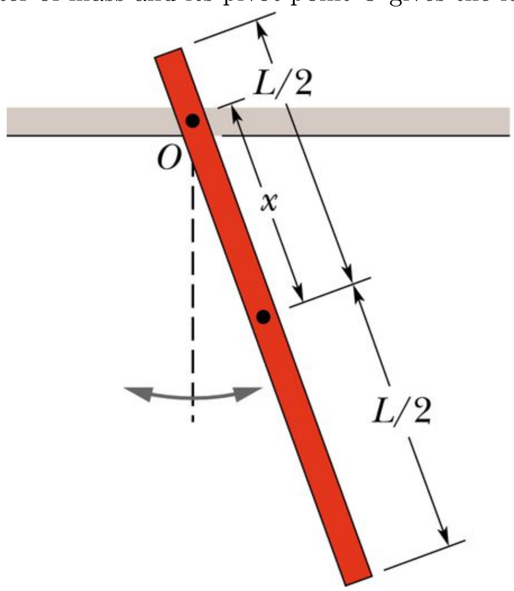
General Physics B1 - Homework Set 4

Due 12/02/2022, 5:00PM sharp. Please hand in your homework via eLearn.

1 points for each problem. Total:5 points

1.Physical Pendulum

A stick of length L = 1.85 m oscillates as a physical pendulum. (a) What value of distance x between the stick's center of mass and its pivot point O gives the least period? (0.5point) (b) What is that least period? (0.5point)



Solution:

(a) According to what we discussed in the course, the period of physical pendulum is $T = 2\pi\sqrt{\frac{I}{mgx}}$. By applying parallel axis theorem, the rotational inertia $I = \frac{1}{12}mL^2 + mx^2$. Therefore, we can get the period $T = 2\pi\sqrt{\frac{\frac{1}{12}mL^2+mx^2}{mgx}} = \frac{2\pi}{\sqrt{g}}\sqrt{\frac{\frac{1}{12}L^2+x^2}{x}}$. The minimum T happened when $\frac{2x^2-\frac{1}{12}L^2-x^2}{x^2} = 0$. This means when $x = \frac{1}{\sqrt{12}}L = 0.534m$ (Answer), this physical pendulum has the shortest period.

(b)Pluged back $x = 0.534m$, the shortest period $T_{min} = 2.07sec$(Answer)

2.Wave speed of a hanging rope

A uniform rope of mass m and length L hangs from a ceiling. (a) Show that the speed of a transverse wave on the rope is a function of y, the distance from the lower end, and is given by $v = \sqrt{gy}$. (0.5point) (b) Show that the time a transverse wave takes to travel the length of the rope is given by $t = 2\sqrt{L/g}$. (0.5point)

Solution:

(a)Since the rope is not massless, the tension of the rope is a function of y and the tension at any spot keep the rope at rest. At position y, the mass of the part of rope below y is $y\frac{m}{L}$ and thus the tension is $\tau = y\frac{m}{L}g$. Thus the wave speed of the rope is $v = \sqrt{\frac{\tau}{\mu}} = \sqrt{\frac{y\frac{m}{L}g}{(\frac{m}{L})}} = \sqrt{gy}$(Answer)

(b)Since the wave speed varies with position y, the time dt for the wave to travel through a small segment dy at position y is $dt = \frac{dy}{v} = \frac{dy}{\sqrt{gy}} = \frac{y^{-\frac{1}{2}}}{\sqrt{g}}$.

Thus the total time $t = \int dt = \int_0^L \frac{y^{-\frac{1}{2}}dy}{\sqrt{g}} = \frac{2}{\sqrt{g}}(y^{\frac{1}{2}})|_0^L = 2\sqrt{\frac{L}{g}}$(Answer)

3. Standing wave

A string oscillates according to the equation $y'(x,t) = (0.5cm)sin[(\frac{\pi}{3}cm^{-1})x]cos[(40\pi s^{-1})t]$. What are the (a) What is the distance between nodes? (0.5point)(b) What is the transverse speed of a particle of the string at the position x = 1.5 cm when $t = \frac{9}{8}$? (0.5point)

Solution:

(a)The distance between nodes is half of the wavelength $\frac{\lambda}{2} = \frac{2\pi}{2k} = 3cm$.

(b)The transverse speed can be written as $u = \frac{\partial y'}{\partial t} = -(0.5cm)(40\pi s^{-1})sin[(\frac{\pi}{3}cm^{-1})x]sin[(40\pi s^{-1})t]$.

Thus plug in $x = 1.5cm$ and $t = \frac{9}{8}$, we can get $u = 0$ (Answer)

4.Doppler effect

A woman is riding a bicycle at 18.0 m/s along a straight road that runs parallel to and right next to some railroad tracks. She hears the whistle of a train that is behind. The frequency emitted by the train is 840 Hz, but the frequency the woman hears is 778 Hz. Take the speed of sound to be 340 m/s. (a) What is the speed of the train, and is the train traveling away from or toward the bicycle? (0.5point) (b) What frequency is heard by a stationary observer located between the train and the bicycle? (0.5point)

Solution:

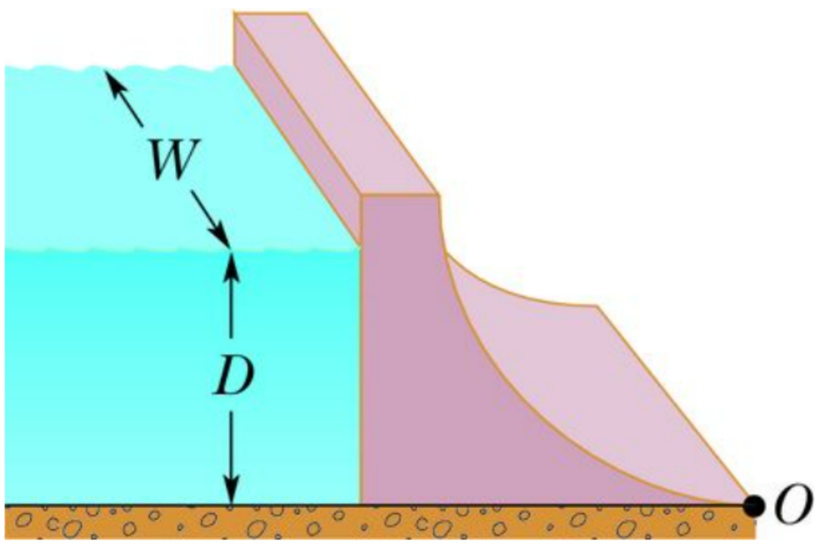
(a)The woman is an observer with a speed of 18m/s and assume the train is with a speed of u. Since the train's whistle comes from behind and the woman is moving forward, the woman(observer) is moving away from the source. If the train is not moving, the frequency change due to observer moving away is $f' = 840(1 - 18/340) = 795Hz$. However, the detected frequency 778Hz is lower than the 795Hz. Thus, the train is also moving away from the woman and we have: $778 = 840\frac{(1-18/340)}{(1+u/340)}$.

Therefore, $u = 7.66m/s$ and the train is moving away from the woman. (Answer)

(b) If the observer is not moving. Then we have $f' = 840/(1 + 7.66/340) = 821Hz$ (Answer)

5. Torque on dam due to water

In the following figure, water stands at depth D = 35.0 m behind the vertical upstream face of a dam of width W = 314 m. Find (a) the net horizontal force on the dam from the gauge pressure of the water (0.5point) and (b) the net torque due to that force about a horizontal line through O parallel to the (long) width of the dam. (0.5point) This torque tends to rotate the dam around that line, which would cause the dam to fail.



Solution:

(a) For a slice of dam with height dh at depth h, the force due to pressure of water is $pdA = \rho gh(Wdh)$. Thus, the net horizontal force on the dam is $\int_0^D \rho ghWdh = \rho gW[\frac{1}{2}h^2]_0^D = \frac{1}{2}\rho gWD^2 = \frac{1}{2}1000(kg/m^3)\cdot 9.8(m/s^2)\cdot 314(m)\cdot (35m)^2 = 1.88 \times 10^9 N$

(b) Following the same analysis of part (a), the torque contributed by the water pressure acting a slice of dam with height dh at depth h is $(D-h)pdA = (D-h)\rho ghWdh$. Here $(D-h)$ is the moment arm of the torque to O. Thus the total torque is $\int_0^D (D-h)\rho ghWdh = \rho gW\int_0^D (D-h)hdh = \rho gW[\frac{1}{2}Dh^2 - \frac{1}{3}h^3]_0^D = \frac{1}{6}\rho gWD^3 = 2.20\times 10^{10} N\cdot m$