Course announcement

- The homework set 2 has been posted on eLearn. Please submit your homework via eLearn by 5PM, 10/21. No late homework will be accepted.
- A review section will be on 10/21
- The first midterm will on 10/25 (Tuesday).

4	10/7(Fri.)	**Energy:** kinetic energy and work
5	10/11(Tue.)	**Energy:** potential energy and conservation of energy
5	10/14(Fri.)	**Gravity:** Law of gravity (Homework2)
6	10/18(Tue.)	**Gravity:** Gravitational energy and gravitational field
6	10/21(Fri.)	**Review I**
7	10/25(Tue.)	Mid Term 1

GENERAL PHYSICS B1
GRAVITY

Gravitational Energy and Gravitational Field
2022/10/18

Today’s topic

- Gravitational Energy
- Gravitational Field
- Gravity Beyond Newton’s law

Universal Gravitation

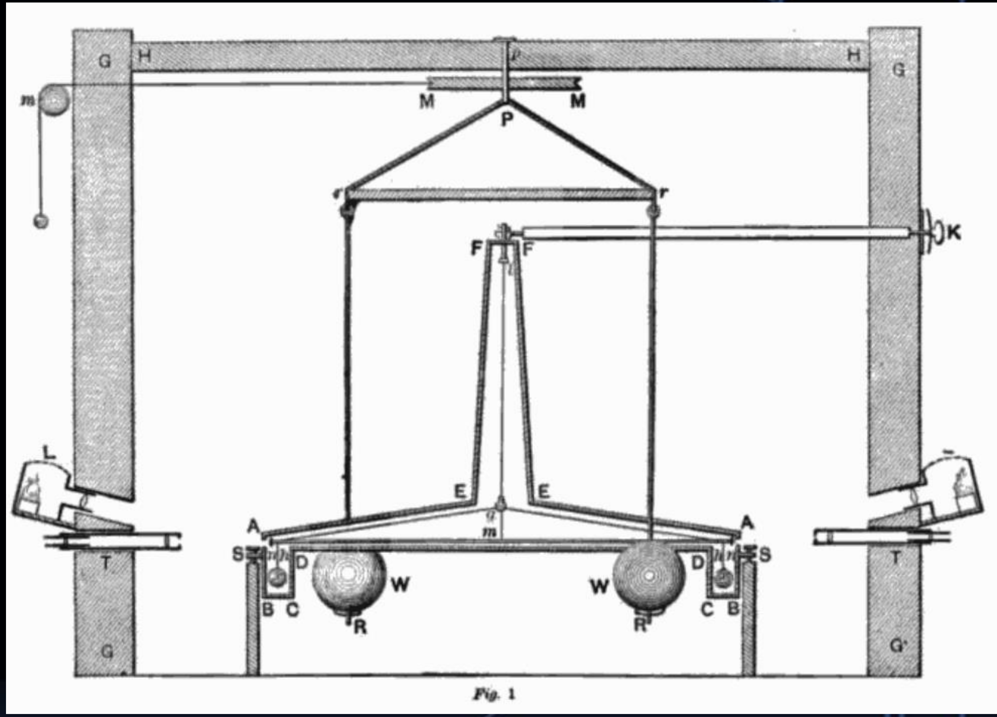
- Newton's **law of universal gravitation** states that any two point particles attract each other with a force that is proportional to the product of their masses and inversely proportional to the square of their separation:

$$F = \frac{Gm_1m_2}{r^2}$$

- $G = 6.67 \times 10^{-11} N \cdot m^2/kg^2$ is the **constant of universal gravitation**.

The Cavendish Experiment: Weighing the Earth

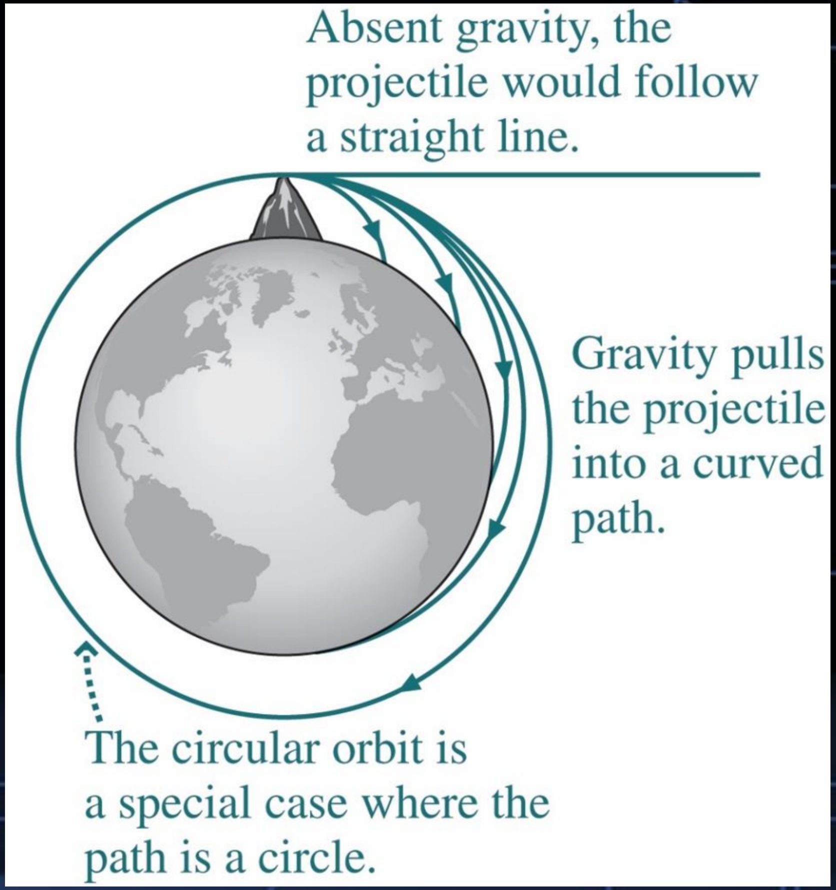
- In 1798, Henry Cavendish made the first experimental measurement of the gravitational constant *G*:
  - In an ingenious experiment, Cavendish measured the small gravitational force between two fixed masses and two masses that were suspended by a thin fiber.
  - His result allowed him to find the Earth's mass!(g=GM/r)



https://en.wikipedia.org/wiki/Cavendish_experiment#/media/File:Cavendish_Experiment.png

Orbital Motion around the Earth

- Orbital motion occurs when gravity is the dominant force.
- Newton explained orbits using universal gravitation and his laws of motion:
  - The force of gravity causes a projectile to deviate from its straight-line path.
  - At a critical speed, the curvature of the projectile's path follows the Earth's curvature and it enters a circular orbit.



Circular Orbits

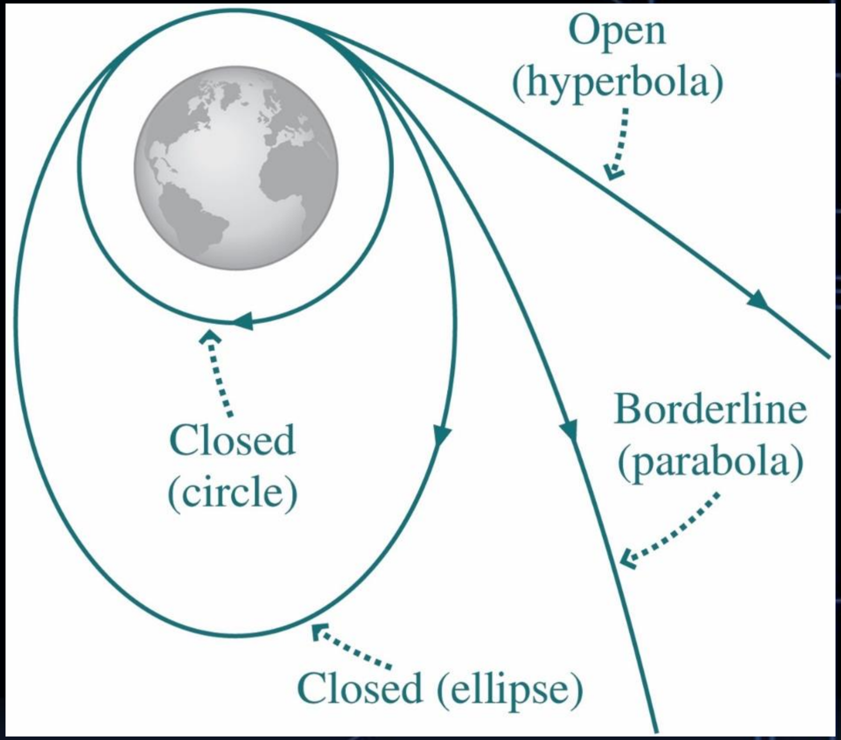
- In a circular orbit, gravity provides the centripetal force needed to keep an object of mass *m* in its circular path about a much more massive object of mass *M*. Therefore:

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

- The orbital speed: $v = \sqrt{\frac{GM}{r}}$
- Orbital period: $T^2 = (\frac{2\pi r}{v})^2 = \frac{4\pi^2 r^3}{GM}$
- This proves Kepler's third law: $T^2 \propto r^3$
- For satellites in low-Earth orbit, the period is about 90 minutes.

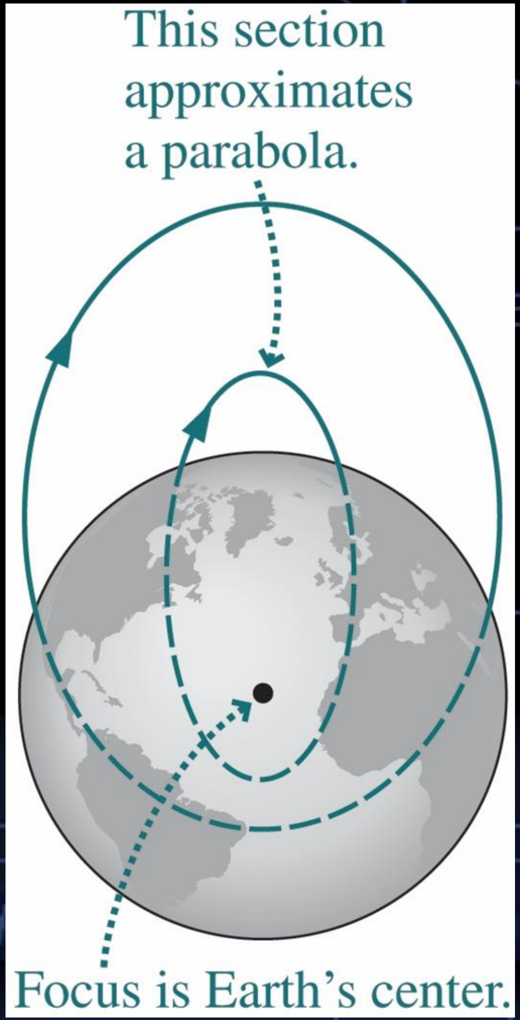
Elliptical Orbits and Open Orbits

- A circular orbit is not the only possibility:
  - Closed (bound) orbits are elliptical.
  - In the special case of a circular orbit, the acceleration of the orbiting object has a constant magnitude and always points toward the center of the orbit.
  - Unbound orbits are hyperbolic or (borderline case) parabolic.



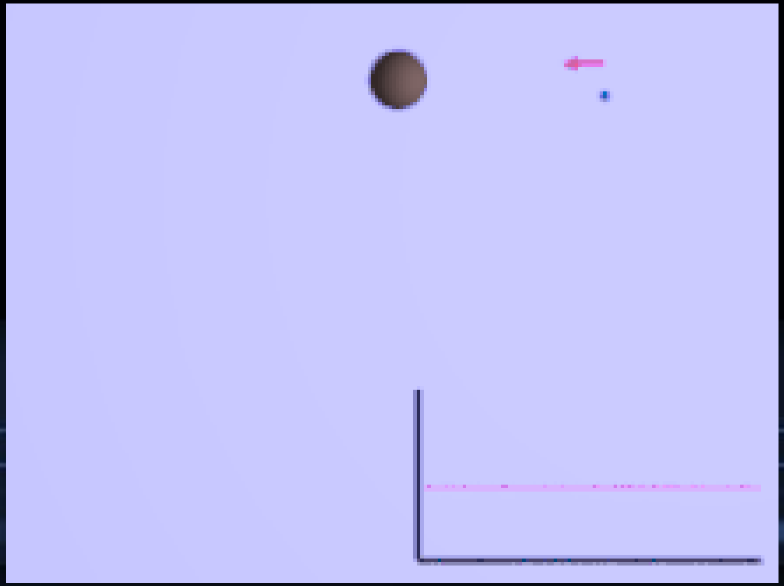
Projectile Motion and Orbits

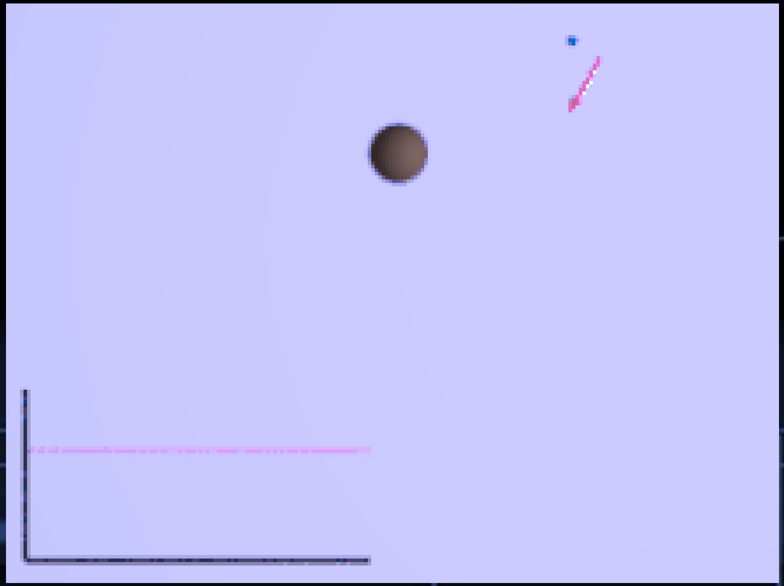
- The "parabolic" trajectories of projectiles near Earth's surface are actually sections of elliptical orbits that intersect Earth.
- The trajectories are parabolic only in the approximation that we can neglect Earth's curvature and the variation in gravity with distance from Earth's center.



Gravity Assist for spacecraft

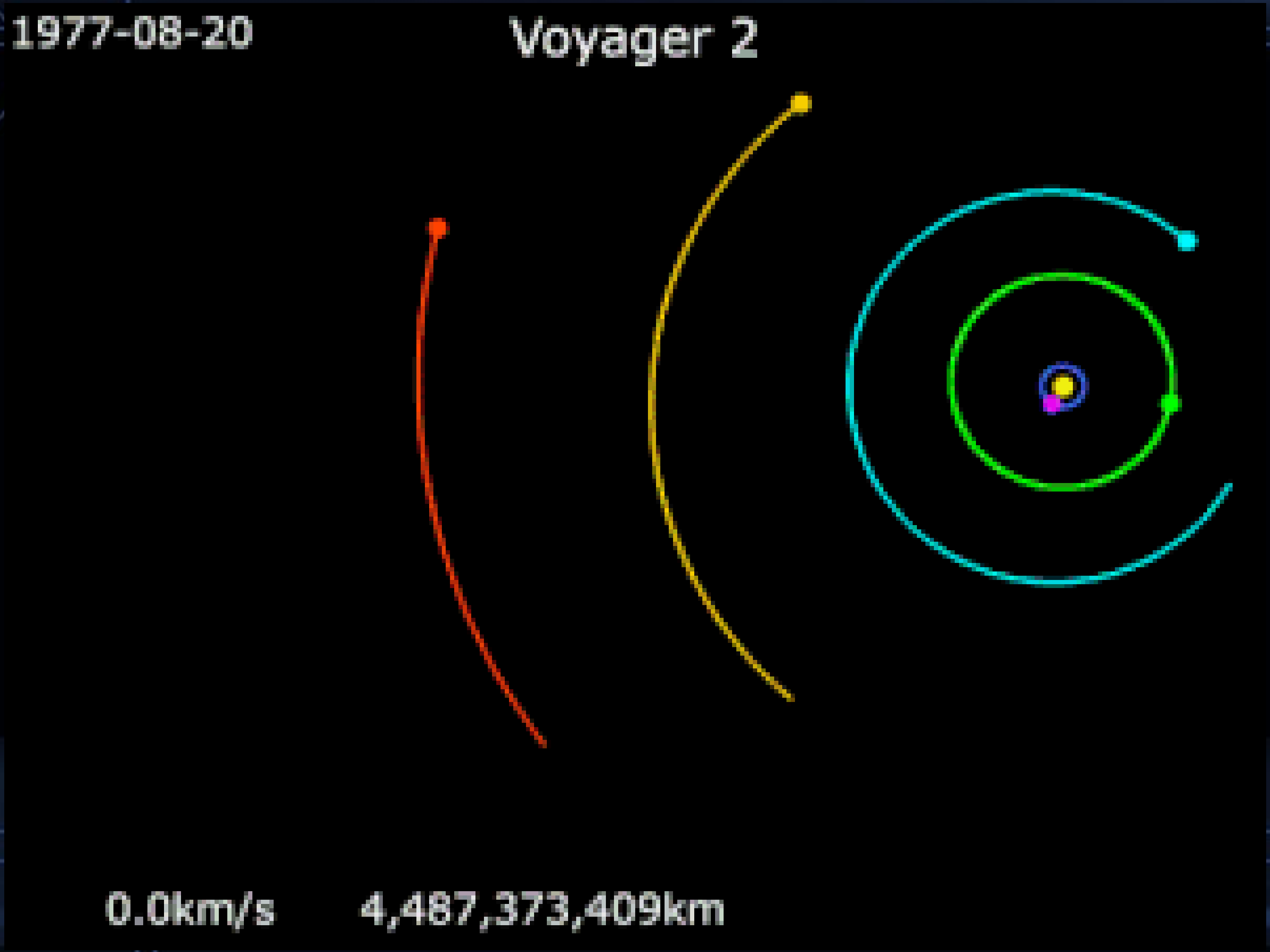
- A **gravitational slingshot**, **gravity assist maneuver**, or **swing-by** is the use of the relative movement and gravity of a planet or other astronomical object to alter the path and speed of a spacecraft.





https://upload.wikimedia.org/wikipedia/commons/8/8b/Swingby_dec_anim.gif

Multiple gravity assist orbits of Voyager 2



Animation of *Voyager 2*'s trajectory from 20 August 1977 to 30 December 2000

Voyager 2 · Earth · Jupiter · Saturn · Uranus · Neptune · Sun

https://upload.wikimedia.org/wikipedia/commons/thumb/1/1d/Animation_of_Voyager_2_trajectory.gif/450px-Animation_of_Voyager_2_trajectory.gif

Gravitational Energy

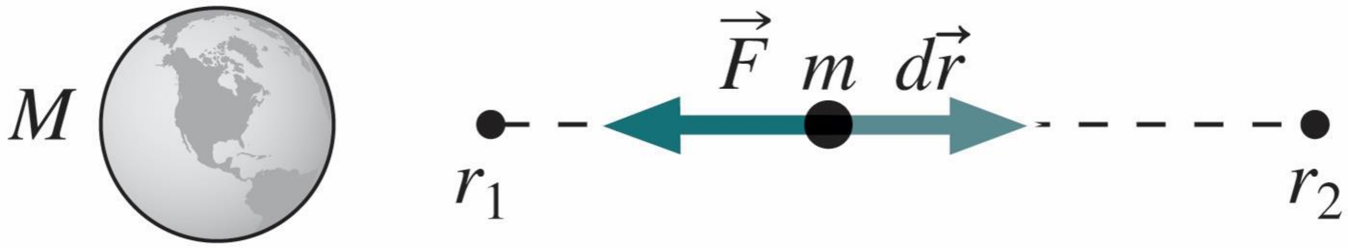
- By observing Newton's law of gravity:

$$F = \frac{Gm_1m_2}{r^2}$$

Gravitational force only depends on position => conservative force!

- The work done by a conservative force become change of potential energy.

Gravitational Energy



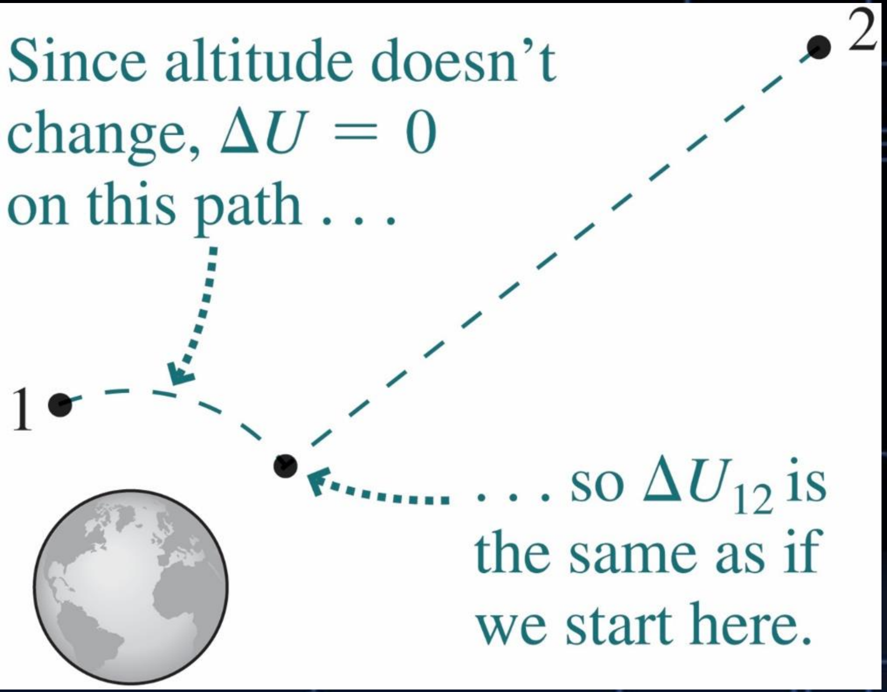
- The potential energy changes over large distances = The work done by gravitational force:

$$W = \int_{r_1}^{r_2} \frac{GMm}{r^2} dr = \left[-\frac{GMm}{r}\right]_{r_1}^{r_2} = GMm\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

Gravitational Energy

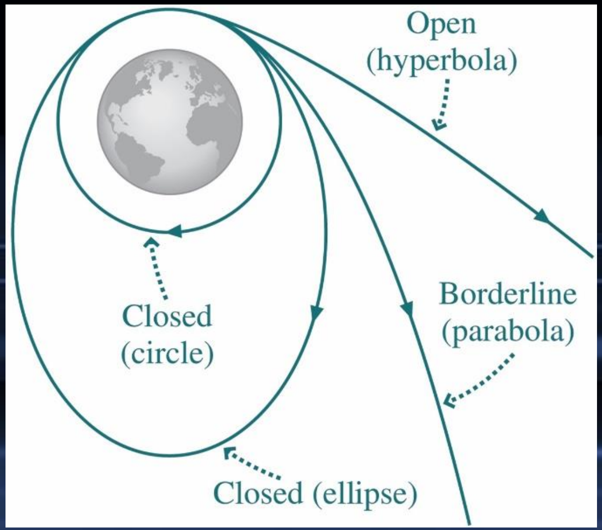
- The result holds regardless of whether the two points are on the same radial line.
- It’s convenient to take the zero of gravitational potential energy at infinity. Then, the gravitational potential energy becomes:

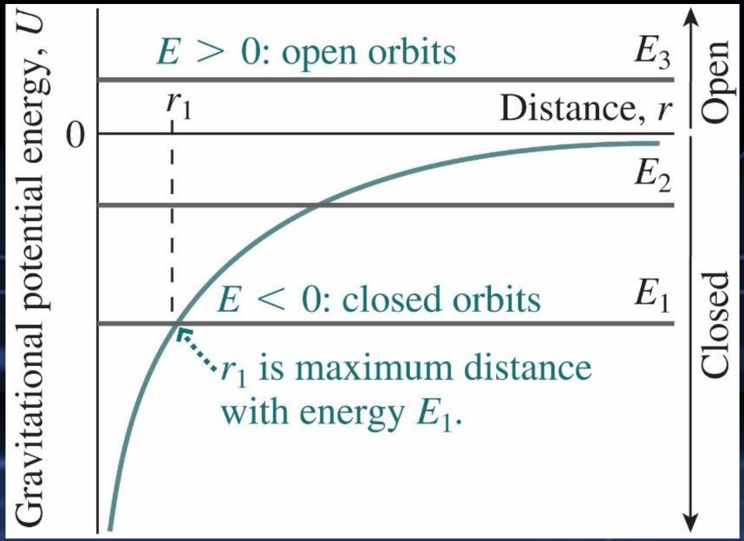
$$U(r) = -\frac{GMm}{r}$$



Energy and Orbits

- The total energy $E = K + U$, the sum of kinetic energy K and potential energy U, determines the type of orbit an object follows:
  - $E < 0$: The object is in a closed (bound), elliptical orbit:
    - Special cases include circular orbits and the straight-line paths of falling objects.
  - $E > 0$: The orbit is open (unbound) and hyperbolic.
  - $E = 0$: The borderline case gives a parabolic orbit.





Escape Speed

- An object with total energy E less than zero is in a bound orbit and cannot escape from the gravitating center.
- With energy E greater than zero, the object is in an unbound orbit and can escape to infinitely far from the gravitating center.

$$0 = K + U = \tfrac{1}{2}mv^2 - \frac{GMm}{r}$$

- Solving for v gives the escape speed:

$$v_{esc} = \sqrt{\frac{2GM}{r}}$$

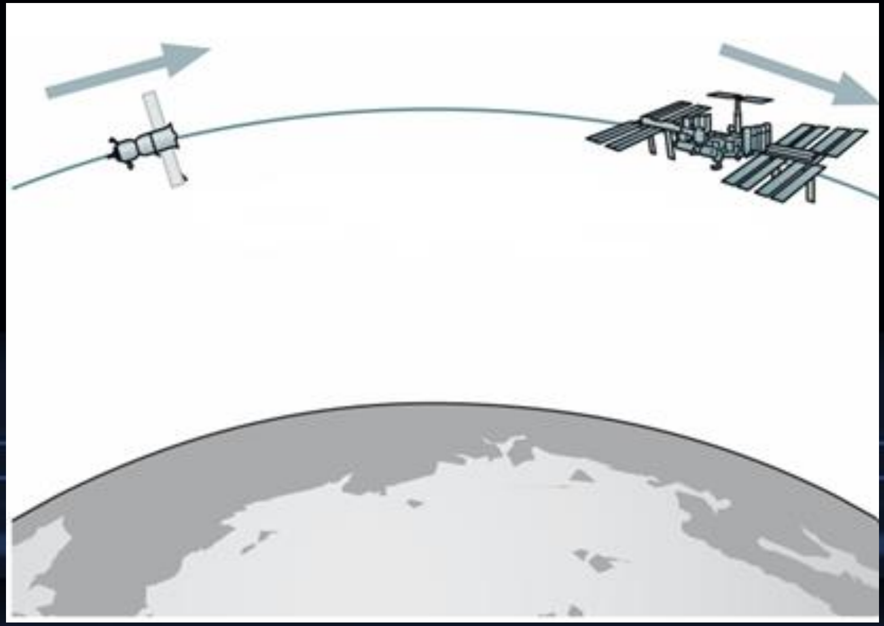
- Escape speed from Earth's surface is about 11 km/s.

Energy in Circular Orbits

- If an object is in a circular orbit due to gravity. We know that the centripetal acceleration is provided by gravity and thus $v^2 = GM/r$. Thus, kinetic energy $K = \frac{1}{2}mv^2 = \frac{GMm}{2r}$
- The potential energy $\mathrm{U} = -\frac{GMm}{r}$
- Thus, the total energy: $\mathrm{E} = \mathrm{U} + K = -\frac{GMm}{2r} = \frac{1}{2}U = -K$
  - This negative energy shows that the orbit is bound.
  - The lower the orbit, the lower the total energy—but the faster the orbital speed.

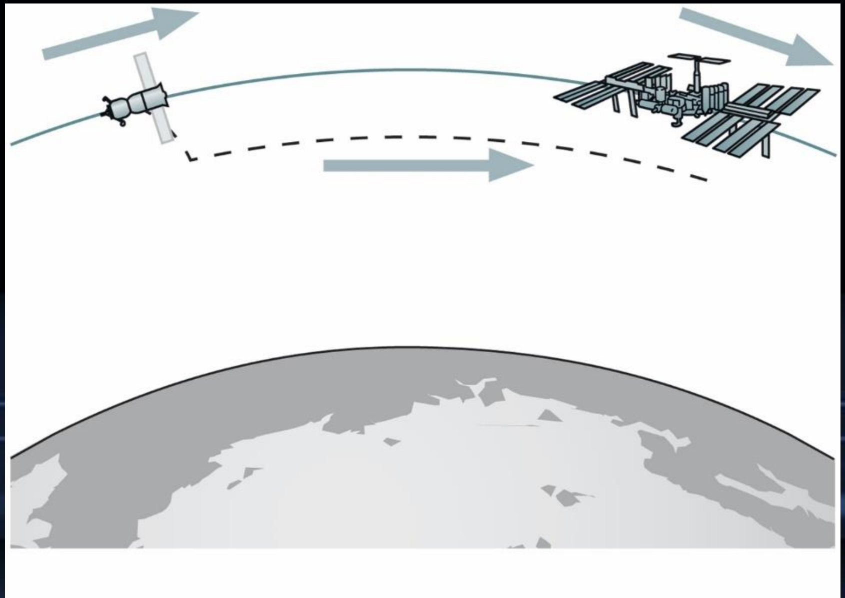
Think about it…

- Astronauts heading for the International Space Station find themselves in the right circular orbit, but well behind the station. How should they do to catch up?



Think about it…

- The lower the orbit, the lower the total energy—but the faster the orbital speed:
  - This means an orbiting spacecraft needs to lose energy to gain speed.



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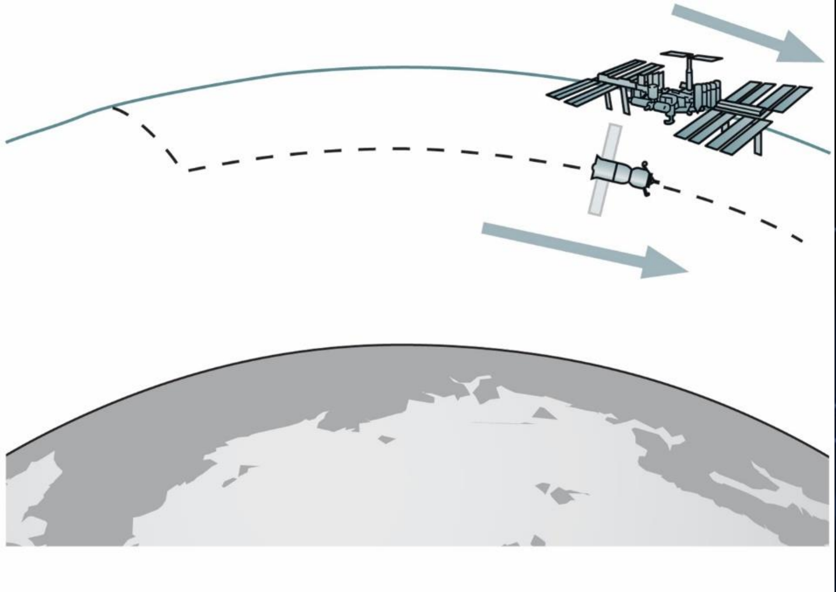

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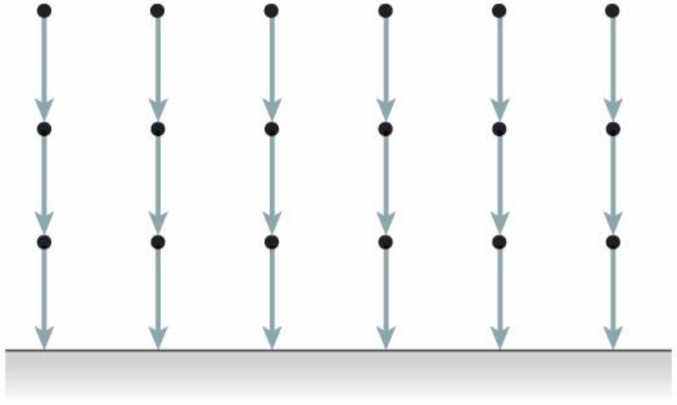
The Gravitational Field

- It's convenient to describe gravitation in terms of a **gravitational field** that results from the presence of mass and that exists at all points in space:
  - A massive object creates a gravitational field in its vicinity and other objects respond to the field **at their immediate locations**.
  - The gravitational field can be visualized with a set of vectors giving its strength (in N/k g; equivalently, m/s^2) and its direction.

$$\vec{g} = -\frac{GM}{r}\hat{r}$$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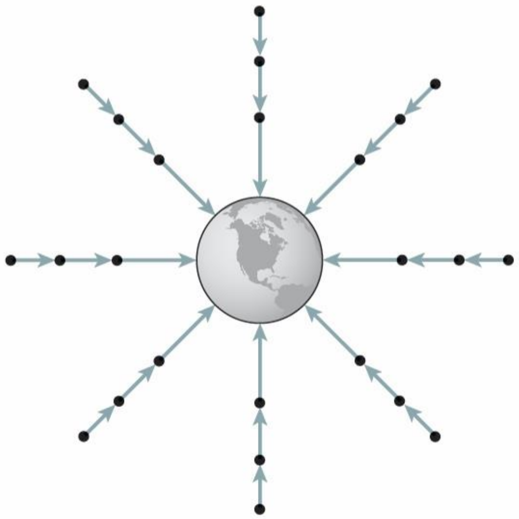
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Summary

- Newton's **law of universal gravitation** describes the attractive force between two point masses m_1 and m_2 located a distance r apart:

$$F = \frac{Gm_1m_2}{r^2}$$

- Motion under gravity includes the following:
  - Closed elliptical and circular orbits when the orbiting object's total energy is less than zero.
  - Open parabolic and hyperbolic orbits when the total energy is zero or greater.
- The **gravitational field** describes the force of gravity in terms of a field that exists at all points in space; an object then responds to the field in its immediate vicinity.