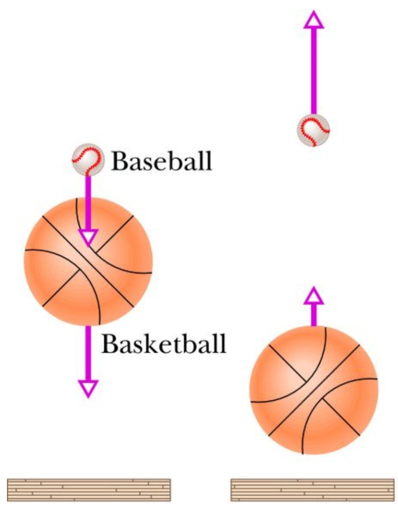
General Physics B1 - Midterm Exam 2 (12/06, 8:00AM~9:50AM)

There are 6 problme sets on two sides. Total:105 points

You may answer in English or Chinese. Please use SI units and take significant figure to the second decimal place for the answers. The graviational acceleration g=$9.8m/s^2$**.**

1.Elastic Collision

A small ball of mass m is aligned above a larger ball of mass M = 0.9 kg (with a slight separation, as with the baseball and basketball of Fig.(a), and the two are dropped simultaneously from a height of h = 1.5 m. (Assume the radius of each ball is negligible relative to h.) (1) If the larger ball rebounds elastically from the floor and then the small ball rebounds elastically from the larger ball, what value of m results in the larger ball stopping when it collides with the small ball?(10points) (2) What height does the small ball then reach ing Fig.(b)?(10points)



(a) Before (b) After

Solution:

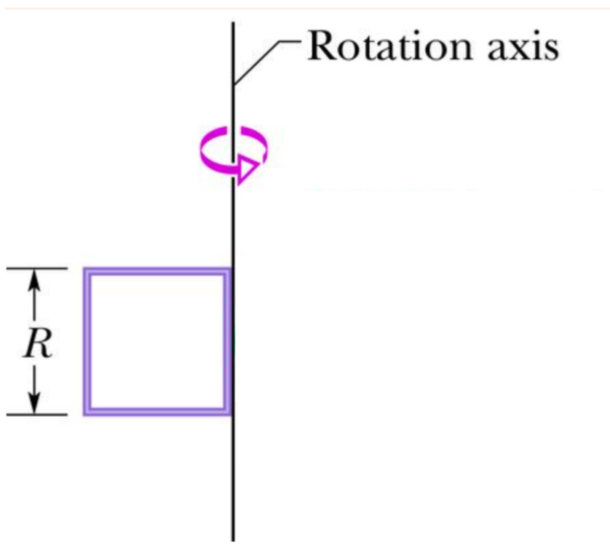
(1) Right before the basketball hit the floor, both basket ball and baseball have a velocity with magnitude $v_0 = \sqrt{2gh} = -5.42m/s$ and the negative sign means the direction of momention is pointing down. After the basketball hit the floor, the basketball will have a velocity with the same magnitude but the direction is upward because it rebounds elastically. Next, the collision process between basketball and baseball is elastic and the final velocity of basketball is zero. Therefore, we will have conservation of energy and conservation of momentum and the expression for final velocity of ellastic collision is valid.

Therefore final velocity of basketball $v_{basketball-f} = \frac{M-m}{M+m}v_0 - \frac{2m}{M+m}v_0 = \frac{M-3m}{M+m}v_0$. Thus, if the final velocity of basketball is zero, the requirement is $m = \frac{M}{3} = 0.30kg$. (Answer)

(2)The final velocity of baseball $v_{baseball-f} = \frac{2M}{M+m}v_0 + \frac{M-m}{M+m}v_0 = \frac{3M-m}{M+m}v_0 = \frac{3M-\frac{M}{3}}{M+\frac{M}{3}}v_0 = 2v_0 = 10.84m/s$. Thus, the final height is $h = \frac{v^2_{baseball-f}}{2g} = 6.00m$. (Answer)

2. Rotational Energy and Angular Momentum

As shown in the following figure, a rigid structure consisting of a square made of four thin bars, each of length R=0.5m and mass m = 2.0 kg. The rigid structure rotates at a constant speed about a vertical axis, with a period of rotation of 2.5 s. calculate (1) the structure's rotational energy (10points) and (2) its angular momentum about that axis.(10points)



(1) For the square, the rotational inertia $I_{square} = mR^2 + 2 \times [\frac{1}{12}mR^2 + m(\frac{R}{2})^2] + 0 = \frac{5}{3}mR^2 = 0.83(kg \cdot m^2)$. The angular frequency $\omega = \frac{2\pi}{T} = 2.51(rad/s)$.

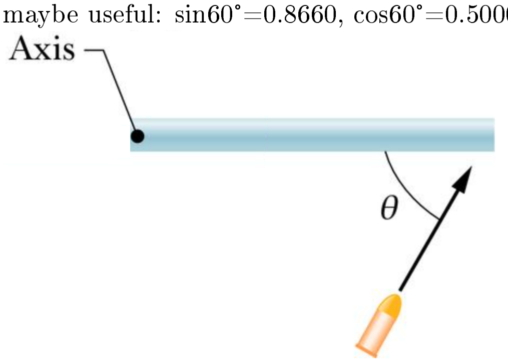
The rotational energy $E = \frac{1}{2}I\omega^2 = 2.61(J)$(Answer)

(2)The angular momentum is $L = I\omega = 2.08(kg \cdot m^2/s)$ with direction pointing up.

3. Rotational Inertia and Conservation of Angular Momentum

In the following figure (an overhead view), a uniform thin rod of length 0.500 m and mass 4.00 kg can rotate in a horizontal plane about a vertical axis through its END. The rod is at rest when a 3.00 g bullet traveling in the rotation plane is fired into the other end of the rod. In the view from above, the bullet's path makes angle $\theta = 60.0°$ with the rod. The bullet lodges in the rod and the angular velocity of the rod is 10 rad/s immediately after the collision. (1)What is the rotational inertia of the rod with the bullet after the collision? (10 points) (2)What is the bullet's speed just before impact? (10points)

maybe useful: sin60°=0.8660, cos60°=0.5000



Solution:

(1)The rotational inertia is given by:$I = \frac{M}{L}\int_0^L x^2 dx = \frac{M}{L}[\frac{1}{3}x^3]_0^L = \frac{1}{3}ML^2 = 0.33(kg \cdot m^2)$ (Answer)

(2)During the process, there is no external torque exerting on the system. Therefore, the angular momentum is conserved:

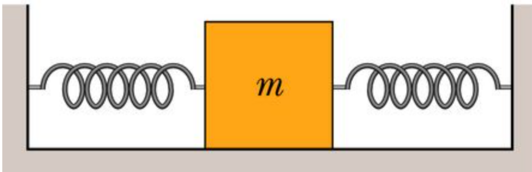
$L_{initial} = rpsin\theta = Lmvsin\theta$

$L_{final} = I\omega = [\frac{1}{3}ML^2 + m(L)^2]\omega$

Thus the velocity of bullet: $v = \frac{[\frac{1}{3}M+m]L\omega}{msin\theta} = 2571.77m/s$ (Answer)

4.Oscillation with two springs

In the following figure, two springs are attached to a block that can oscillate over a frictionless floor. If the left spring is removed, the block oscillates at a frequency of 40 Hz. (1) If the mass is $m = 0.10kg$, what is the srping constant of the right spring. (5points) (2) If, instead, the spring on the right is removed, the block oscillates at a frequency of 50 Hz. At what frequency does the block oscillate with both springs attached? (10points)



Solution:

(1)The oscillation frequency of a mass m attached to one spring with spring constant k is $f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$. Therefore, the spring constant$k = m(2\pi f)^2 = 6316.55(N/m)$. (Answer)

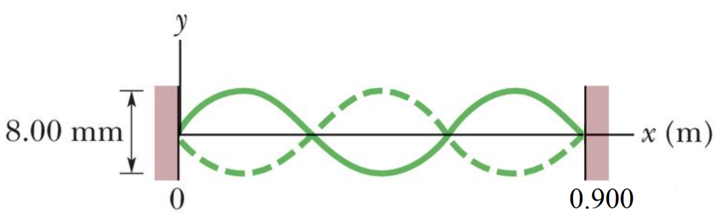
(2)For the setup in the figure, the two springs exert forces that have the same direction and sum up at all time. Therefore, we can conclude that $f_{tot} = \frac{1}{2\pi}\sqrt{\frac{k_1+k_2}{m}}$.

Thus, $f_{tot}^2 = (\frac{1}{2\pi}\sqrt{\frac{k_1+k_2}{m}})^2 = (\frac{1}{2\pi})^2\frac{k_1+k_2}{m} = f_1^2 + f_2^2$.

And $f_{tot} = \sqrt{f_1^2 + f_2^2} = 64.03Hz$.

5. Standing Wave

In the following figure shows resonant oscillation of a string of mass $m = 2.500g$ and length $L = 0.900m$ and that is under tension $\tau = 325.0N$. (1)What is the wavelength λ of the transverse waves producing the standing wave pattern? (5points) and (2)what is the harmonic number n? (5points) (3)What is the frequency f of the transverse waves? (5points)(4)What is the maximum magnitude of the transverse velocity u_m of the element oscillating at coordinate x = 0.20 m? (5points)



Solution:

(1)From the figure, we know that $L = \frac{3}{2}\lambda$. Therefore, $\lambda = \frac{2L}{3} = 0.60m$ (Answer)

(2)Since there are three half-wavelength, the harmonic number $n = 3$(Answer)

(3)The frequency $f = \frac{v}{\lambda} = \frac{1}{\lambda}\sqrt{\frac{\tau}{m/L}} = 570.09Hz$ (Answer)

(4)Note that we have $y_m = 2mm$, $\omega = 2\pi f = 3581.97rad/s$,

The maximum transverse velocity $u_m = -2y_m\omega sinkx = -2y_m\omega sin\frac{2\pi}{\lambda}x = -2 \cdot 0.002 \cdot 3582 \cdot sin(\frac{2\pi}{0.6} \cdot 0.2) = 12.41m/s$ (Answer)

6.Draining a Tank

A large, open tank is filled to a height h=0.5m with liquid of density $\rho = 1000kg/m^3$. Find the speed of the liquid emerging from a small hole at the base of the tank? (10points)

Solution:

The fluid at the top of the tank and just outside the exit hole are both at atmospheric pressure, p_a. The tank is large and we can assume the fluid velocity at the top is nearly zero. Assuming y = 0 at the hole and y = h at the top of the fluid, we can find the exit speed using Bernoulli's equation: $p_a + \rho gh = p_a + \frac{1}{2}\rho v^2$. Thus, $v = \sqrt{2gh} = 3.13(m/s)$ (Answer)