Course announcement

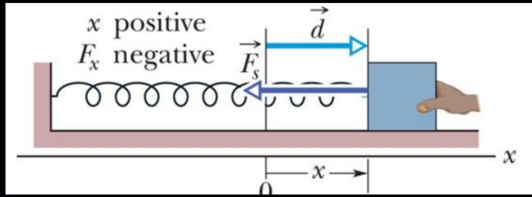
- The homework set 2 will be posted on eLearn on 10/14 (Friday) at 8AM. Please submit your homework via eLearn. No late homework will be accepted.
- The first midterm will on 10/25 (Tuesday).

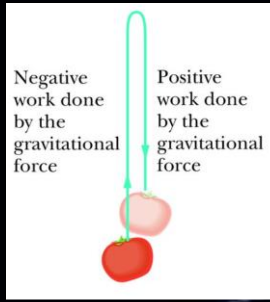
4	10/7(Fri.)	**Energy:** kinetic energy and work
5	10/11(Tue.)	**Energy:** potential energy and conservation of energy
5	10/14(Fri.)	**Gravity:** Law of gravity (Homework2)
6	10/18(Tue.)	**Gravity:** Gravitational energy and gravitational field
6	10/21(Fri.)	**Review I**
7	10/25(Tue.)	Mid Term 1

Conservative and non-conservative force in 1D

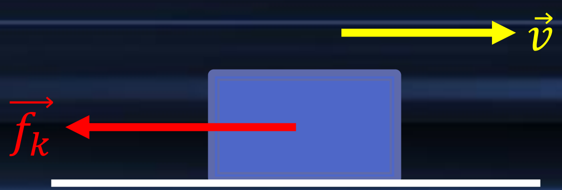
In one dimension, the force can be divided into two different categories:

- Conservative force: the force is only depending on position. e.g. spring force, gravitational force.





- Non-conservative force: the force depending not only on position. e.g. frictional force, force exerting by hand.



Conservative and non-conservative force in 1D

In one dimension, the force can be divided into two different categories:

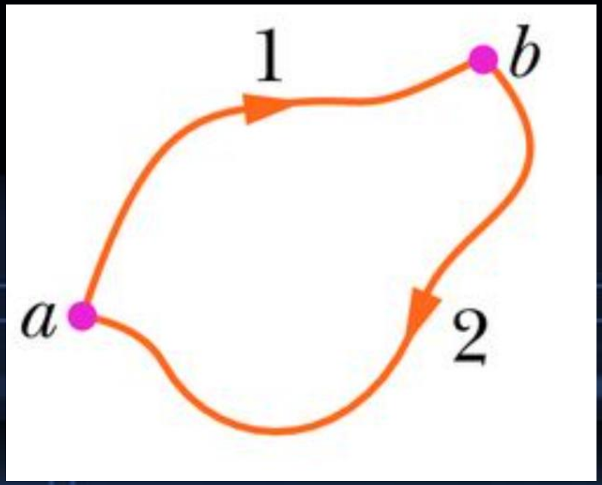
- Conservative force: the force is only depending on position. e.g. spring force, gravitational force.
- The work done by the conservative force is changing in potential energy with a minus sign, which only depends on initial and final position.

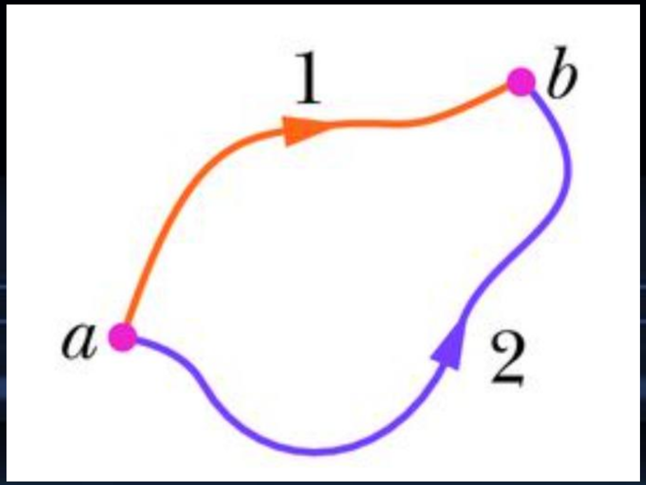
$$W = -\Delta U = -(U(x_2) - U(x_1))$$

Formal definition of conservative force in 3D

- The net work done by a conservative force $\overrightarrow{F_c}$ on a particle moving around any closed path is zero.

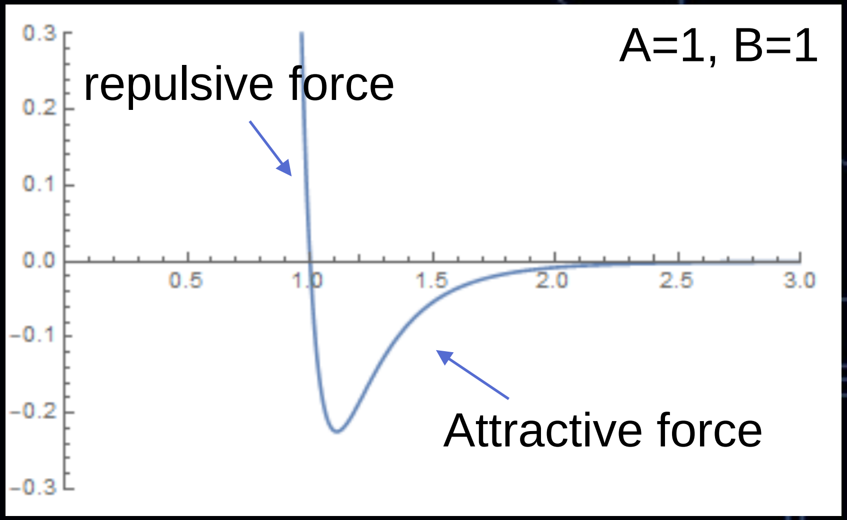
$$\oint \overrightarrow{F_c} \cdot d\vec{r} = 0$$





The work done by $\overrightarrow{F_c}$ does not depend on path, but only the position of a and b.

Example: Van der Waals force

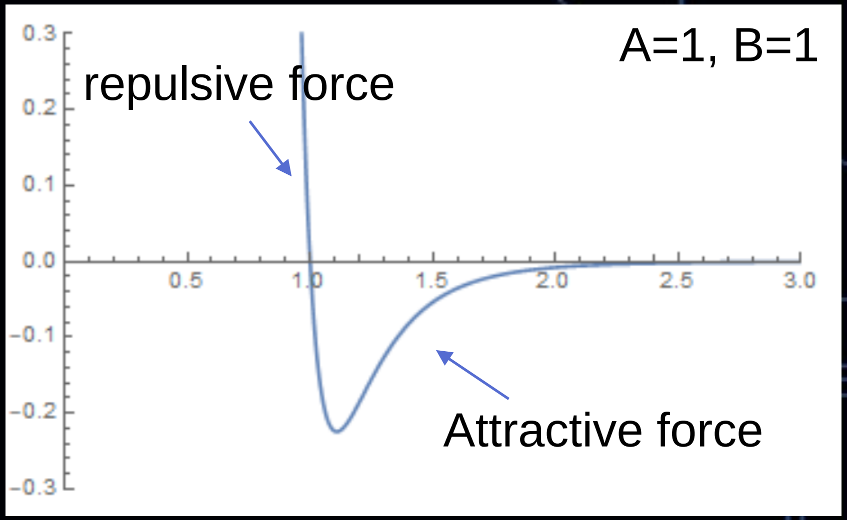


$$F(r) = (\frac{A}{r^{13}} - \frac{B}{r^7})\hat{r}$$

- First, assume we only move in 1D.
- The work done by external force is W_e, and the work done by the force between atom is W_a. Since there is no kinetic energy change, we have $W_e + W_a = 0$. Thus

$$W_e = -W_a$$

Example: Van der Waals force



$$F(r) = \left(\frac{A}{r^{13}} - \frac{B}{r^{7}}\right)\hat{r}$$

$$W_e = -W_a = -\int_{r=\infty}^{r=R} F(r)dr$$

$$= -\int_{r=\infty}^{r=R} \left(\frac{A}{r^{13}} - \frac{B}{r^{7}}\right) dr = -\left(\frac{-A}{12r^{12}} + \frac{B}{6r^{6}}\right)\Bigg|_{r=\infty}^{r=R} = \left(\frac{A}{12R^{12}} - \frac{B}{6R^{6}}\right)$$

Potential energy in 3D

Consider the work done by conservative force in 3D, then:

$$W_c = \int_1^2 \overrightarrow{F_c} \cdot d\vec{r} = -\Delta U = U(\overrightarrow{r_2}) - U(\overrightarrow{r_1})$$

Use Van der Waals force as example:

$$W_c = \int_1^2 \overrightarrow{F_c} \cdot d\vec{r} = \int_1^2 \left(\frac{A}{r^{13}} - \frac{B}{r^7}\right) \hat{r} \cdot d\vec{r} = \int_1^2 \left(\frac{A}{r^{13}} - \frac{B}{r^7}\right) \frac{\vec{r}}{r} \cdot d\vec{r}$$

Potential energy in 3D

$$\vec{r} \cdot d\vec{r} = \frac{1}{2} d(\vec{r} \cdot \vec{r}) = \frac{1}{2} d(r^2) = \frac{1}{2}(2rdr) = rdr$$

Thus we go back to just like 1D case:

$$-\Delta U = W_c = \int_1^2 \left(\frac{A}{r^{13}} - \frac{B}{r^7}\right) dr = \left(\frac{A}{12r^{12}} - \frac{B}{6r^6}\right)_1^2$$

With 1:$r = \infty$ and 2: $\mathrm{r} = \mathrm{R}$

$$U(R) = \left(\frac{A}{12R^{12}} - \frac{B}{6R^6}\right)$$

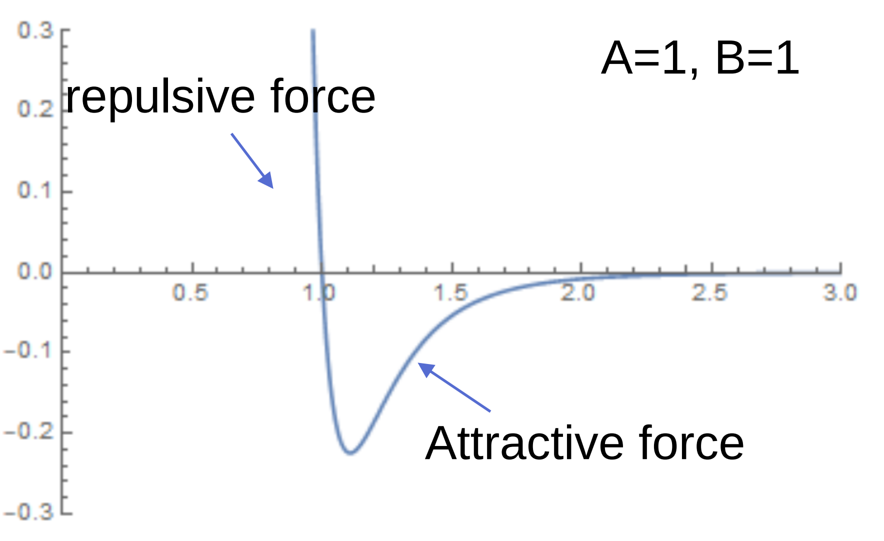
Comparison of function of force and potential

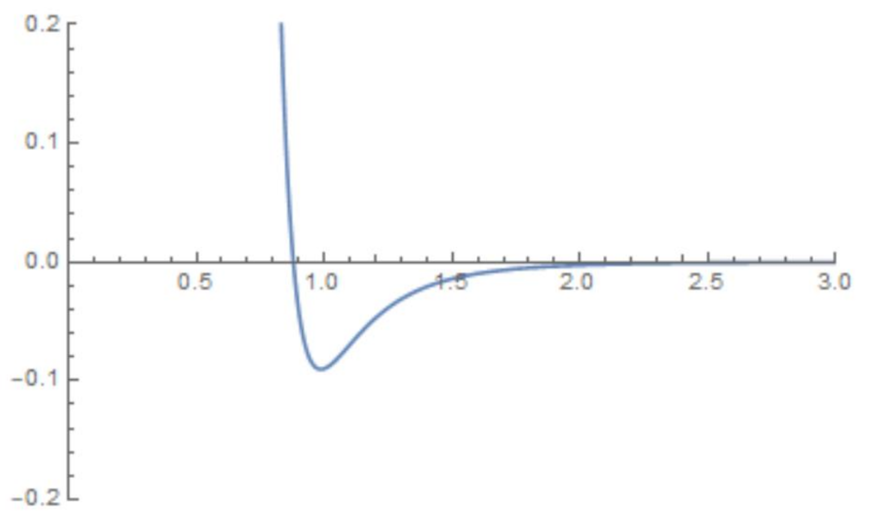
- $\overrightarrow{F(r)} = (\frac{A}{r^{13}} - \frac{B}{r^7})\hat{r}$

- $U(R) = \left(\frac{A}{12R^{12}} - \frac{B}{6R^6}\right)$

- Notice that:

$$\overrightarrow{F(r)} = -\frac{dU}{dr}\hat{r}(= -\nabla U)$$





Conservation of mechanical energy in 3D

- Considering there is only a conservative force $\vec{F_c}$ doing work W_c on an object, then we have:

$$W_c = \int_1^2 \vec{F_c} \cdot d\vec{r} = \Delta K = K_2 - K_1$$

- With the definition $W_c = -\Delta U$, we have:

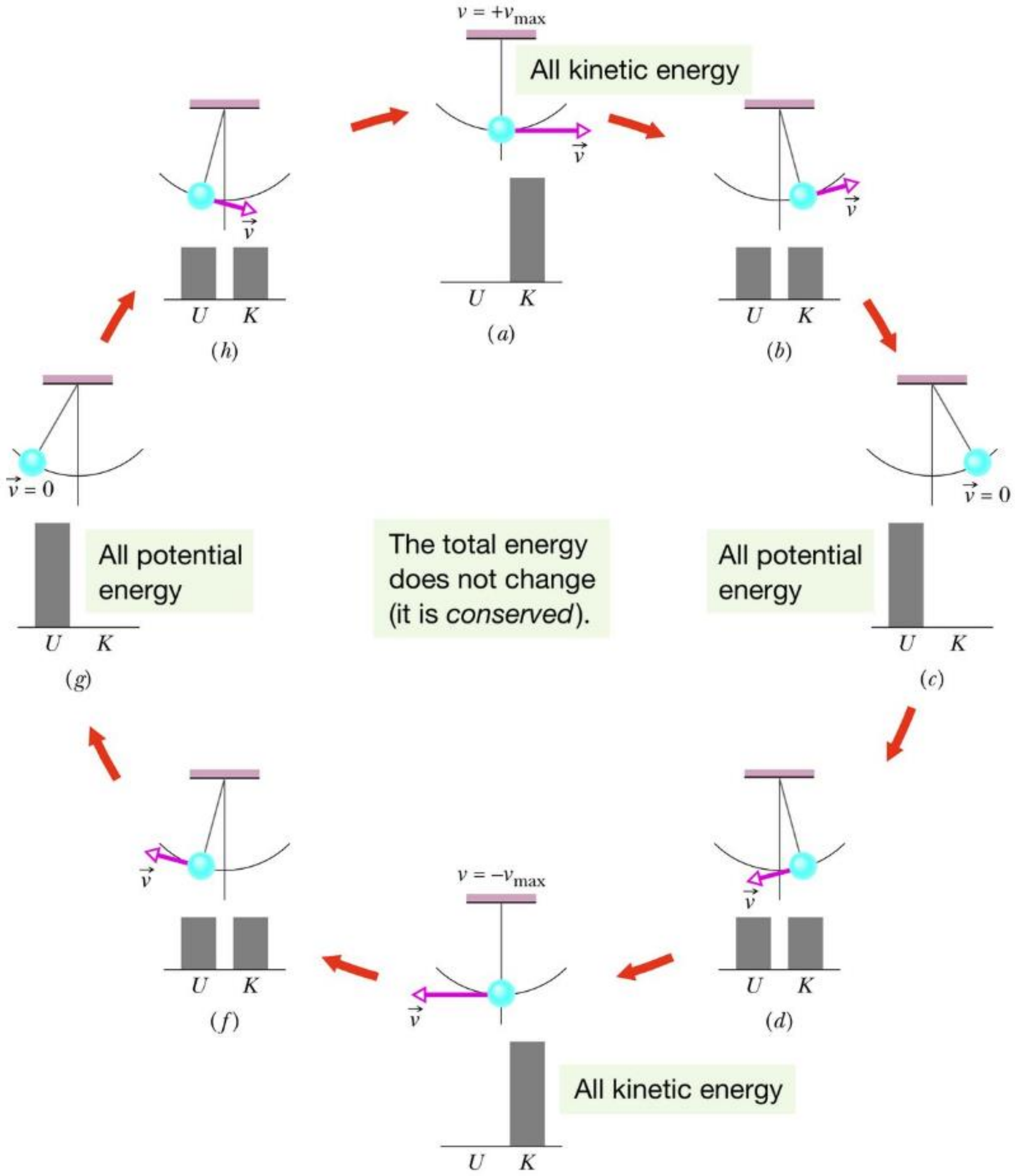
$$W_c = -\Delta U = -(U_2 - U_1) = K_2 - K_1$$

$$K_1 + U_1 = K_2 + U_2 \equiv E_{mec}$$

Conservation of mechanical energy

$$K_1 + U_1 = K_2 + U_2 = E_{mec}$$

In an isolated system where only conservative forces cause energy changes, the kinetic energy and potential energy can change, but their sum, the mechanical energy E_{mec} of the system, cannot change.



$v = +v_{max}$
All kinetic energy
U K
(a)
U K
(b)
$\vec{v} = 0$
All potential energy
U K
(c)
U K
(d)
$v = -v_{max}$
All kinetic energy
U K
U K
(f)
$\vec{v} = 0$
All potential energy
U K
(g)
U K
(h)
The total energy does not change (it is *conserved*).

Conservation of energy

- Now we consider the case that there are both conservative forces $\overrightarrow{F_c}$ and by non-conservative forces $\overrightarrow{F_{nc}}$ on an object. Then we have equation of motion:

$$\overrightarrow{F_c} + \overrightarrow{F_{nc}} = m\vec{a}$$

- Let's integrate equation of motion respect to displacement:

$$\int_1^2 (\overrightarrow{F_c} + \overrightarrow{F_{nc}}) \cdot d\vec{r} = \int_1^2 m \frac{d\vec{v}}{dt} \cdot d\vec{r}$$

Conservation of energy

$$\underbrace{\int_1^2 \overrightarrow{F_c} \cdot \mathrm{d}\vec{r}}_{W_c = -\Delta U} + \underbrace{\int_2^2 \overrightarrow{F_{nc}} \cdot d\vec{r}}_{W_{nc}} = \underbrace{\int_1^2 m \frac{d\vec{v}}{dt} \cdot d\vec{r}}_{\Delta K}$$

Work done by non-conservative force = change of mechanical energy

$$W_{nc} = \Delta K + \Delta U$$

This is conservation of energy

Summary

- Conservation of mechanical energy: Considering there is only a conservative force $\vec{F_c}$ doing work W_c on an object, then we have:

$$W_c = -\Delta U = -(U_2 - U_1) = K_2 - K_1$$

$$K_1 + U_1 = K_2 + U_2 \equiv E_{mec}$$

- Conservation of energy:

Work done by non-conservative force = change of mechanical energy

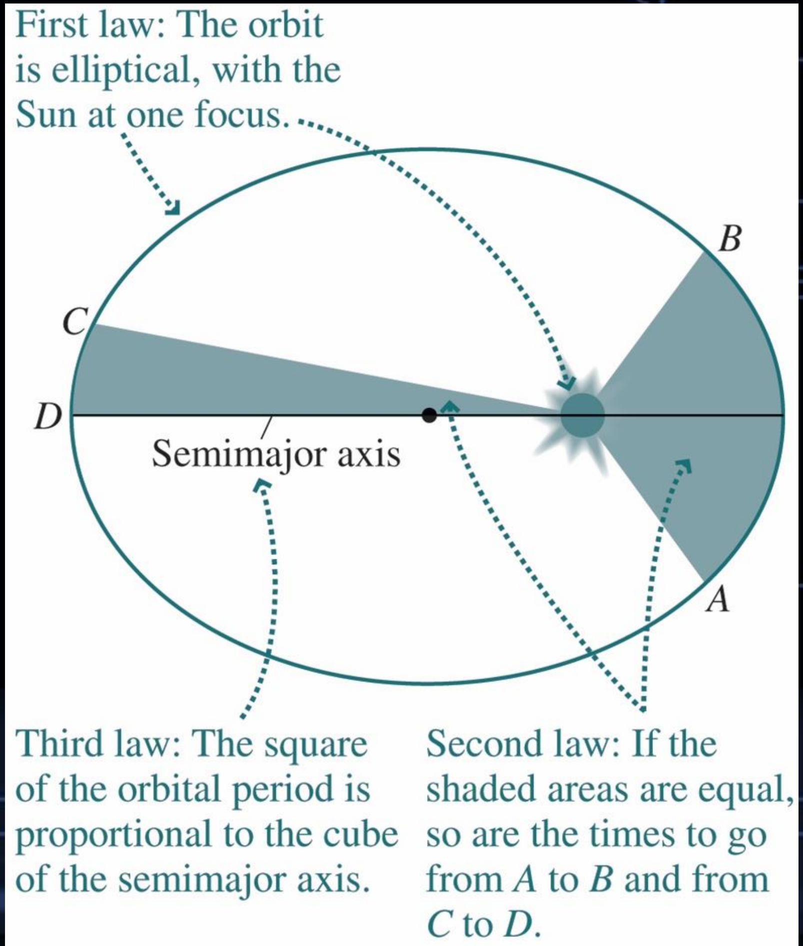
- $W_{nc} = \Delta K + \Delta U$

GENERAL PHYSICS B1
GRAVITY

Law of Gravity
2022/10/14

Toward a Law of Gravity

- Newton's theory of gravity was the culmination of over a century of work:
  - In 1543, Nicolaus Copernicus published a heliocentric theory of the solar system.
  - Subsequently, Tycho Brahe accurately measured planetary motions. After his death in 1601, his work was brilliantly summarized by Johannes Kepler in three laws of planetary motion.



Universal Gravitation

- Newton's **law of universal gravitation** states that any two point particles attract each other with a force that is proportional to the product of their masses and inversely proportional to the square of their separation:

$$F = \frac{Gm_1m_2}{r^2}$$

- $G = 6.67 \times 10^{-11} N \cdot m^2/kg^2$ is the **constant of universal gravitation**.

Think about it…

- If the distance between two objects is cut in half, the gravitational force between them is

(a) 1/2

(b) 2 times

(c) 1/4

(d) 4 times

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Think about it…

- On the surface of the Earth(radius of the Earth is R_E), the gravitational acceleration is g. What is the gravitational acceleration for an object that is R_E away from the surface of the Earth?

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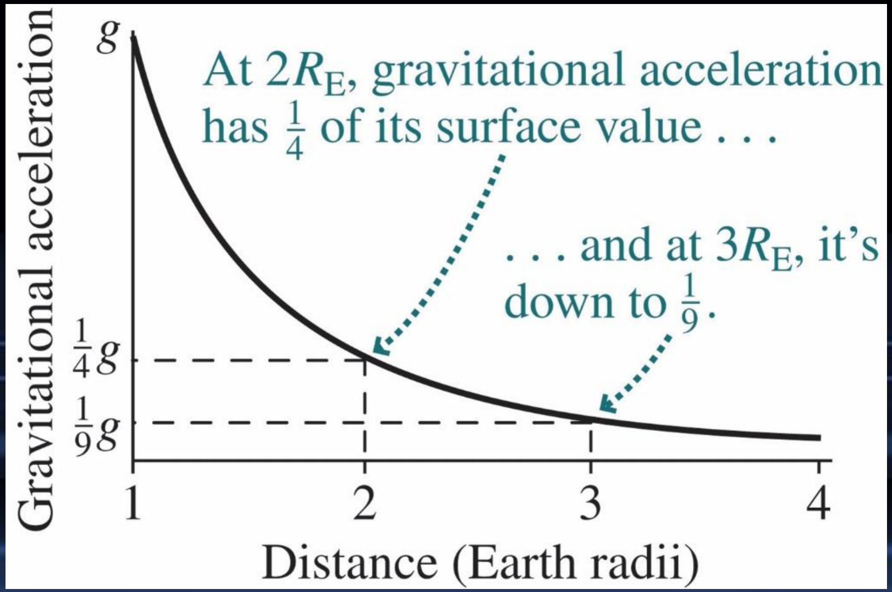

The Cavendish Experiment: Weighing the Earth

- In 1798, Henry Cavendish made the first experimental measurement of the gravitational constant *G*:
  - In an ingenious experiment, Cavendish measured the small gravitational force between two fixed masses and two masses that were suspended by a thin fiber.
  - His result allowed him to find the Earth’s mass!(g=GM/r)

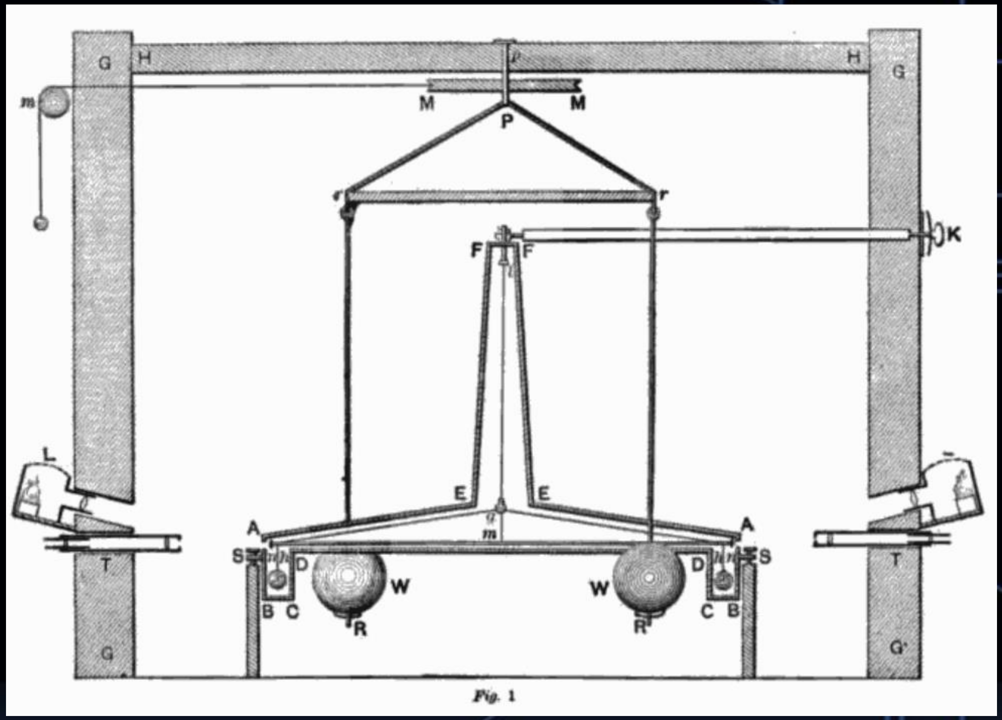

Fig. 1

https://en.wikipedia.org/wiki/Cavendish_experiment#/media/File:Cavendish_Experiment.png

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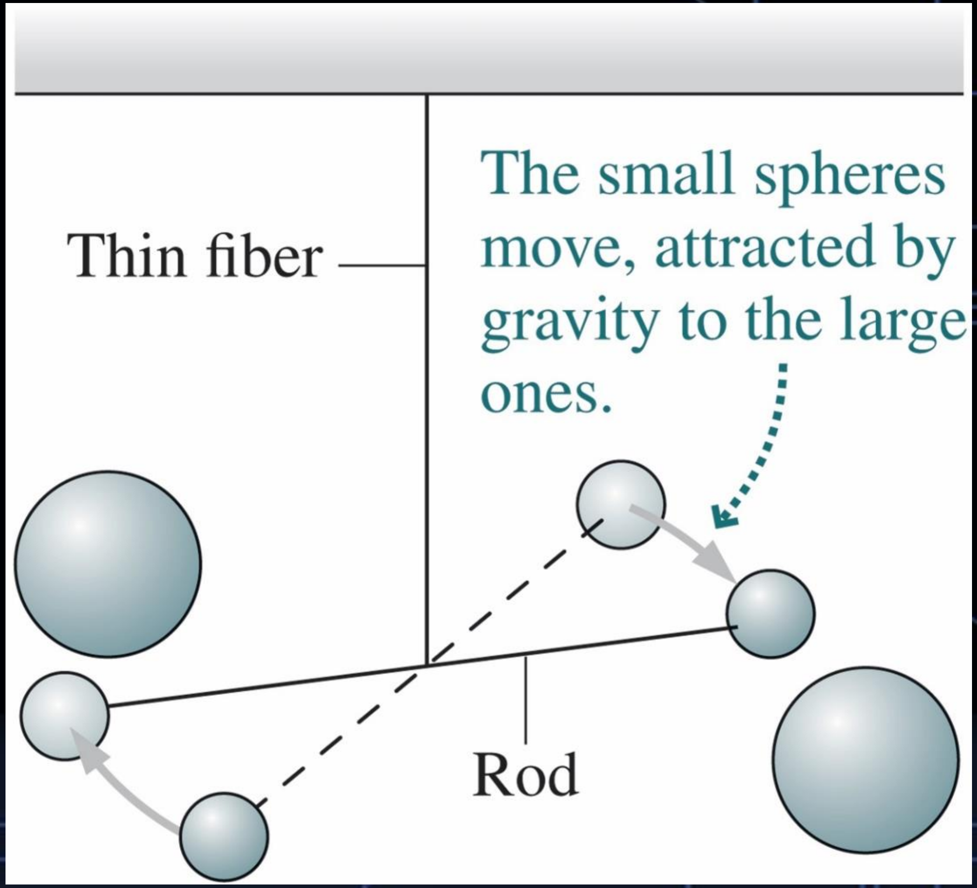
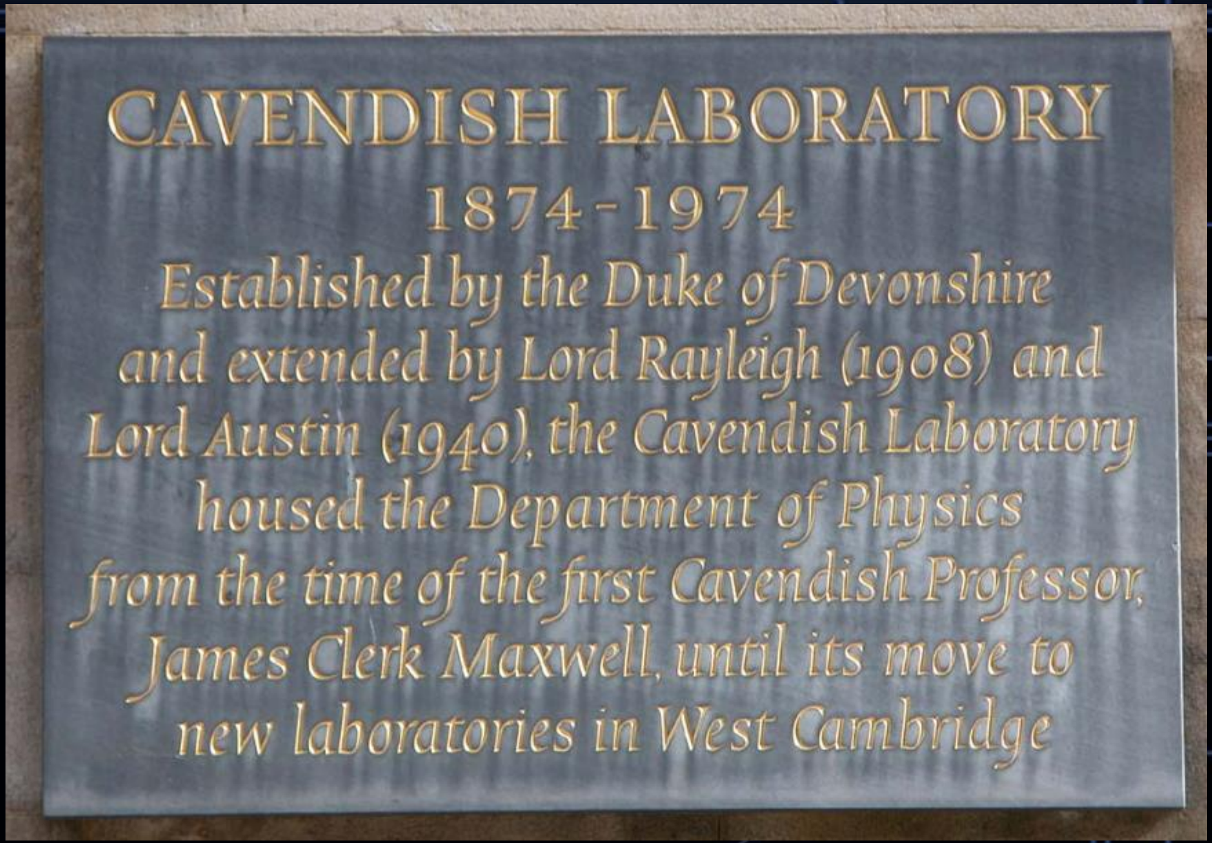


The Cavendish Experiment: Weighing the Earth

https://makeagif.com/gif/cavendish-gravity-experiment-time-lapse-version-1-QZGrGo

https://www.daviddarling.info/encyclopedia/C/Cavendish_Laboratory.html

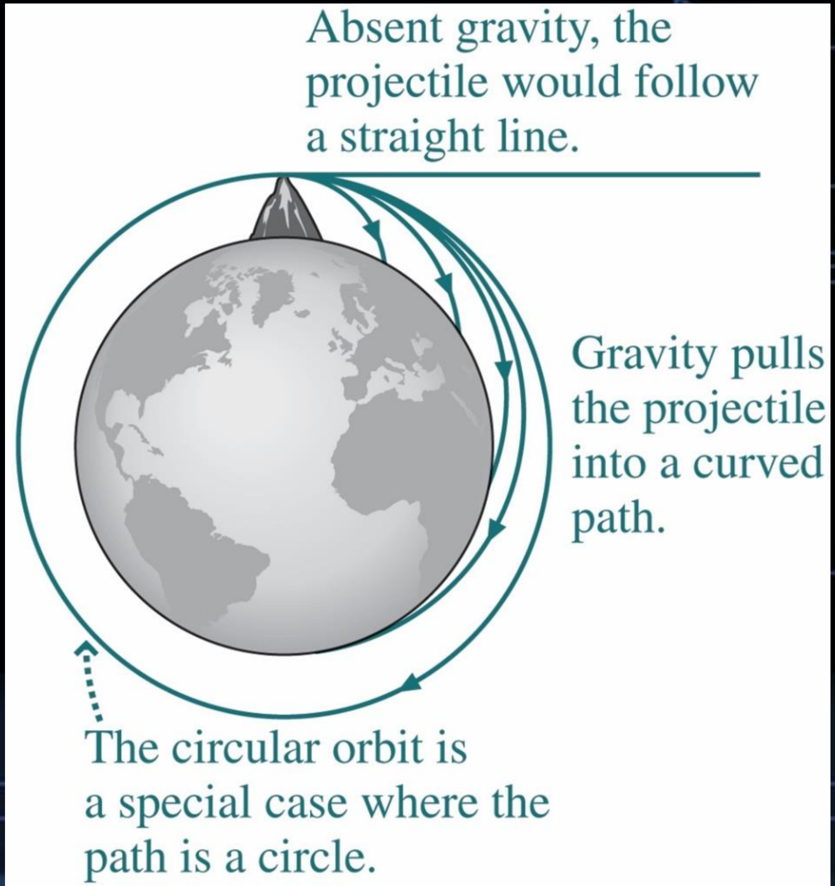


https://en.wikipedia.org/wiki/Cavendish_Laboratory#/media/File:Cavendish-plaque_retouch_b.jpg

As of 2019, 30 Cavendish researchers have won Nobel Prizes

Orbital Motion around the Earth

- Orbital motion occurs when gravity is the dominant force.
- Newton explained orbits using universal gravitation and his laws of motion:
  - The force of gravity causes a projectile to deviate from its straight-line path.
  - At a critical speed, the curvature of the projectile's path follows the Earth's curvature and it enters a circular orbit.



Circular Orbits

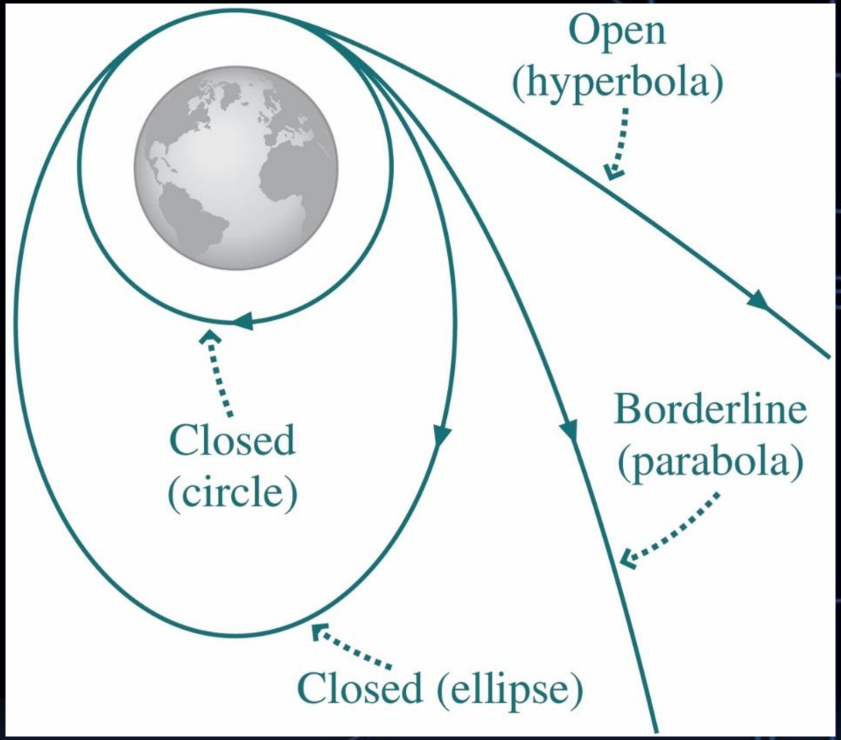
- In a circular orbit, gravity provides the centripetal force needed to keep an object of mass *m* in its circular path about a much more massive object of mass *M*. Therefore:

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

- The orbital speed: $v = \sqrt{\frac{GM}{r}}$
- Orbital period: $T^2 = (\frac{2\pi r}{v})^2 = \frac{4\pi^2 r^3}{GM}$
- This proves Kepler's third law: $T^2 \propto r^3$
- For satellites in low-Earth orbit, the period is about 90 minutes.

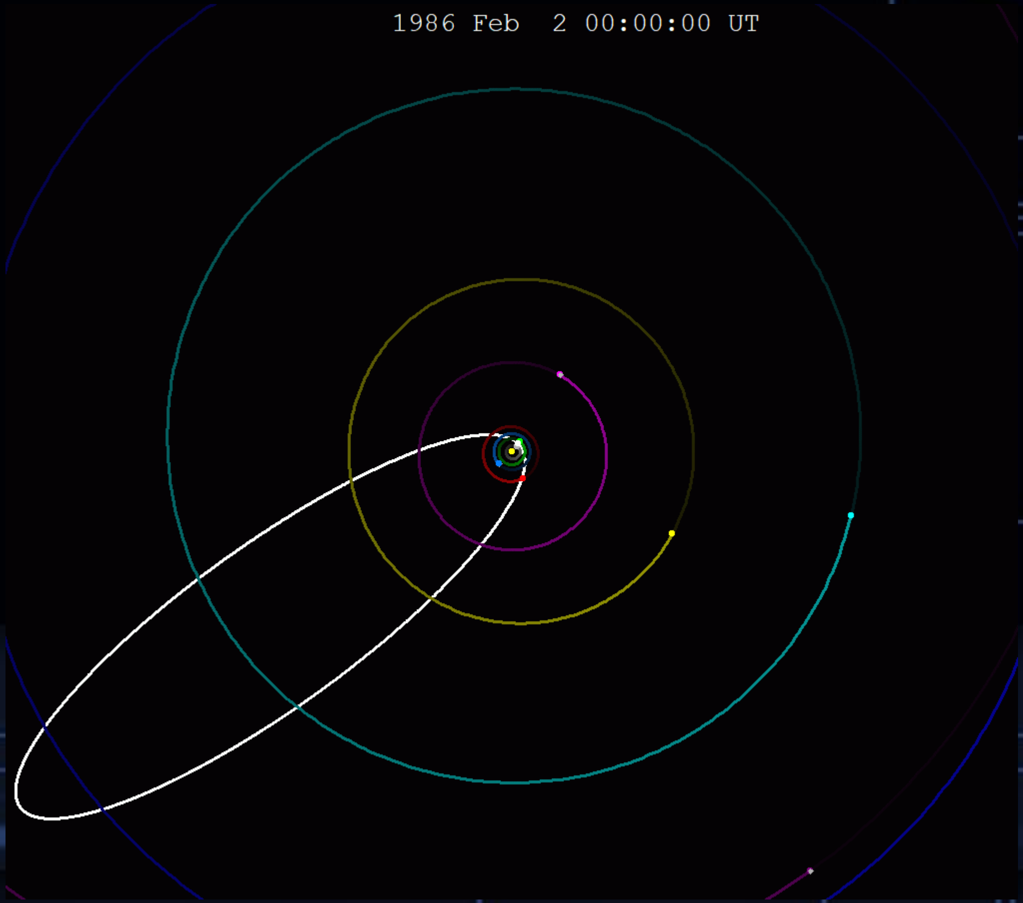
Elliptical Orbits and Open Orbits

- A circular orbit is not the only possibility:
  - Closed (bound) orbits are elliptical.
  - In the special case of a circular orbit, the acceleration of the orbiting object has a constant magnitude and always points toward the center of the orbit.
  - Unbound orbits are hyperbolic or (borderline case) parabolic.



Halley's Comet: elliptical orbit with 75 years period

Halley's Comet on 8 March 1986



Summary

- Newton's **law of universal gravitation:** $F = \frac{Gm_1m_2}{r^2}$

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