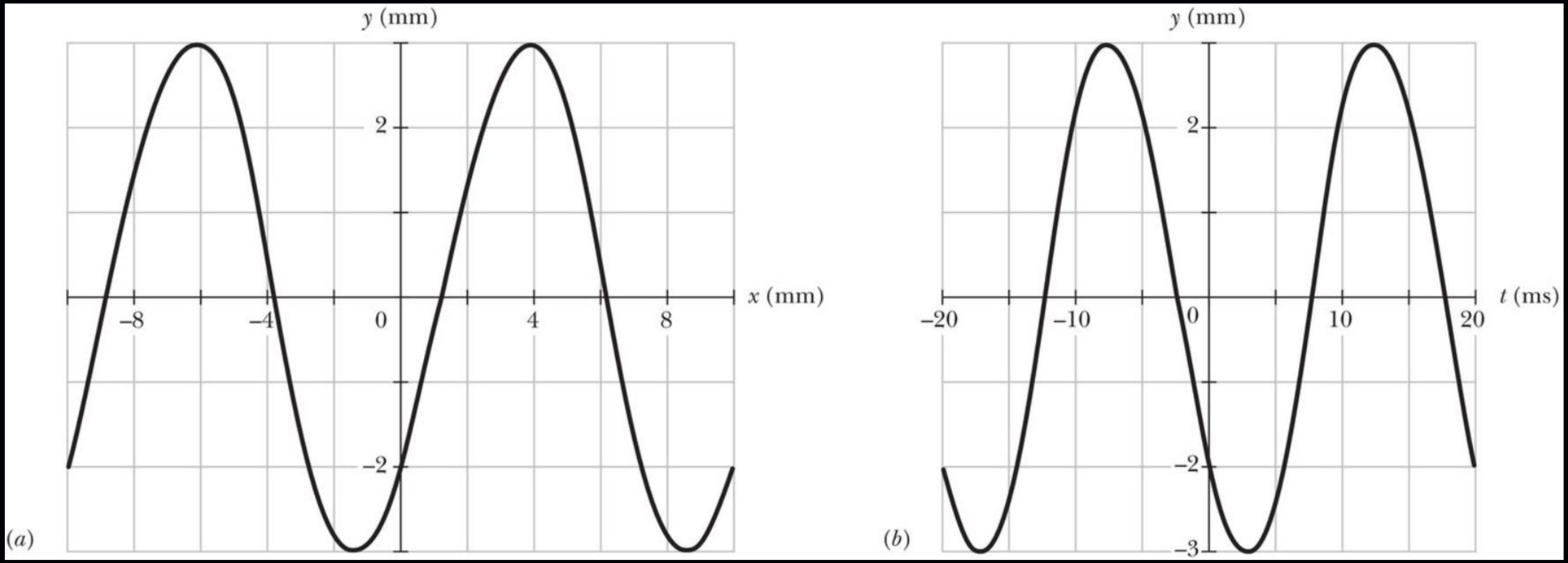
is a wave traveling to negative x direction with negative wave speed $v = -\frac{\omega}{k}$.

Example

A transverse wave traveling along an x axis has the form given by $y(x,t) = y_m \sin(kx \pm \omega t + \phi)$. Figure (a) gives the displacements of string elements as a function of x, all at time t = 0. Figure (b) gives the displacements of the element at x = 0 as a function of t. Find the values of the quantities .

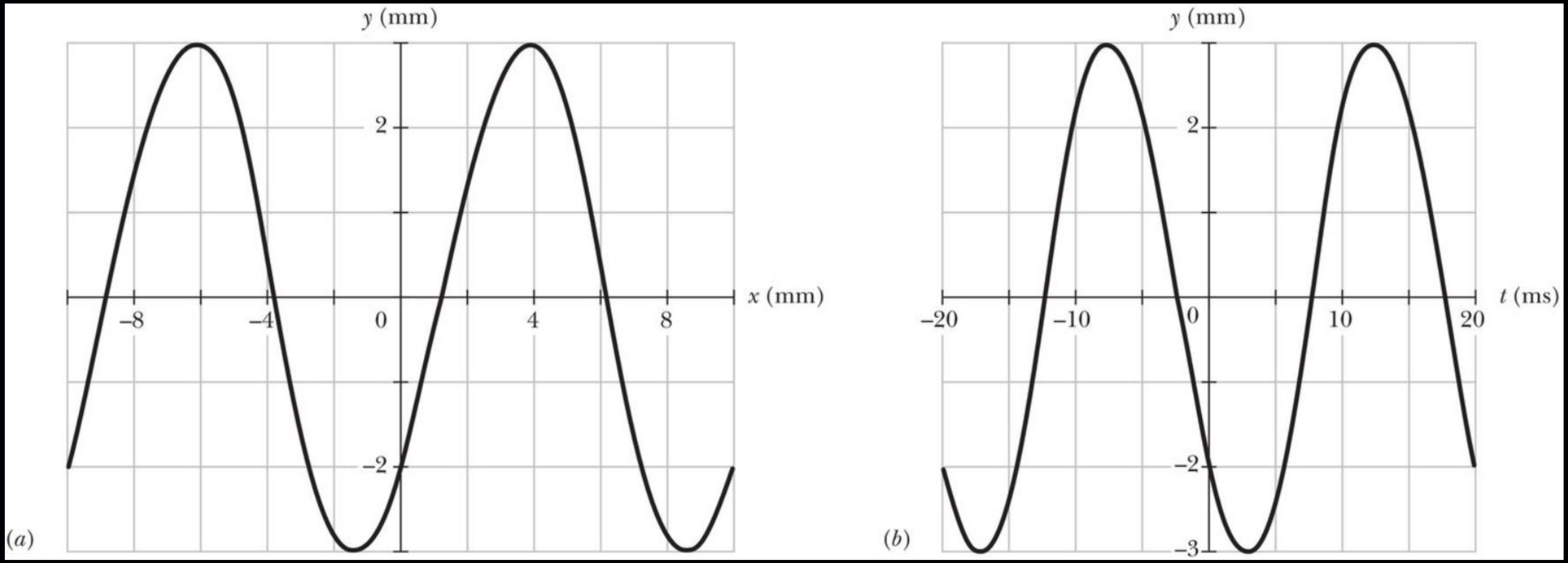


Example



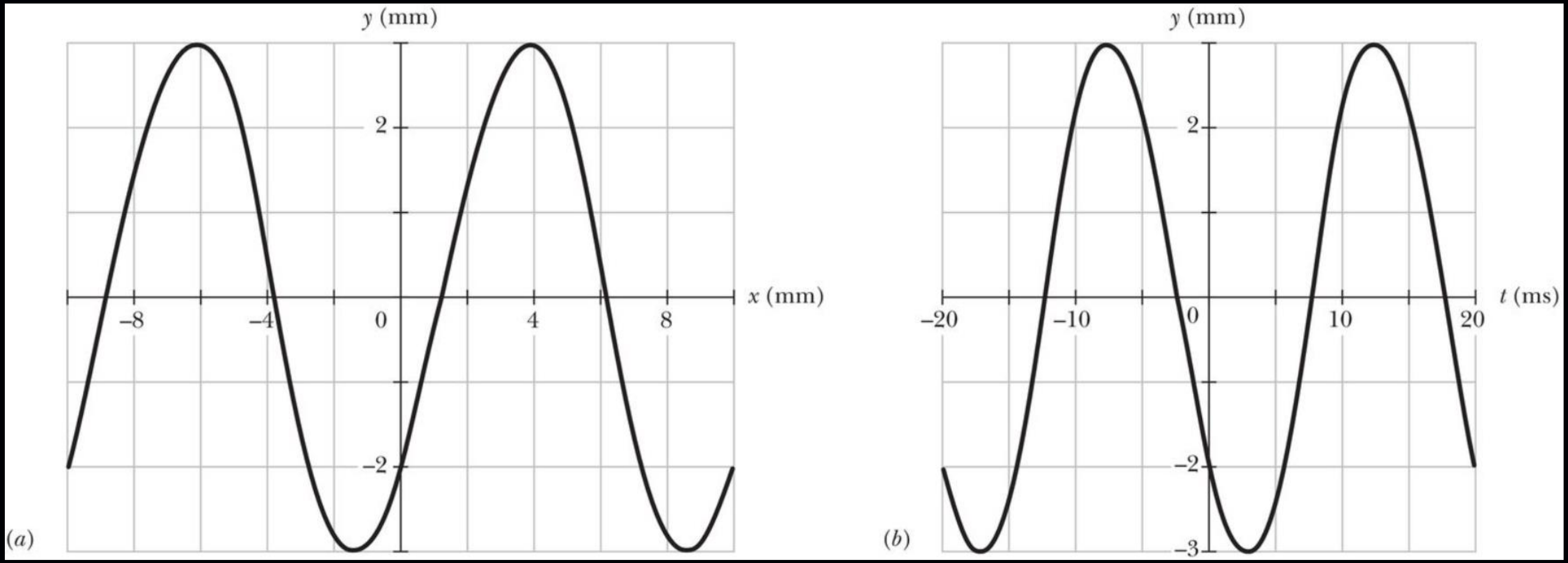
- Amplitude of wave y_m: The wave is oscillating between $+y_m$ and $-y_m$. In both figure, the wave oscillating between +3mm and -3mm. Therefore $y_m = 3mm$

Example



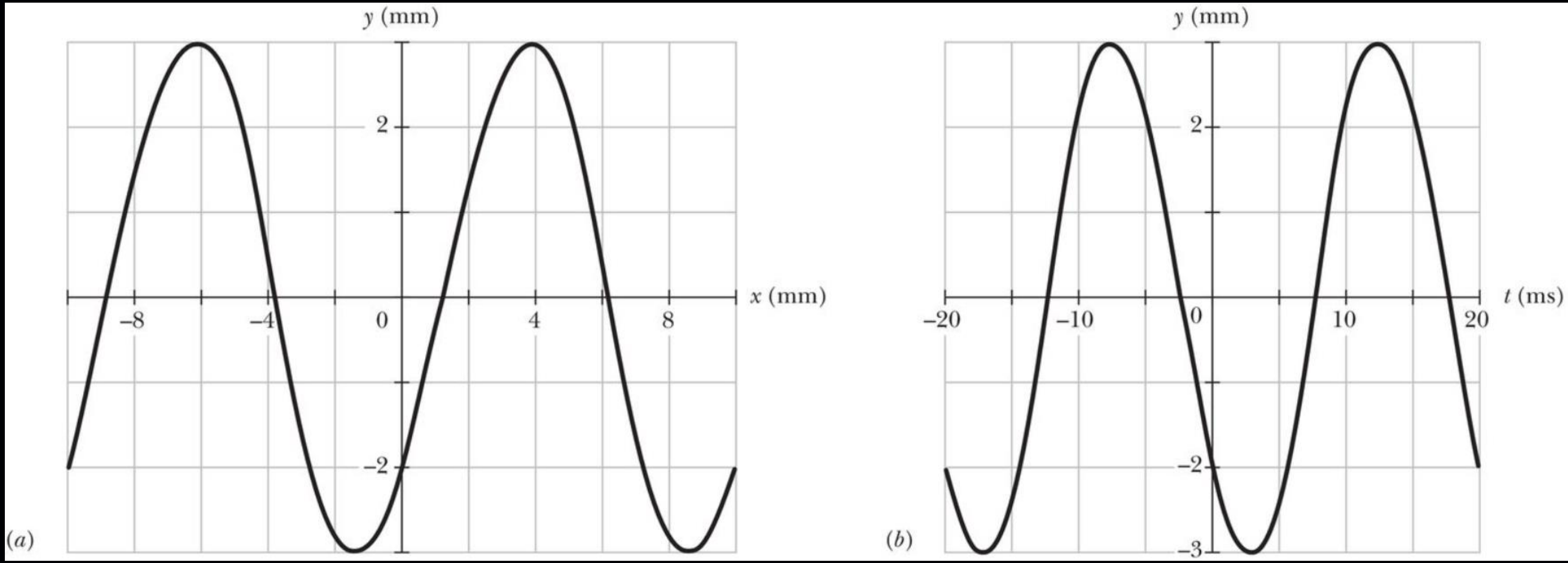
- Angular wave number k: We will find out wavelength λ first and use $k = \frac{2\pi}{\lambda}$ to find out angular wave number. From figure (a), we can find out $\lambda = 10mm$. Therefore $k = 200\pi\ rad/m$

Example



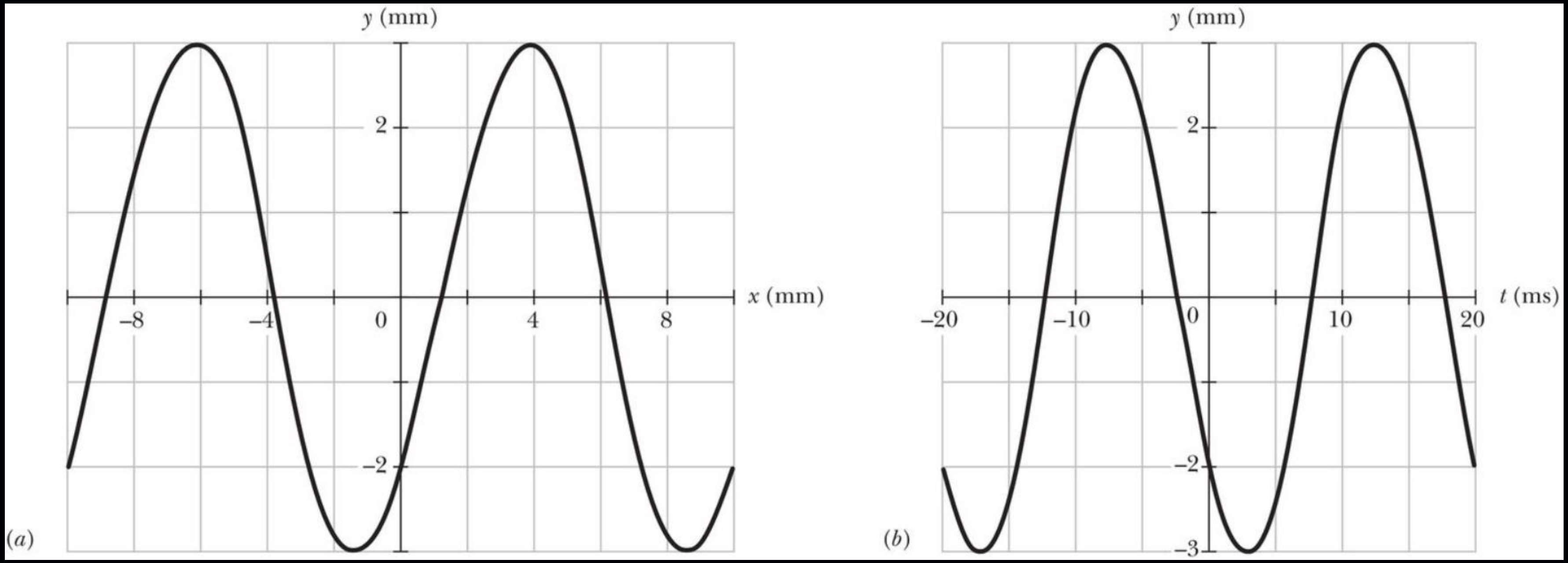
- Angular wave frequency ω: We will find out period T first and use $\omega = \frac{2\pi}{T}$ to find out angular wave frequency. From figure (b), we can find out $\mathrm{T} = 20ms$. Therefore $\omega = 100\pi\ rad/s$

Example



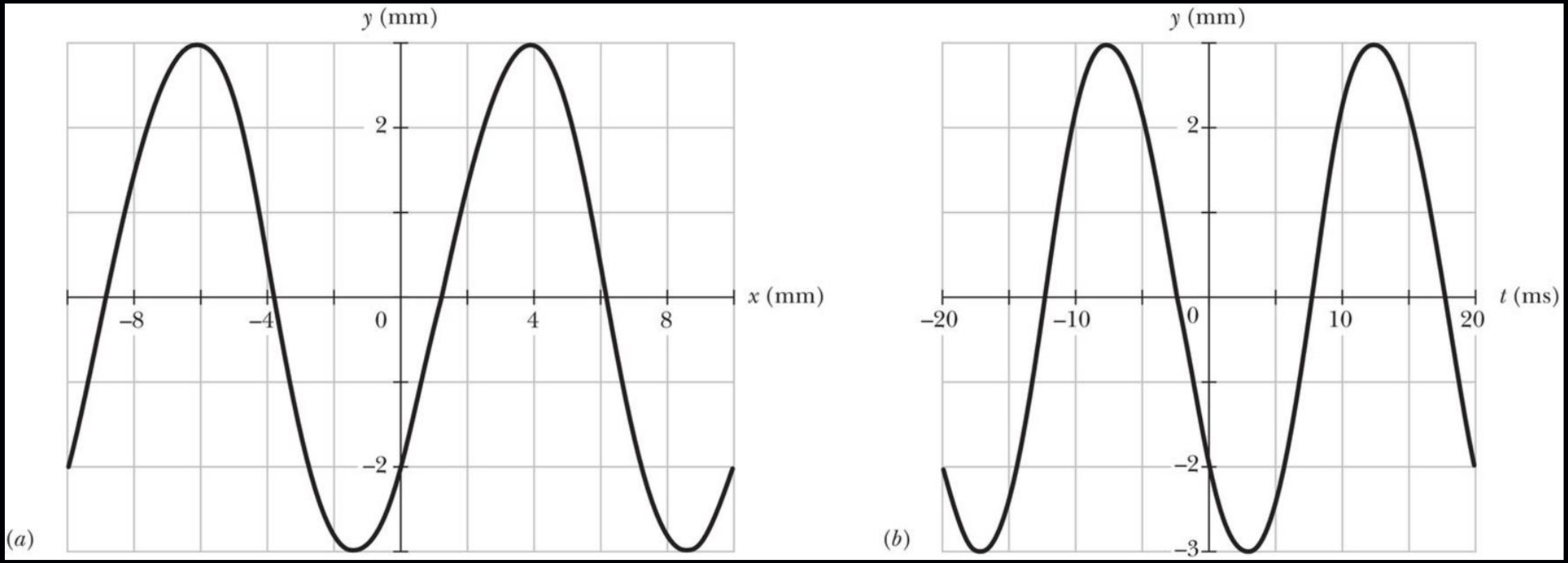
- Direction of travel: From (b), we know that the point x=0 is oscillating toward negative first when t is increasing from 0. This can only happen when the wave is moving to the right in figure (a). Thus, it should be $kx - \omega t$

Example



- Phase constant ϕ: From (a), we know that at $y(x = 0, t = 0) = 3\sin(\phi) = -2mm$. This gives $\phi = sin^{-1}\left(-\frac{2}{3}\right) = -0.73rad$

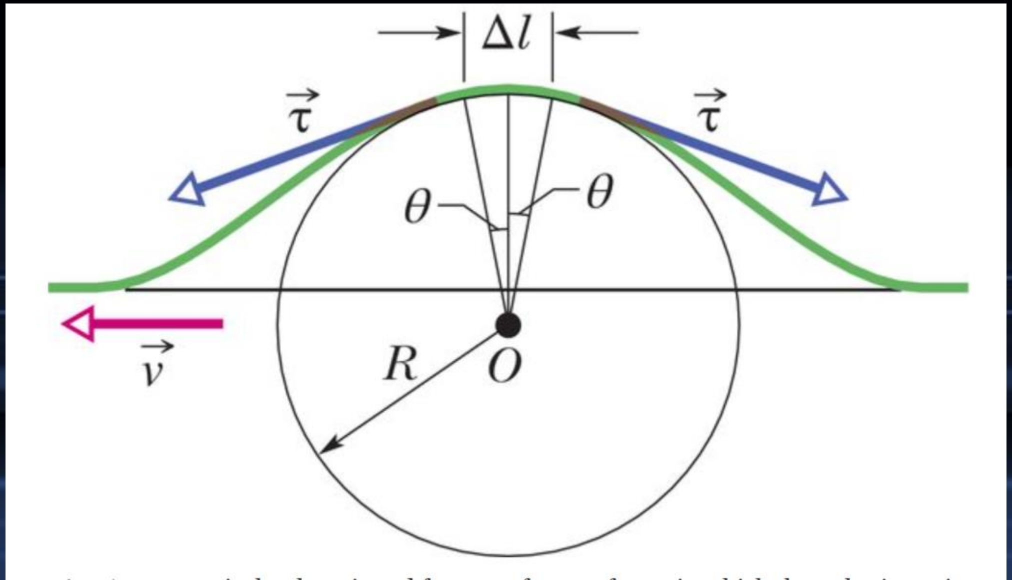
Example



- Therefore we have $y(x,t) = 3mm\,\sin(200\pi x - 100\pi t - 0.73)$

Wave speed on a stretched string

Consider a small string element of length Δl within the pulse, an element that forms an arc of a circle of radius R and subtending an angle 2θ at the center of that circle. A force $\vec{\tau}$ with a magnitude equal to the tension in the string pulls tangentially on this element at each end.



Wave speed on a stretched string

Considering it as a uniform circular motion, the force provide a centripetal acceleration:$a = \frac{v^2}{R}$ to the mass $\Delta m = \mu \Delta l$.

Thus, we have $\tau \frac{\Delta l}{R} = \mu \Delta l a = \mu \Delta l \frac{v^2}{R}$

There fore we can get $v = \sqrt{\frac{\tau}{\mu}}$, which is the wave speed of the stretched string.

Principle of superposition

- Principle of superposition: when several effects occur simultaneously, their net effect is the sum of the individual effects.
- Suppose that two waves travel simultaneously along the same stretched string. Let y1(x, t) and y2(x, t) be the displacements that the string would experience if each wave traveled alone. The displacement of the string when the waves overlap is then the algebraic sum:

$$y'(x,t) = y_1(x,t) + y_2(x,t)$$

Math description: interference of waves in the same directions

- The resultant wave: $y'(x,t) = y_1(x,t) + y_2(x,t) = y_m \sin(kx - \omega t) + y_m \sin(kx - \omega t + \phi)$

Displacement

$$y'(x,t) = \left[2y_m \cos \tfrac{1}{2}\phi\right] \sin\left(kx - \omega t + \tfrac{1}{2}\phi\right)$$

Magnitude gives amplitude

Oscillating term

Superposition of two waves in opposite directions

- Consider two waves: $y_1(x,t) = y_m \sin(kx - \omega t)$ and $y_2(x,t) = y_m \sin(kx + \omega t)$.
- By principle of superposition:

$$y'(x,t) = y_1(x,t) + y_2(x,t)$$

- The resultant wave:

$$y'(x,t) = [2y_m \sin(kx)] cos\omega t$$

Mathematical form of standing wave

Displacement

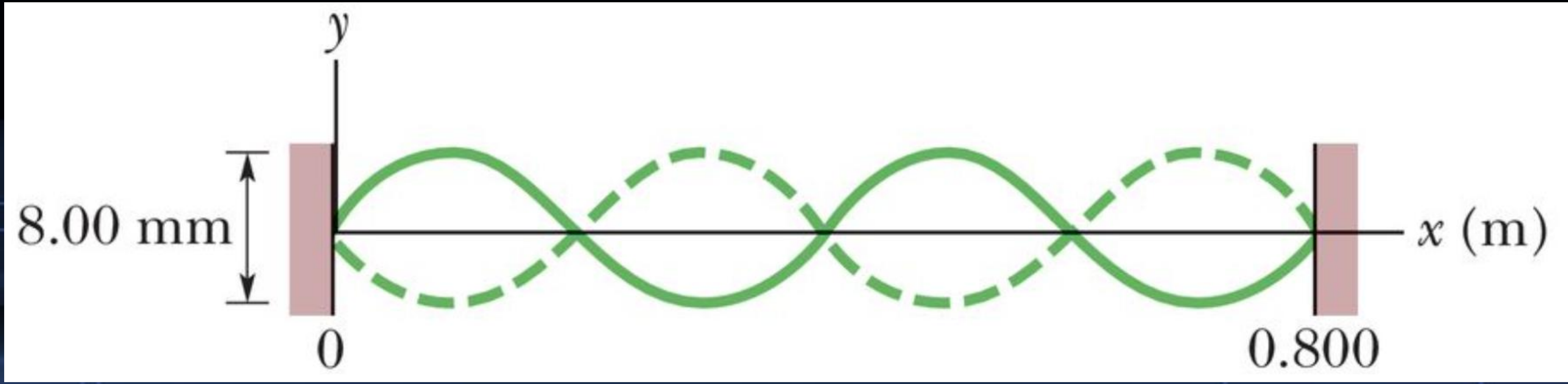
$$y'(x,t) = [2y_m \sin kx] \cos \omega t$$

Magnitude gives amplitude at position x

Oscillating term

Example

resonant oscillation of a string of mass m = 2.500 g and length L = 0.800 m and that is under tension τ = 325.0 N. (a)What is the wavelength λ of the transverse waves producing the standing wave pattern, and what is the harmonic number n?

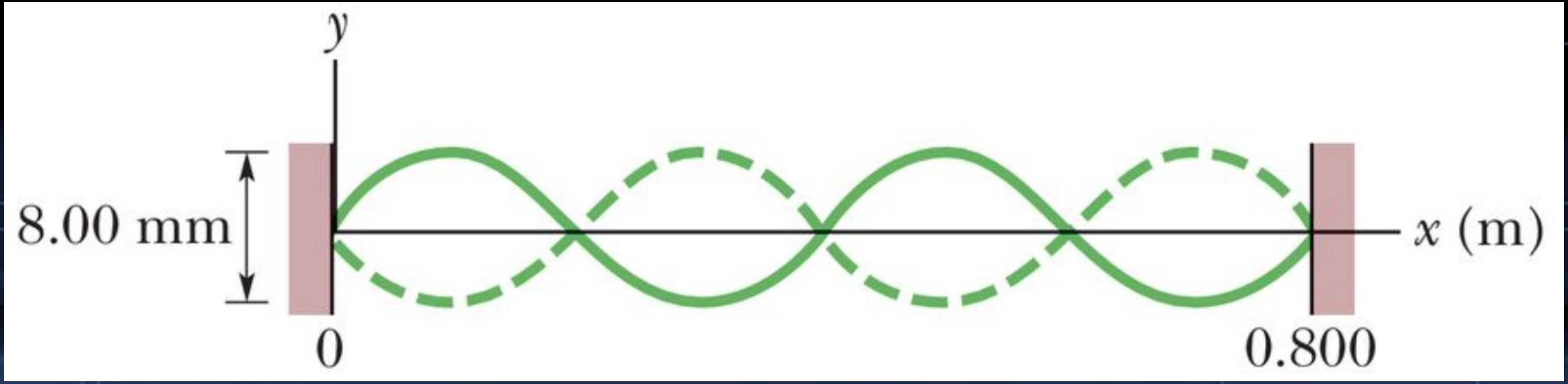


Example

$$y'(x,t) = [2y_m \sin(kx)]cos\omega t$$

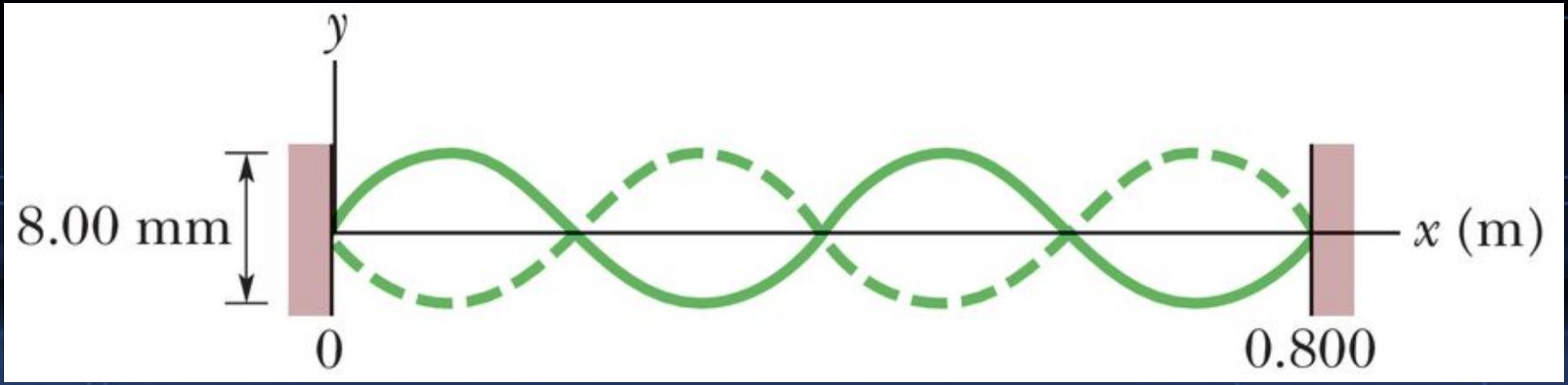
We can find in the plot that $L = 2\lambda$. Thus $\lambda = 0.4m$

By counting the or half-wavelengths, the correspond harmonic is n=4



Example

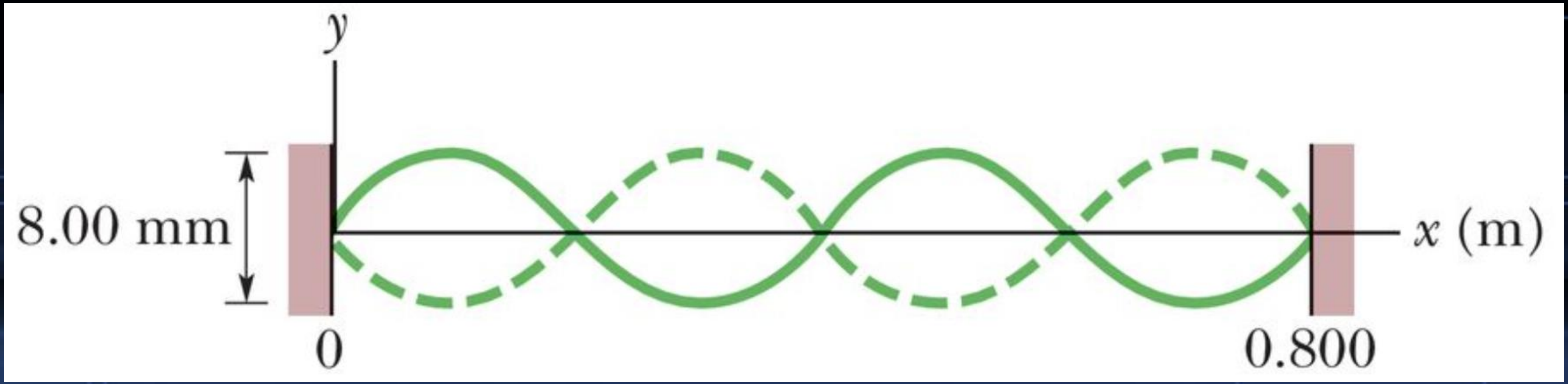
(b)What is the frequency f of the transverse waves and of the oscillations of the moving string elements?



Example

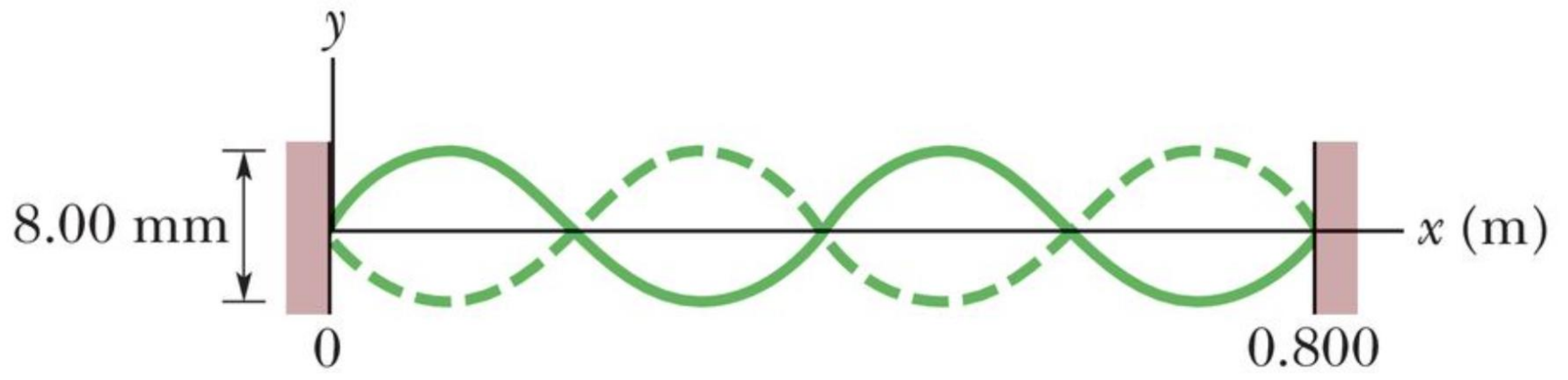
With the wave speed on string: $v = \sqrt{\frac{\tau}{\mu}} = \sqrt{\frac{\tau L}{m}} = 322.49m/s$

The resonance frequency $f = \frac{v}{\lambda} = 806.2Hz$



Example

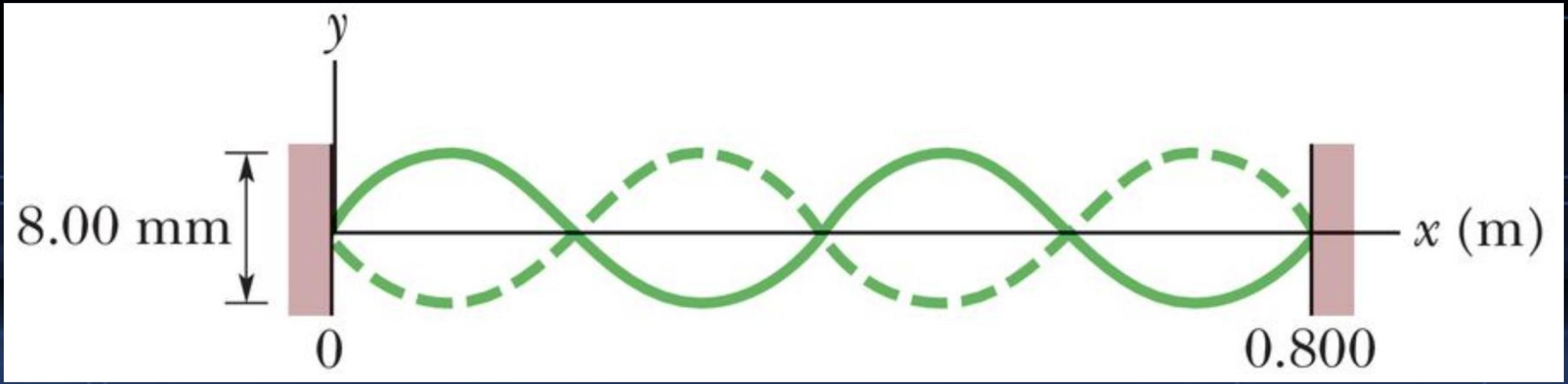
(c) What is the maximum magnitude of the transverse velocity um of the element oscillating at coordinate x = 0.180 m?



Example

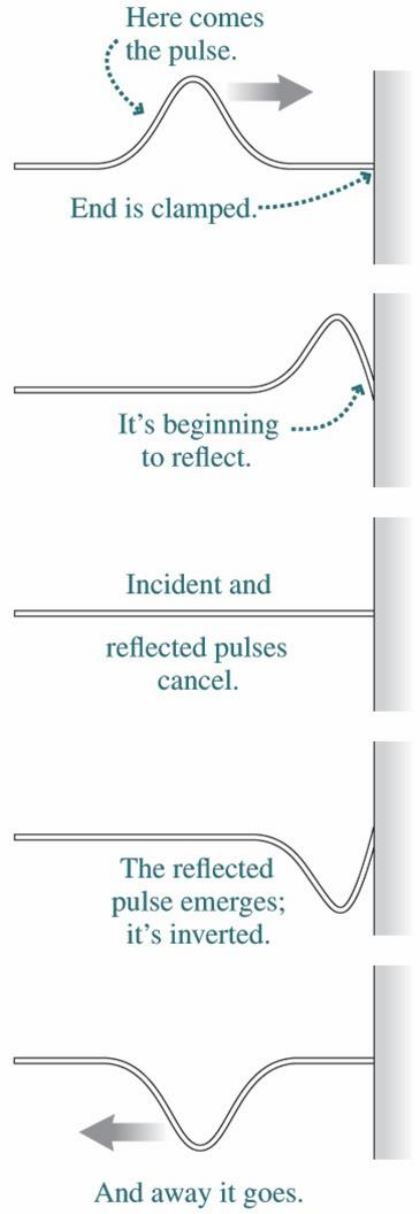
By taking derivative of displacement respect to time, we have: $u(t) = \frac{\partial}{\partial t}[2y_m sinkx]cos\omega t = [-2y_m \omega sinkx]sin\omega t$

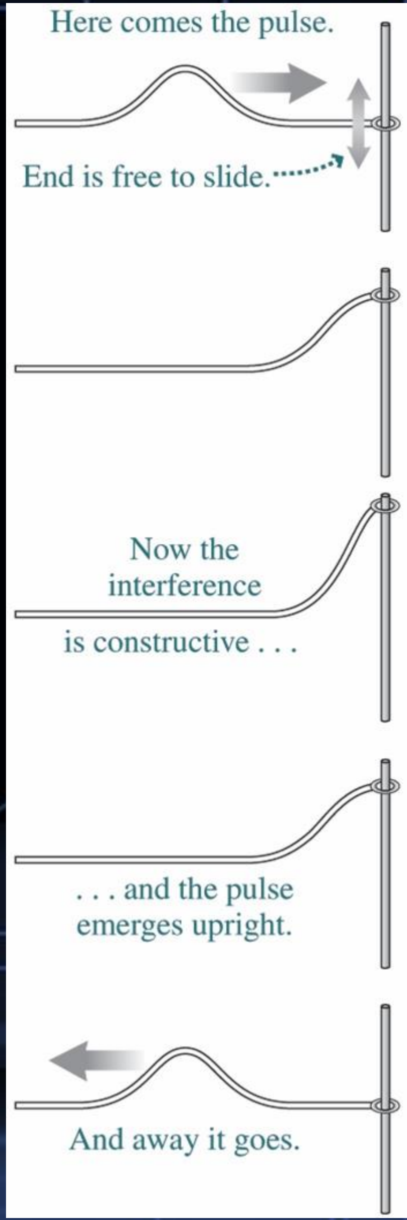
Therefore, at x=0.18m, the maximum $u = 6.26m/s$



Reflect on the boundary

- There are two ways of pulse can reflect from the end of the string:
- (a) hard end: the reflected pulse is inverted from the incident pulse.
- (b) soft end: the pulse is not inverted by the reflection.

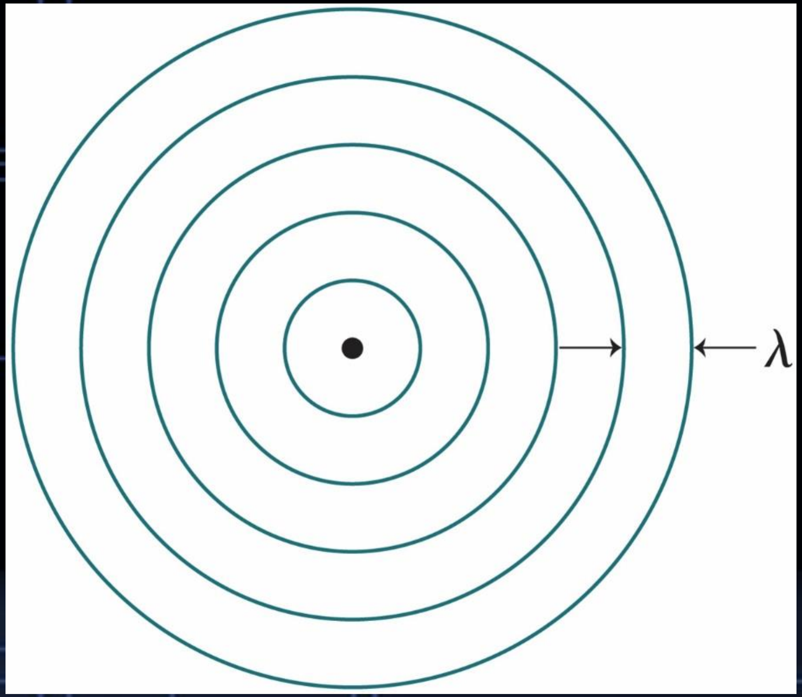


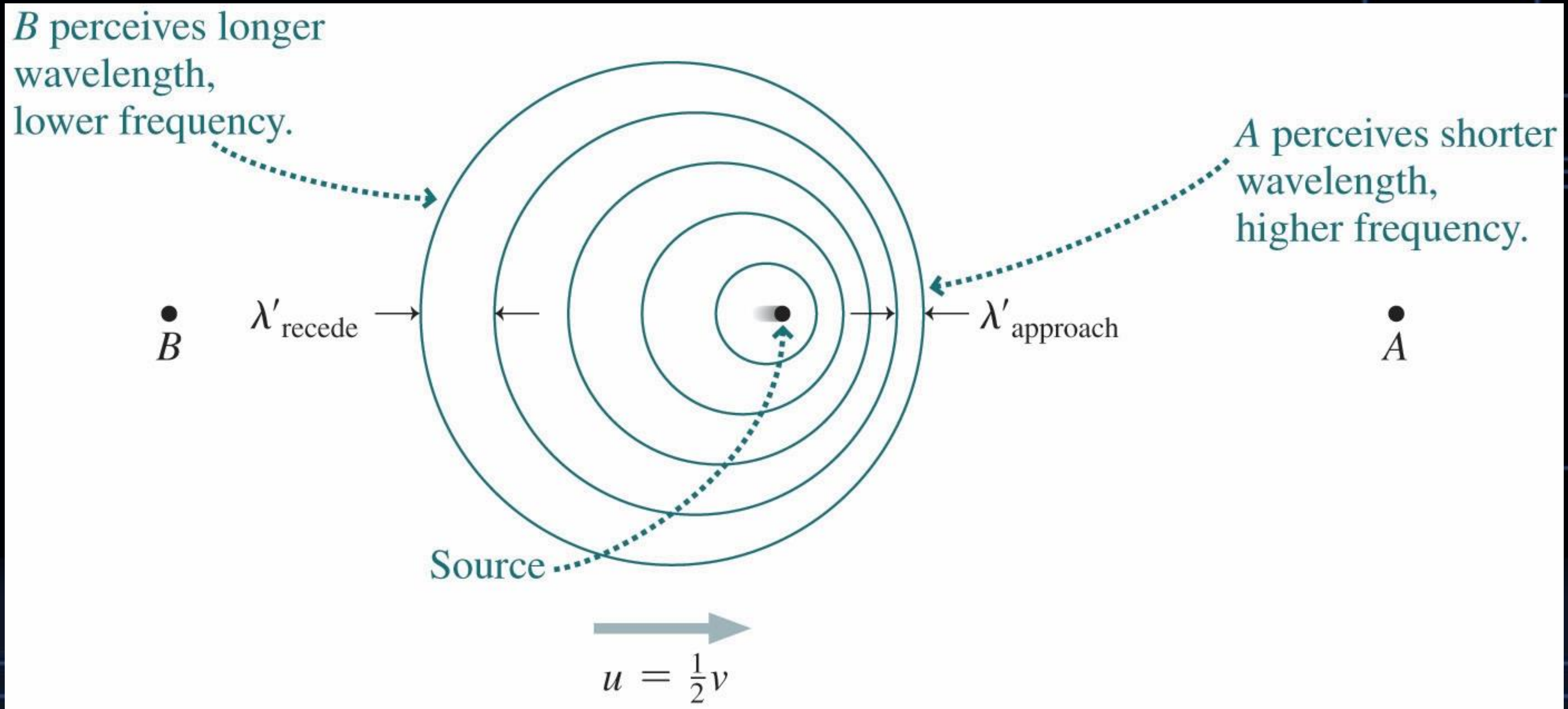


The Doppler Effect: Moving Source

- When a wave source moves through the wave medium, a stationary observer experiences a shift in wavelength and frequency:
  - If the wave moves with a speed *v* and the source moves with a speed *u*, the shifted frequency is: $f' = f/(1 \pm u/v)$
    - The frequency decreases for a receding source.
    - The frequency increases for an approaching source.

The Doppler Effect: Moving Source





The Doppler Effect: Moving Observers

- When the wave source is stationary, an observer moving with a speed u will experience a Doppler shift in frequency (but no shift in wavelength) that is given by:

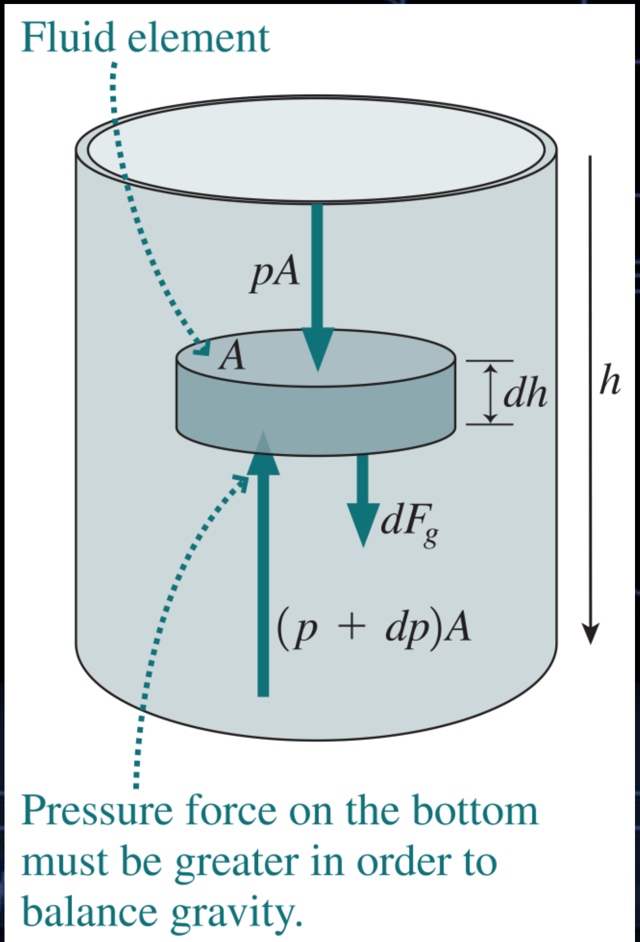
$$f' = f\left(1 \pm u/v\right)$$

- At low speeds (u is small compared to v), the formula for a source moving with a speed u gives nearly the same result as the formula for an observer moving with a speed u.

Hydrostatic equilibrium with gravity

- In the presence of gravity, the pressure in a static fluid must increase with depth:
  - This allows an upward pressure force to balance the downward gravitational force.
  - This condition is hydrostatic equilibrium.
  - Details depend on the nature of the fluid:
    - Incompressible fluids like liquids have constant density; for them, pressure as a function of depth h is as follows: $p = p_0 + \rho g h$

where p_0 is the pressure at the surface

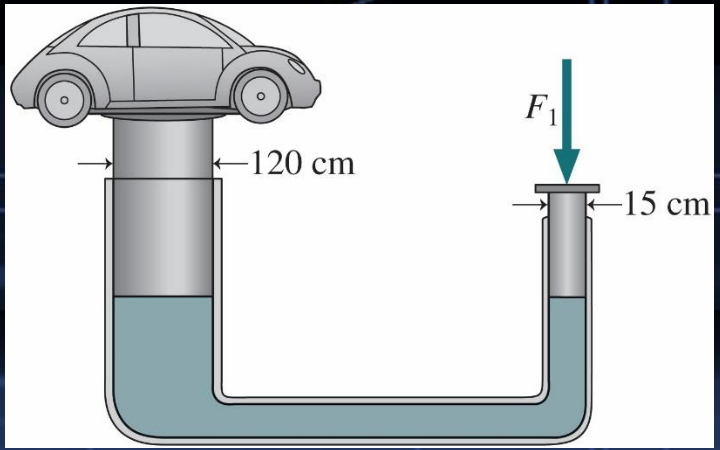


Pascal's Law

- A pressure increase anywhere is felt through out the fluid: Pascal's law.
- Pascal's law's application: hydraulic press.
- Example: lift a car with hydraulic press as shown

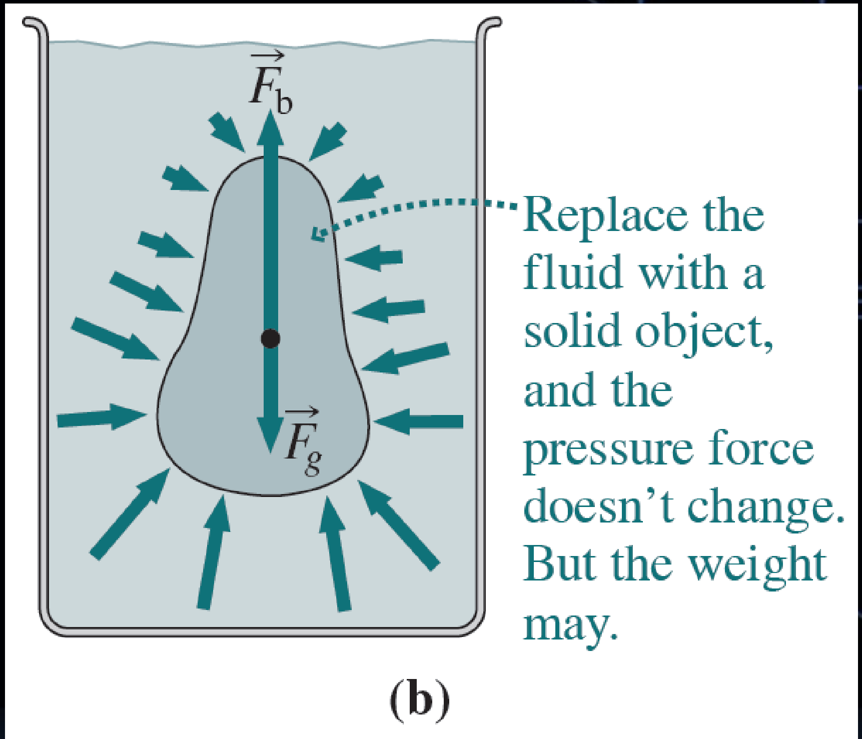
$$m_{car}g = p\pi(60)^2 = \frac{F_1}{\pi(7.5)^2}\pi(60)^2$$

- Thus: $F_1 = \frac{m_{car}g}{64}$



Archimedes' Principle and Buoyancy

- Replacing the fluid with an object of the same shape doesn't change the force due to the pressure differences:
  - Therefore, the object experiences an upward force equal to the weight of the original fluid.
  - This is the **buoyancy force**.
  - **Archimedes' principle** states that the buoyancy force is equal to the weight of the displaced fluid: $F_b = \rho_f g V_f$



Example

- The average density of a typical arctic iceberg is 0.86 that of sea water. What fraction of an iceberg's volume is submerged?

Example

- The average density of a typical arctic iceberg is 0.86 that of sea water. What fraction of an iceberg's volume is submerged?

The weight of iceberg is equal to buoyancy force:

Weight of iceberg: $m_{ice}g = \rho_{ice}V_{ice}g$

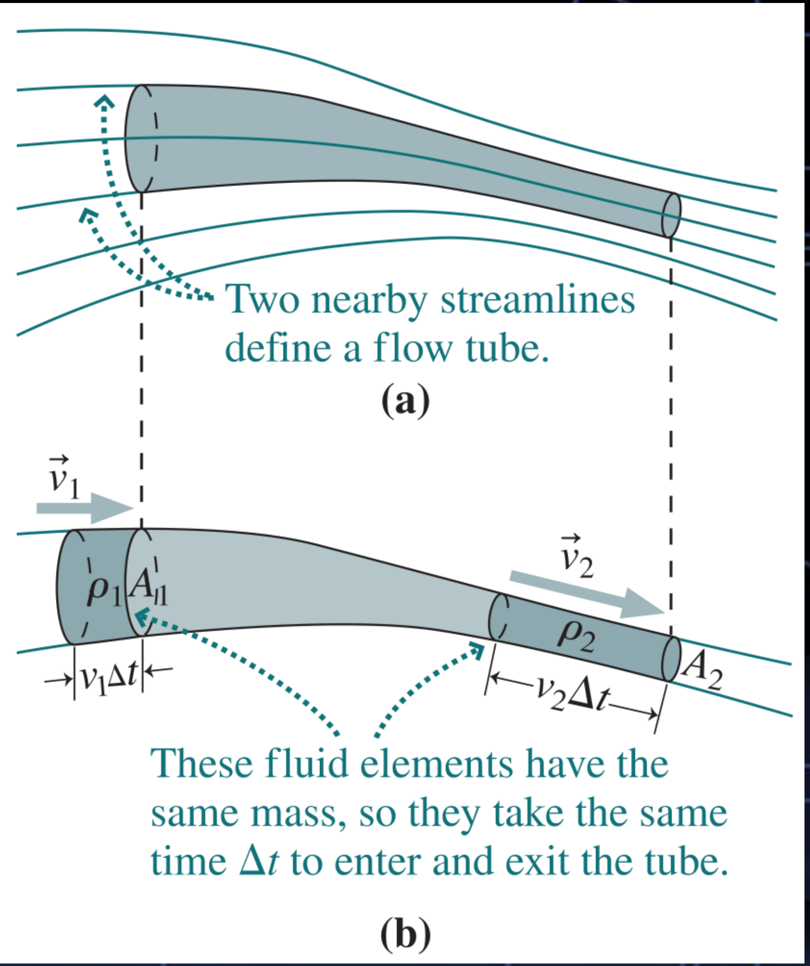
buoyancy force: $W_{water} = \rho_{water}gV_{sub}$

Thus:

$$\frac{V_{sub}}{V_{ice}} = \frac{\rho_{ice}}{\rho_{water}} = 0.86$$

Conservation of Mass: The Continuity Equation

- The **continuity equation** expresses conservation of mass in a moving fluid:
  - It follows from considering a **flow tube**, usually an imaginary tube bounded by nearby streamlines:
    - The flow tube may also be an actual physical tube or pipe.



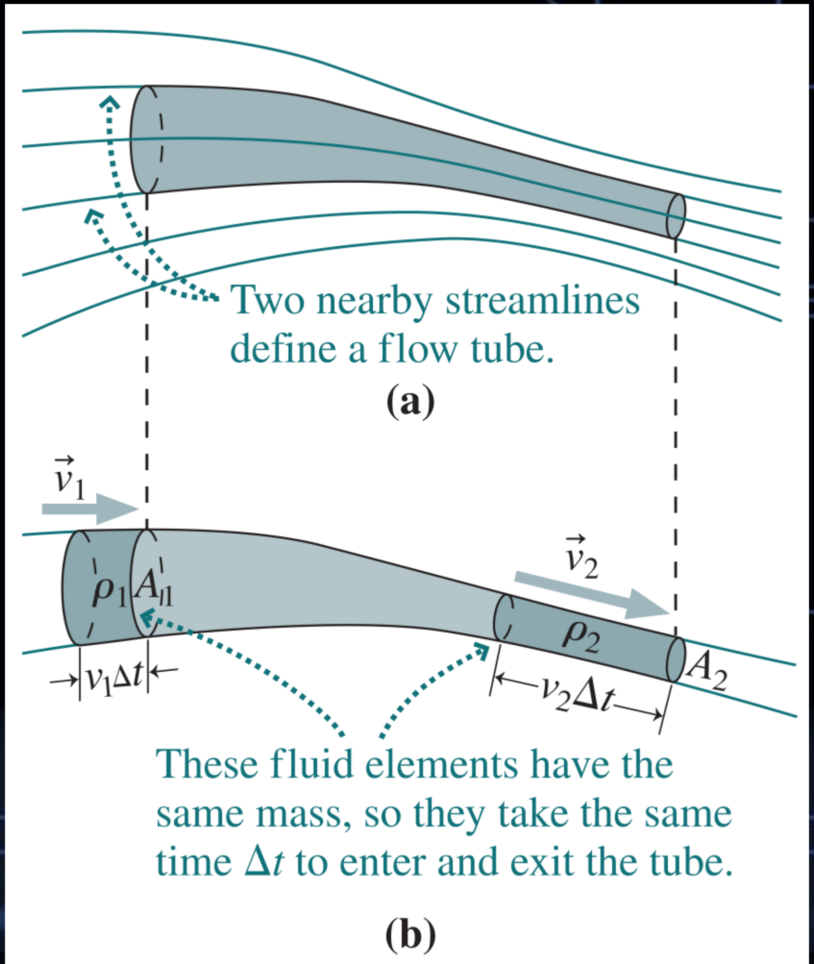
Conservation of Mass: The Continuity Equation

- The continuity equation reads:

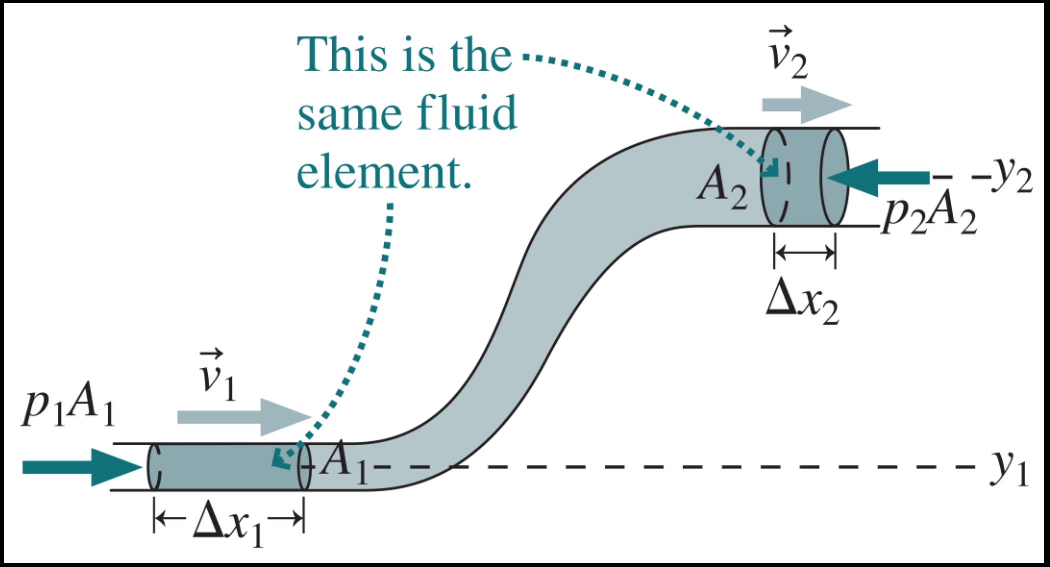
$$\rho v A = \text{constant}$$

where ρ is the density, v is the flow speed, and A is the cross-sectional area; the quantities are evaluated at points along the same flow tube.

  - The quantity ρvA is is the mass flow rate.
  - For incompressible fluids, density is constant and the continuity equation reduces to vA = constant:
    - Here vA is the volume flow rate.



Conservation of Energy: Bernoulli's Equation

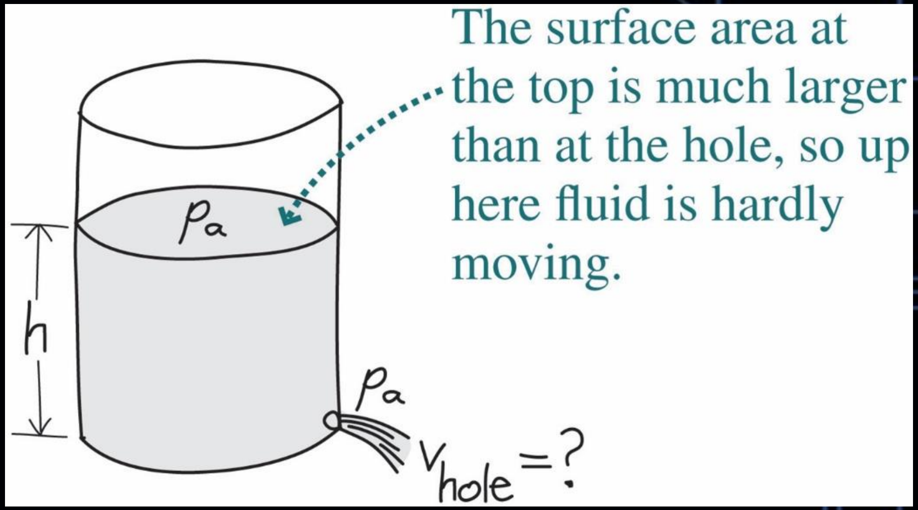


Neglecting fluid friction (viscosity) and in the absence of mechanical pumps and turbines that add or remove energy from the incompressible fluid, Bernoulli's equation reads

$$p + \frac{1}{2}\rho v^2 + \rho g h = \text{constant}$$

Example: Draining a Tank

- A large, open tank is filled to a height *h* with liquid of density *ρ*. Find the speed of the liquid emerging from a small hole at the base of the tank?



Example: Draining a Tank

- The fluid at the top of the tank and just outside the exit hole are both at atmospheric pressure, p_a.
- Because the tank is large, the fluid velocity at the top is nearly zero.
- Assuming $y = 0$ at the hole and $y = h$ at the top of the fluid, we can find the exit speed using Bernoulli's equation:

$$p_a + \rho g h = p_a + \frac{1}{2}\rho v_{hole}^2$$

- Solving for v_{hole}, we obtain: $v_{hole} = \sqrt{2gh}$