Course announcement

- Solution of homework set 1 has been posted on eLearn.
- The homework set 2 will be posted on eLearn on 10/14 (Friday) at 8AM. Please submit your homework via eLearn. No late homework will be accepted.
- The first midterm will on 10/25 (Tuesday).

4	10/7(Fri.)	**Energy:** kinetic energy and work
5	10/11(Tue.)	**Energy:** potential energy and conservation of energy
5	10/14(Fri.)	**Gravity:** Law of gravity (Homework2)
6	10/18(Tue.)	**Gravity:** Gravitational energy and gravitational field
6	10/21(Fri.)	**Review I**
7	10/25(Tue.)	Mid Term 1

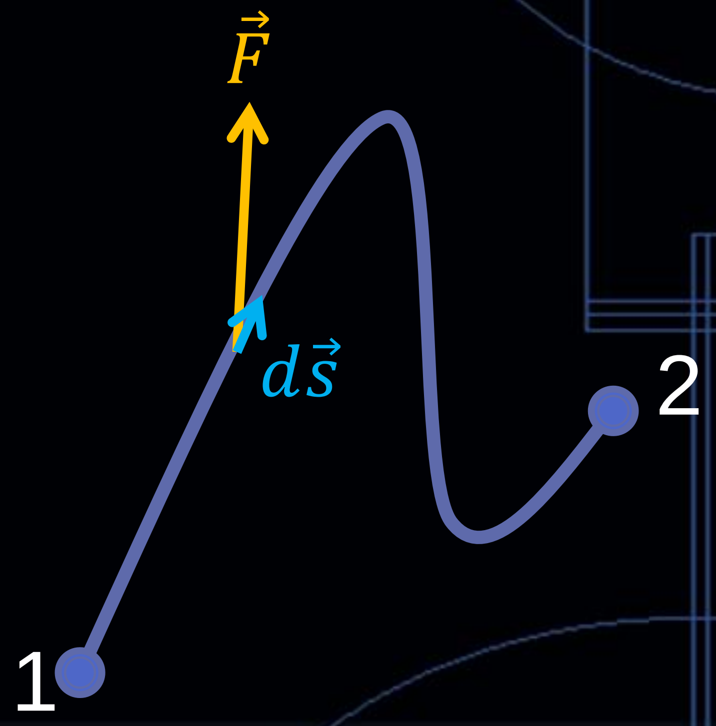
GENERAL PHYSICS B1
ENERGY

Potential Energy and Conservation of Energy
2022/10/11

Work on an object with a curve trajectory

- If the force acting on an object that moves in a curve. Then, the work done from position 1 to position two can be expressed as:

$$W = \int_1^2 \vec{F} \cdot d\vec{s}$$



Relationship between work and kinetic energy

- Since we introduce Newton's second law and work, what is the relationship between them?

Let's start from equation of motion - Newton's second law:

$$\overrightarrow{F_{NET}} = m\vec{a} = m\frac{d\vec{v}}{dt}$$

To link the force with work, let's integral respect displacement

$$\int_{1}^{2} \vec{F} \cdot d\vec{s} = \int_{1}^{2} m\frac{d\vec{v}}{dt} \cdot d\vec{s}$$

Relationship between work and kinetic energy

- Since we introduce Newton's second law and work, what is the relationship between them?

The left hand side is work W done by the force from 1 to 2.

There for we have

$$W = \int_1^2 m \frac{d\vec{v}}{dt} \cdot d\vec{s} = \int_1^2 m d\vec{v} \cdot \frac{d\vec{s}}{dt} = \int_1^2 m\vec{v} \cdot d\vec{v}$$

Note that: $d(\vec{v} \cdot \vec{v}) = (d\vec{v}) \cdot \vec{v} + \vec{v} \cdot (d\vec{v}) = 2\vec{v} \cdot (d\vec{v})$

Relationship between work and kinetic energy

- Since we introduce Newton's second law and work, what is the relationship between them?

Thus we have:

$$W = \int_1^2 \frac{1}{2} m d(\vec{v} \cdot \vec{v}) = \int_1^2 d(\frac{1}{2} m v^2) = K_2 - K_1$$

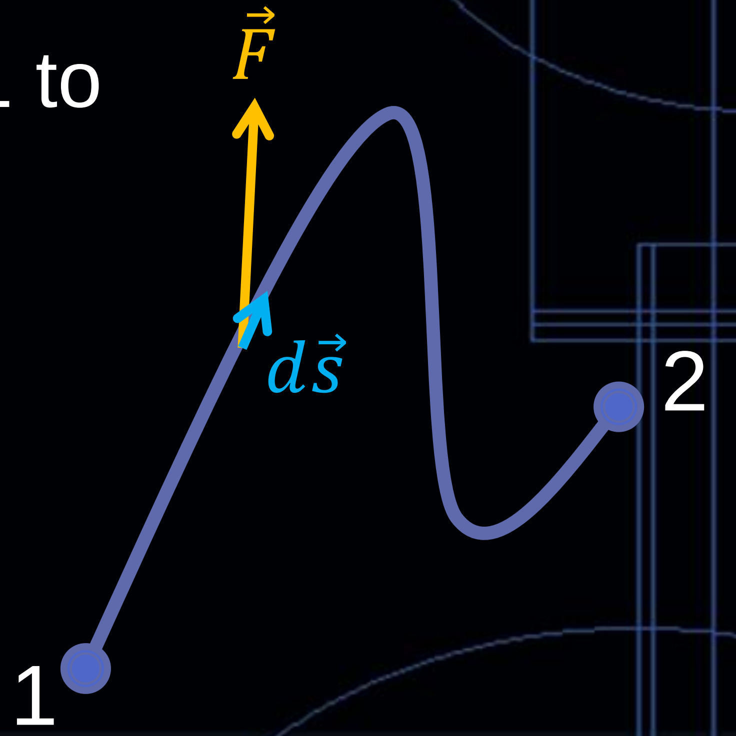
Work done by general force in 3D

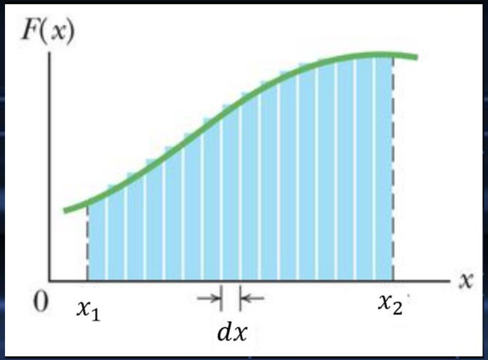
- We defined the work done from position 1 to position two can be expressed as:

$$W = \int_1^2 \vec{F} \cdot d\vec{s}$$

$$= \int_1^2 \left(F_x\hat{\imath} + F_y\hat{\jmath} + F_z\hat{k}\right) \cdot \left(dx\hat{\imath} + dy\hat{\jmath} + dz\hat{k}\right)$$

$$= \int_1^2 \left(F_x dx + F_y dy + F_z dz\right)$$





in x, y, z three different direction!

Work and Work-kinetic energy theorem

- **Work** involves force applied over distance:

-In one dimension: $W = \vec{F} \cdot \vec{s}$.

-In three dimension: $W = \int_1^2 \vec{F} \cdot d\vec{s}$

- **Work-kinetic energy theorem**: The net work done on an object equal to the change in kinetic energy.

$$\mathrm{W} = K_2 - K_1$$

Today’s topic

- Power
- Potential energy and conservative force
- Conservation of mechanical energy
- Conservation of energy

Power

- The time rate at which work is done by a force is said to be the power due to the force.
- If a force does an amount of work W in an amount of time Δt, the average power due to the force during that time interval is $P_{avg} = \frac{W}{\Delta t}$
- The instantaneous power P is the instantaneous time rate of doing work $\mathrm{P} = \frac{dW}{dt}$
- The SI unit of power is the joule per second. This unit is used so often that it has a special name, the watt (W)

Power

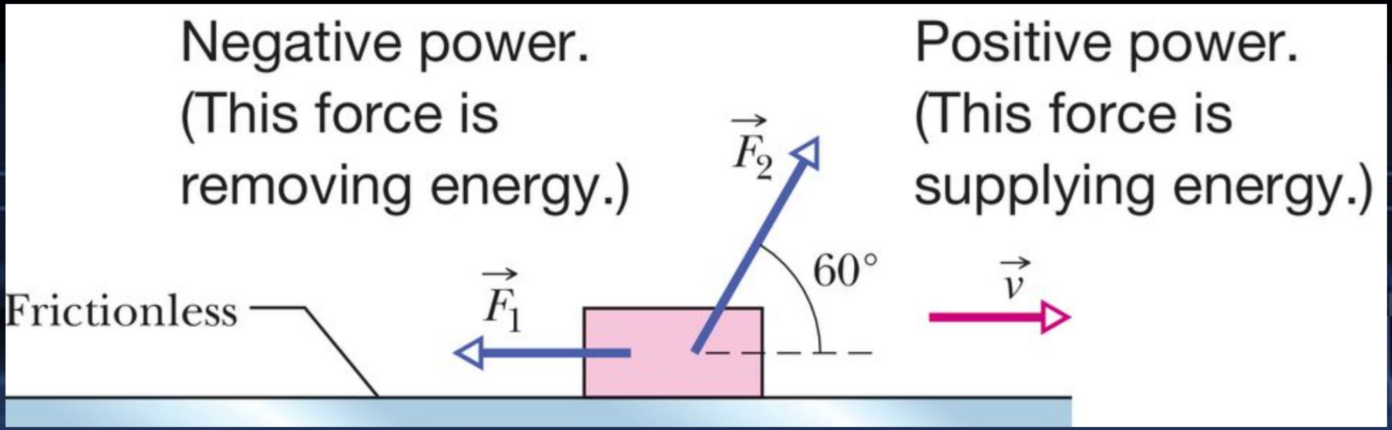
- By definition of instantaneous Power, we have: $\mathrm{P} = \frac{dW}{dt}$
- Assuming the work is done by a constant force. Plug in the definition of work,

$$\mathrm{P} = \frac{dW}{dt} = \frac{d(\vec{F}\cdot\vec{s})}{dt} = \vec{F}\cdot\frac{d\vec{s}}{dt} = \vec{F}\cdot\vec{v}$$

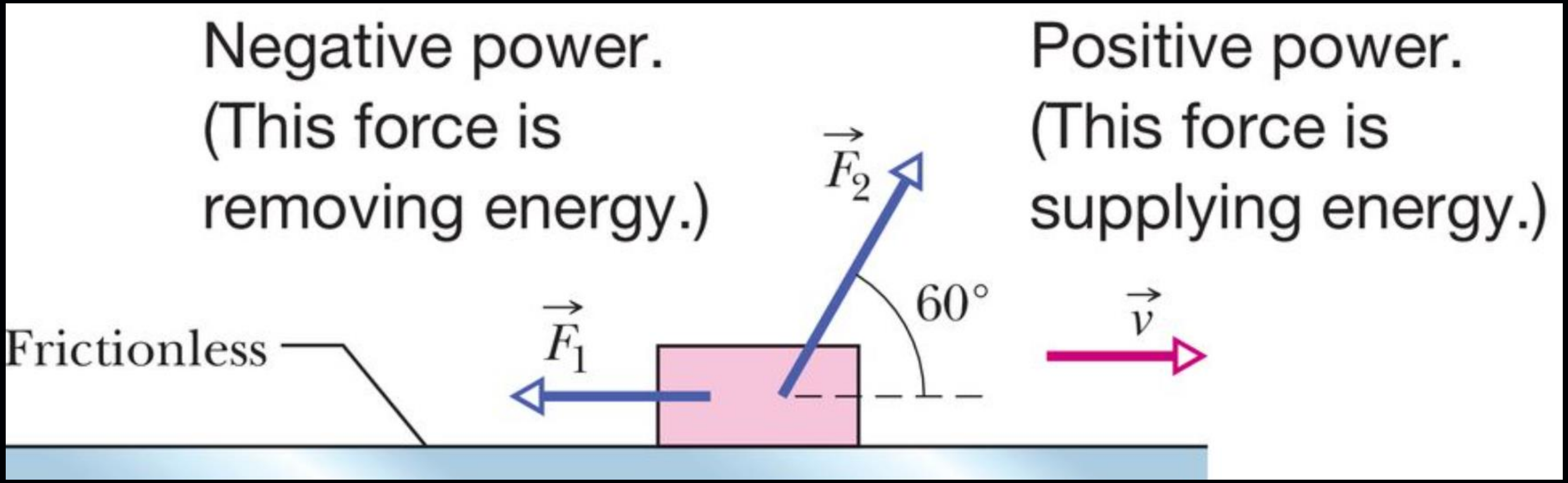
which described the instantaneous of power.

Example of Power

Two constant forces $\overrightarrow{F_1}$ and $\overrightarrow{F_2}$ acting on a box as the box slides rightward across a frictionless floor. Force $\overrightarrow{F_1}$ is horizontal, with magnitude 2.0 N; force $\overrightarrow{F_2}$ is angled upward by 60° to the floor and has magnitude 4.0 N. The speed v of the box at a certain instant is 3.0 m/s. What is the power due to each force acting on the box at that instant, and what is the net power?



Example of Power

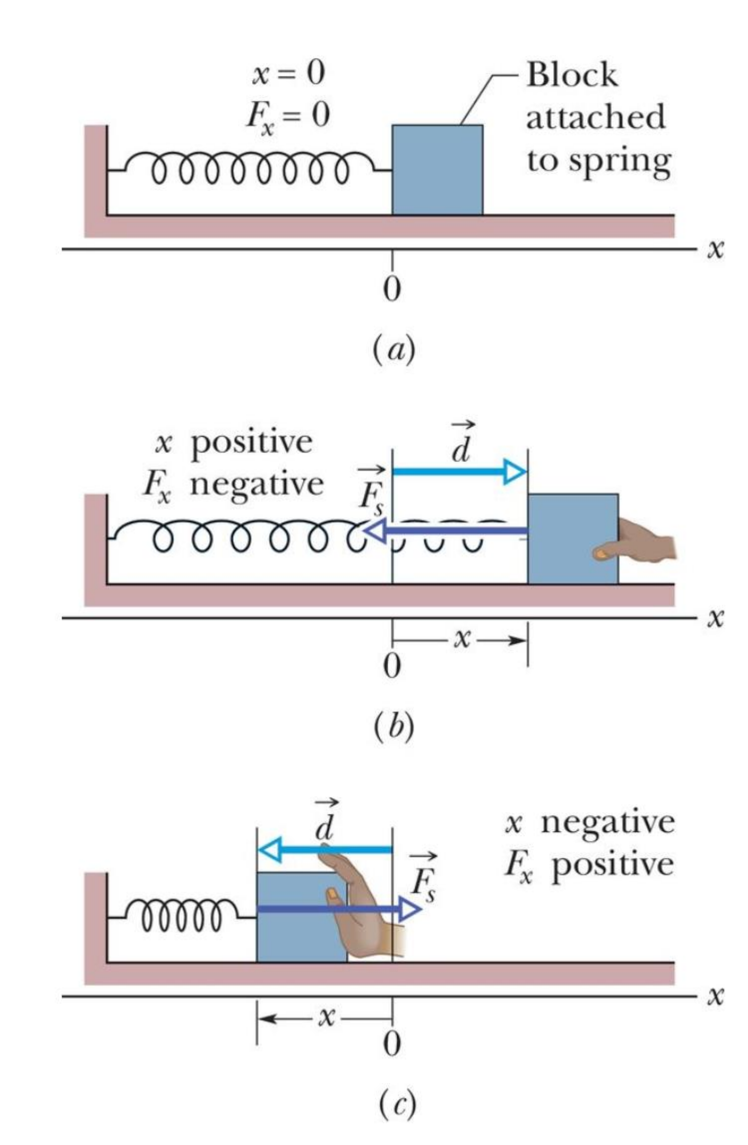


$$P_1 = F_1 v cos 180^\circ = -6W$$
$$P_2 = F_2 v cos 60^\circ = 6W$$
$$P_{NET} = P_1 + P_2 = 0W$$

Review of spring force

Let's review spring force in 1D:

- In plot (a), a spring in its relaxed state—that is, neither compressed nor extended: $\vec{F} = F_x = 0$
- In plot (b), the spring pulls on the block toward the left: $\vec{F} = F_x = -\mathrm{kd}$
- In plot (b), the spring pushes on the block toward the right: $\vec{F} = F_x = \mathrm{kd}$
- We have: $\vec{F} = -\mathrm{k}\vec{d}$ (Hooke's law)

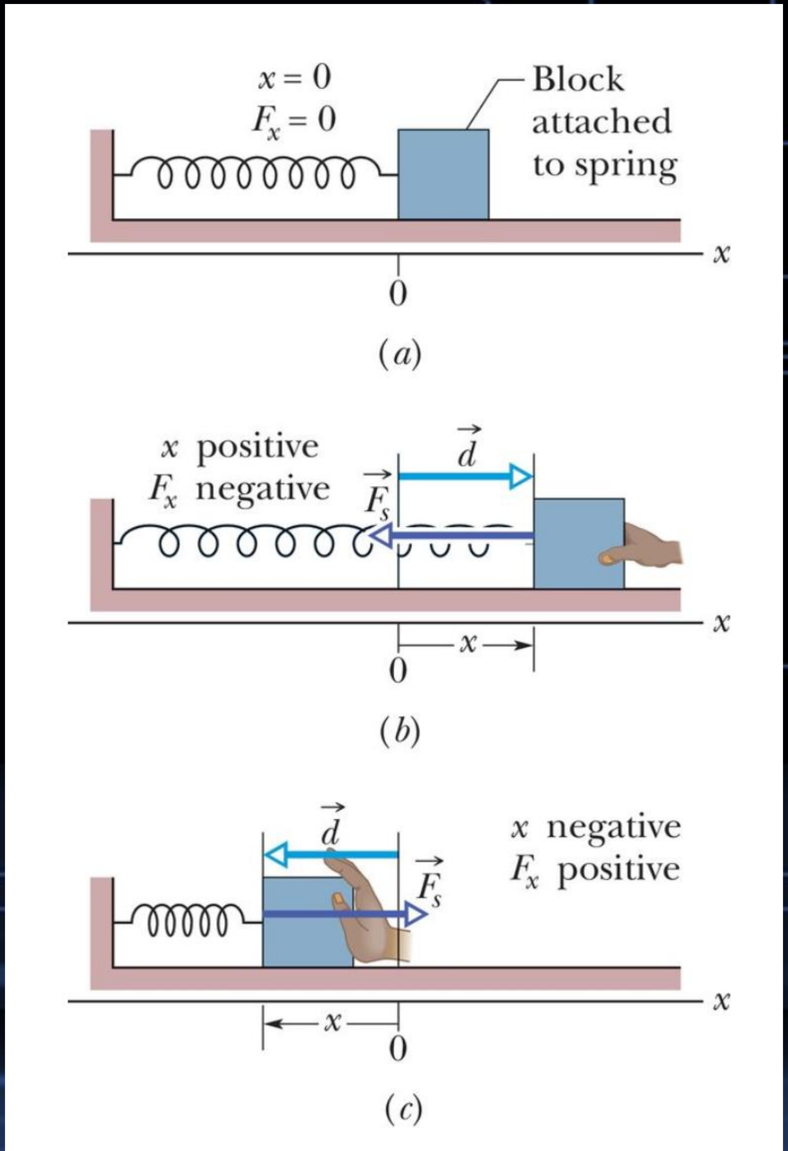


Work done by a spring force

Let's find the work done by a spring force

- Two assumption: (1) the spring is massless (2) the spring is an ideal spring, i.e., Hooke's law is valid at all positions
- Follow the definition of work:

$$W_s = \int_{x=x_1}^{x=x_2} \vec{F_s} \cdot d\vec{x}$$



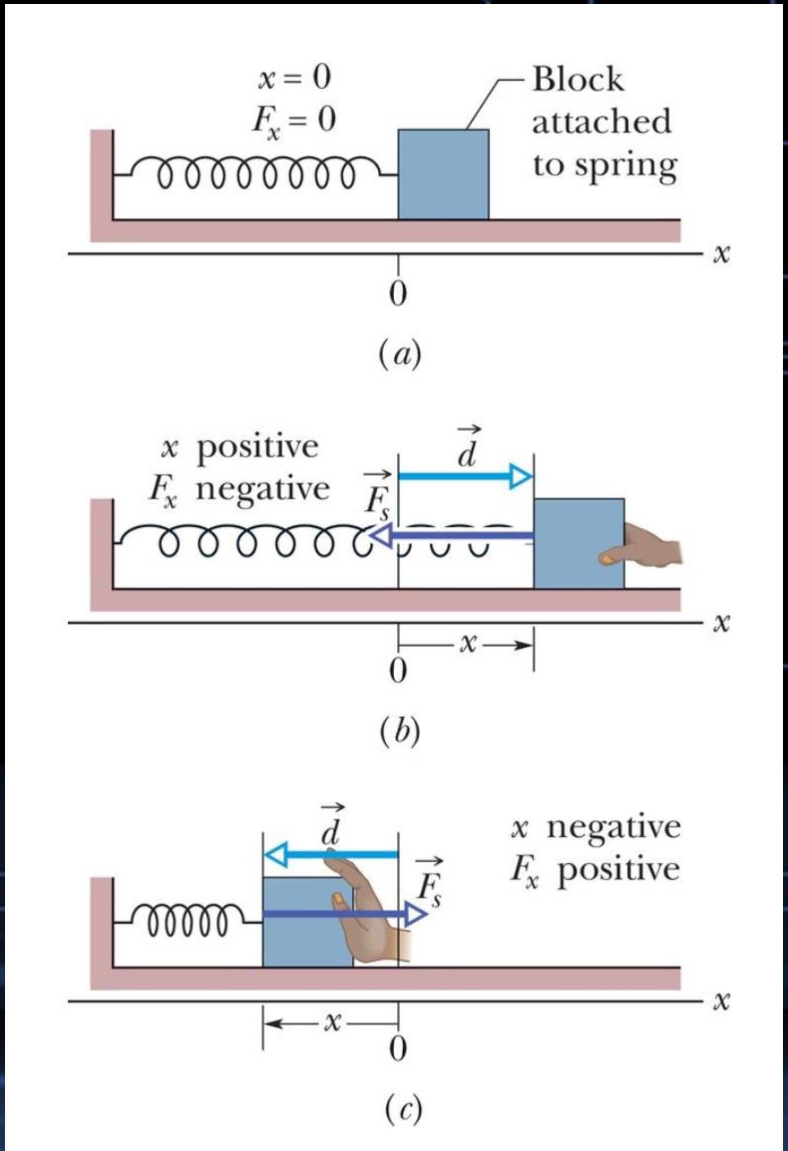
Work done by a spring force

- Following the definition of work:

$$W_s = \int_{x=x_1}^{x=x_2} \vec{F_s} \cdot d\vec{x}$$

$$= \int_{x=x_1}^{x=x_2} (-kx)dx = -k \int_{x=x_1}^{x=x_2} xdx$$

$$= -(\frac{1}{2}kx_2^2 - \frac{1}{2}kx_1^2)$$



Work-kinetic energy theorem of a spring force

- The work done by a spring force from position x_1 to x_2 :

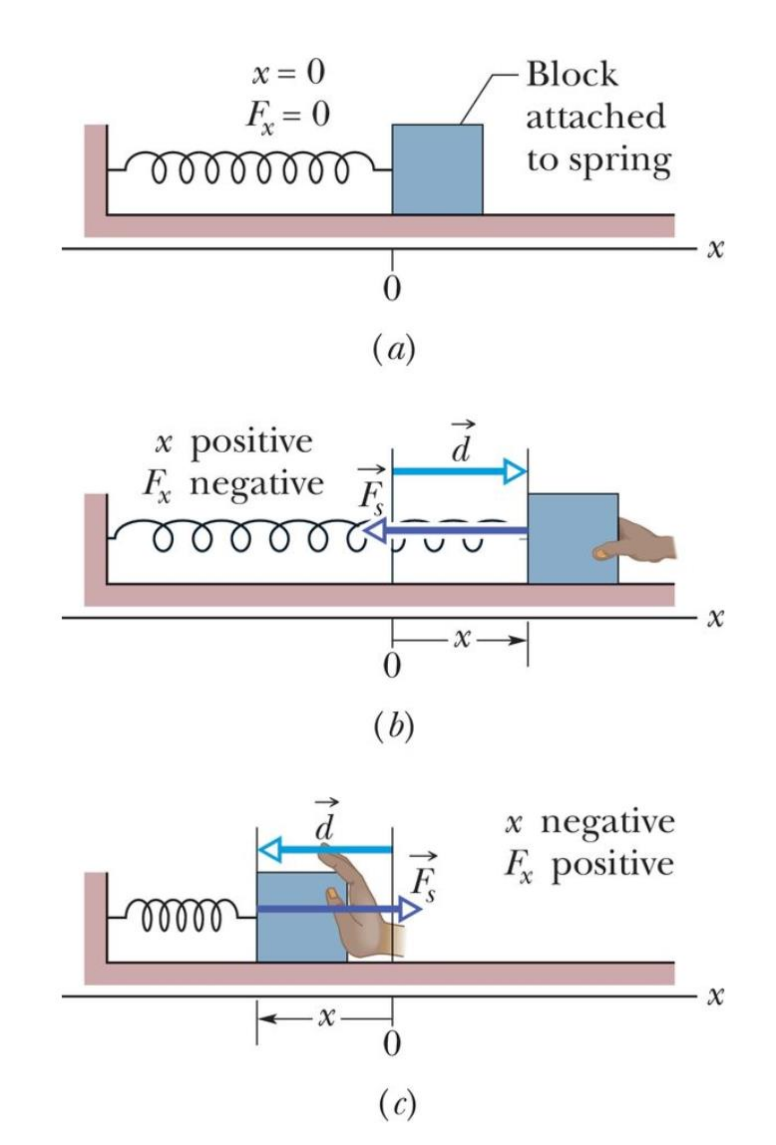
$$W_s = -(\frac{1}{2}kx_2^2 - \frac{1}{2}kx_1^2)$$

- With work-kinetic energy theorem :

$$W_s = K_2 - K_1 = -\left(\frac{1}{2}kx_2^2 - \frac{1}{2}kx_1^2\right)$$

$$K_1 + \left(\frac{1}{2}kx_1^2\right) = K_2 + \left(\frac{1}{2}kx_2^2\right)$$

A constant at any position!



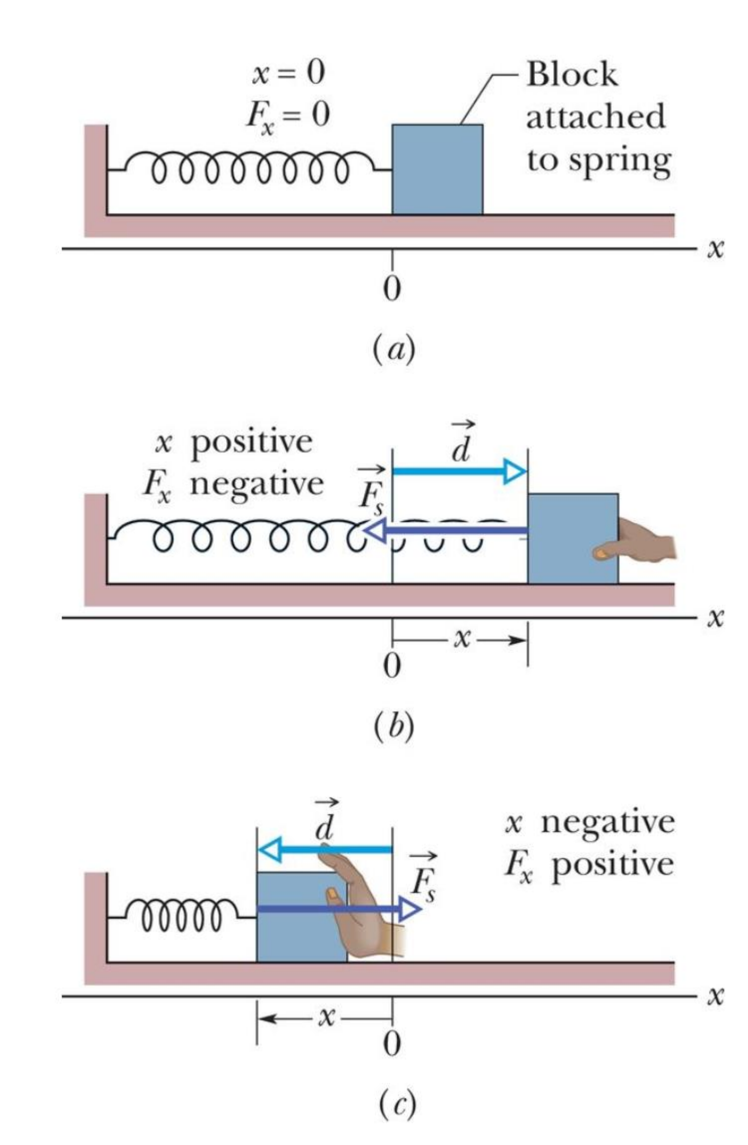
Work of external force and spring force

Now suppose that we displace the block along the x axis while continuing to apply a force $\overrightarrow{F_a}$ to it. The work done by $\overrightarrow{F_a}$ is W_a and the work done by spring for is W_s from position x_1 to x_2. Then, we will have:

$$W_a + W_s = W_a - \left(\frac{1}{2}kx_2^2 - \frac{1}{2}kx_1^2\right) = K_2 - K_1$$

$$W_a + K_1 + \left(\frac{1}{2}kx_1^2\right) = K_2 + \left(\frac{1}{2}kx_2^2\right)$$

The work done by external force add in to the constant!



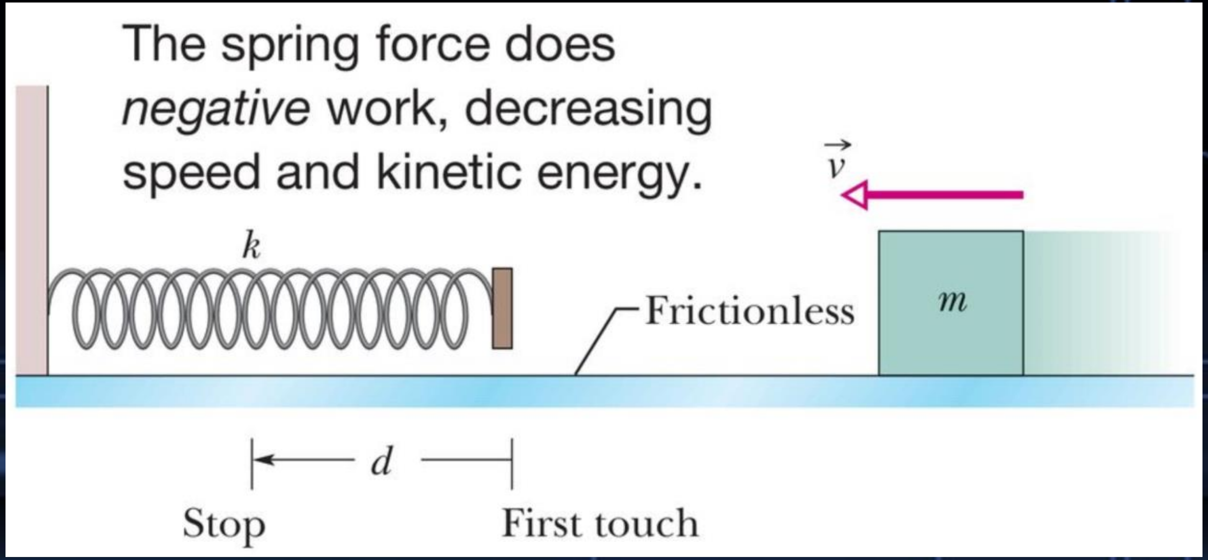
Example of work done by spring force

A mass m = 0.40 kg slides across a horizontal frictionless counter with speed v = 0.50 m/s. It then runs into and compresses a spring of spring constant k = 750 N/m. When the mass is momentarily stopped by the spring, by what distance d is the spring compressed?

$$K_1 + \left(\frac{1}{2}kx_1^2\right) = K_2 + \left(\frac{1}{2}kx_2^2\right)$$

$$\frac{1}{2}mv^2 + 0 = 0 + \left(\frac{1}{2}kx_2^2\right)$$

$$x_2 = v\sqrt{\frac{m}{k}} = 1.2cm$$



First look of potential energy

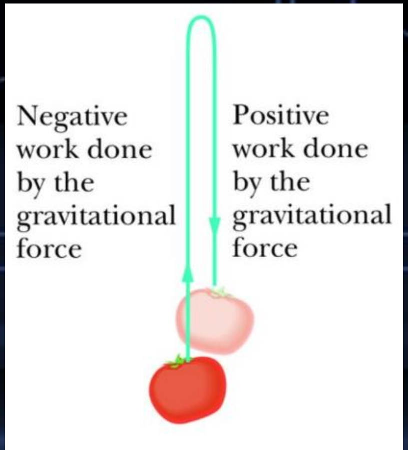
- In spring force, we found applying the work-kinetic energy theorem will lead to:

$$K_1 + \left(\frac{1}{2}kx_1^2\right) = K_2 + \left(\frac{1}{2}kx_2^2\right)$$

- Apply the same treatment to gravitational force:

$$W_g = \int_1^2 (-mg)dy = -mg(y_2 - y_1) = K_2 - K_1$$

$$K_1 + (mgy_1) = K_2 + (mgy_2)$$



First look of potential energy

- In the example of spring force and gravitational force in 1D cases, we found sum of kinetic energy and some kind of energy is a constant (conserved!).
- This is the potential energy associated with the spring and gravitational force. We can define the work done by spring and gravitational force leads to:

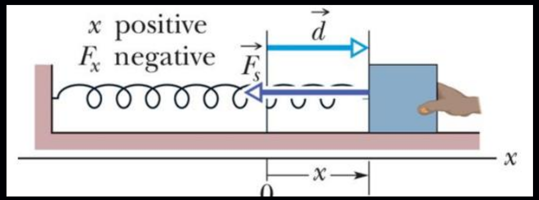
$$W = -\Delta U$$

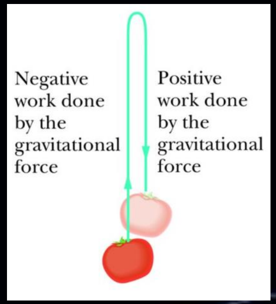
Thus, the results that we saw is conservation of energy!

Conservative and non-conservative force in 1D

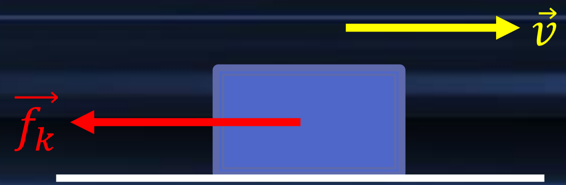
In one dimension, the force can be divided into two different categories:

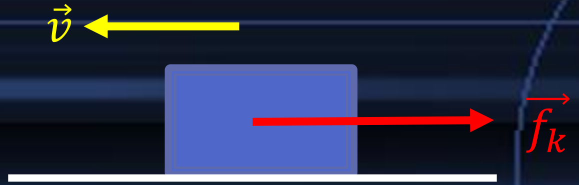
- Conservative force: the force is only depending on position. e.g. spring force, gravitational force.





- Non-conservative force: the force depending not only on position. e.g. frictional force, force exerting by hand.





Conservative and non-conservative force in 1D

In one dimension, the force can be divided into two different categories:

- Conservative force: the force is only depending on position. e.g. spring force, gravitational force.
- The work done by the conservative force is changing in potential energy with a minus sign, which only depends on initial and final position.

$$W = -\Delta U = -(U(x_2) - U(x_1))$$

Example: Van der Waals force

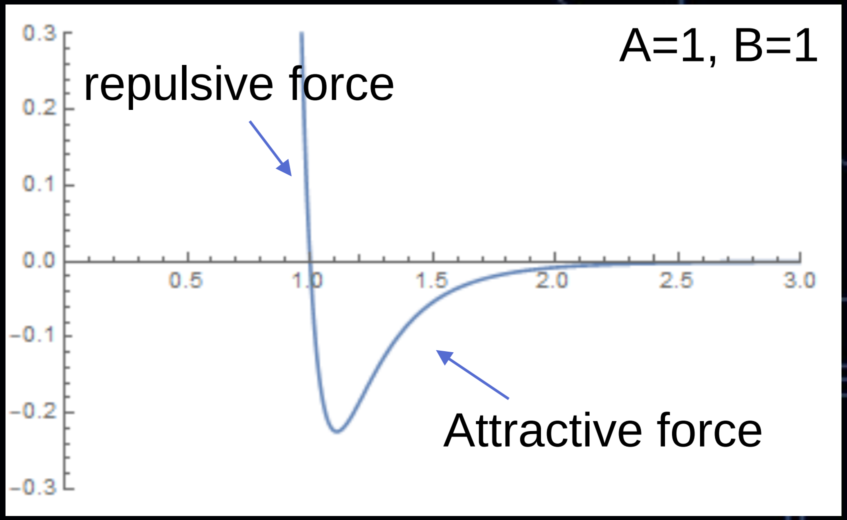
It is found that two neutral atoms have a force exerting to each other with the form:

$$F(r) = (\frac{A}{r^{13}} - \frac{B}{r^{7}})\hat{r}$$

where A and B are constant, and r is the distant between two atoms. Now one atom is fixed, you bring another atom from infinite far to a position R by an external force. In the end both atoms have relative velocity = 0. What is the work done by the external force?

Example: Van der Waals force

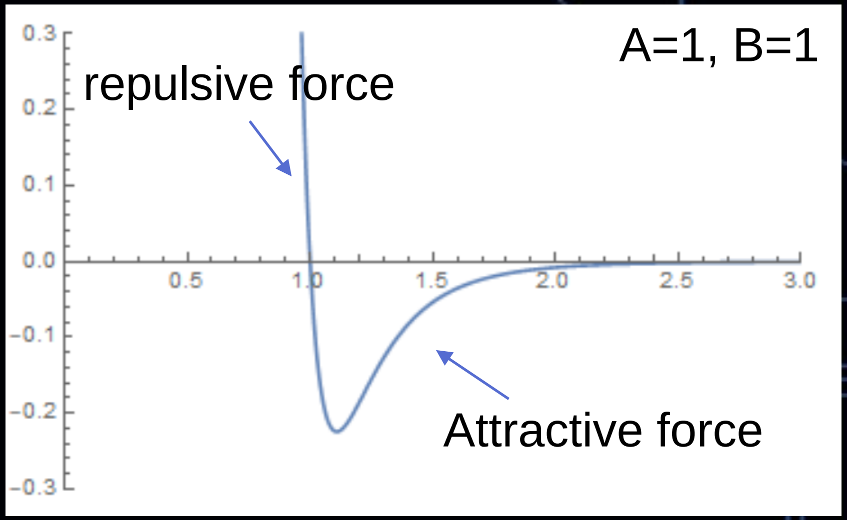
$$F(r) = (\frac{A}{r^{13}} - \frac{B}{r^7})\hat{r}$$



- First, assume we only move in 1D.
- The work done by external force is W_e, and the work done by the force between atom is W_a. Since there is no kinetic energy change, we have $W_e + W_a = 0$. Thus

$$W_e = -W_a$$

Example: Van der Waals force



$$F(r) = \left(\frac{A}{r^{13}} - \frac{B}{r^{7}}\right)\hat{r}$$

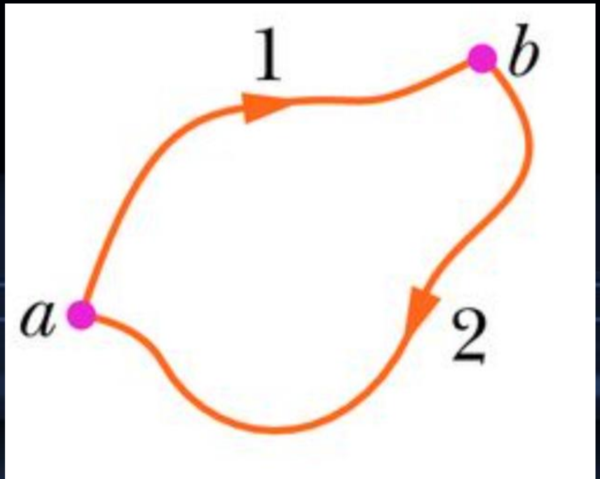
$$W_e = -W_a = -\int_{r=\infty}^{r=R} F(r)dr$$

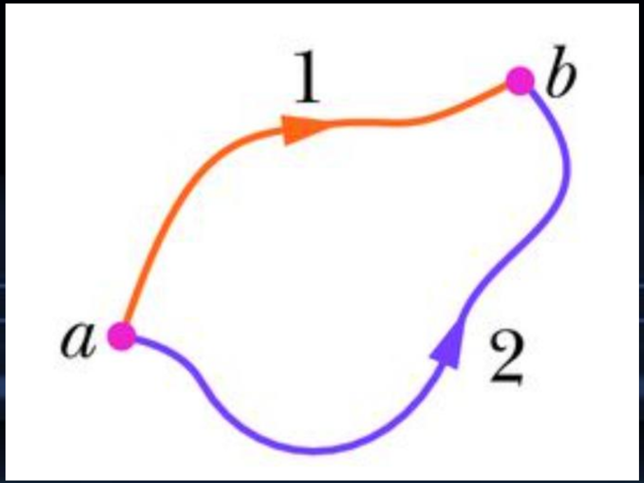
$$= -\int_{r=\infty}^{r=R} \left(\frac{A}{r^{13}} - \frac{B}{r^{7}}\right) dr = -\left(\frac{-A}{12r^{12}} + \frac{B}{6r^{6}}\right)\Bigg|_{r=\infty}^{r=R} = \left(\frac{A}{12R^{12}} - \frac{B}{6R^{6}}\right)$$

Formal definition of conservative force in 3D

- The net work done by a conservative force $\overrightarrow{F_c}$ on a particle moving around any closed path is zero.

$$\oint \overrightarrow{F_c} \cdot d\vec{r} = 0$$





The work done by $\overrightarrow{F_c}$ does not depend on path, but only the position of a and b.

Potential energy in 3D

Consider the work done by conservative force in 3D, then:

$$W_c = \int_1^2 \overrightarrow{F_c} \cdot d\vec{r} = -\Delta U = U(\overrightarrow{r_2}) - U(\overrightarrow{r_1})$$

Use Van der Waals force as example:

$$W_c = \int_1^2 \overrightarrow{F_c} \cdot d\vec{r} = \int_1^2 \left(\frac{A}{r^{13}} - \frac{B}{r^7}\right)\hat{r} \cdot d\vec{r} = \int_1^2 \left(\frac{A}{r^{13}} - \frac{B}{r^7}\right)\frac{\vec{r}}{r} \cdot d\vec{r}$$

Potential energy in 3D

$$\vec{r} \cdot d\vec{r} = \frac{1}{2} d(\vec{r} \cdot \vec{r}) = \frac{1}{2} d(r^2) = \frac{1}{2}(2rdr) = rdr$$

Thus we go back to just like 1D case:

$$-\Delta U = W_c = \int_1^2 \left(\frac{A}{r^{13}} - \frac{B}{r^7}\right) dr = \left(\frac{A}{12r^{12}} - \frac{B}{6r^6}\right)_1^2$$

With 1:$r = \infty$ and 2: $\mathrm{r} = \mathrm{R}$

$$U(R) = \left(\frac{A}{12R^{12}} - \frac{B}{6R^6}\right)$$

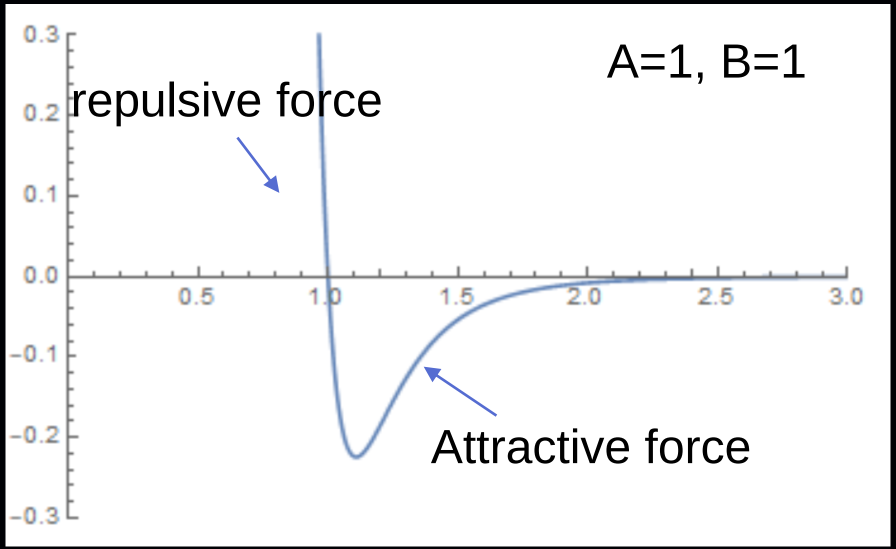
Comparison of function of force and potential

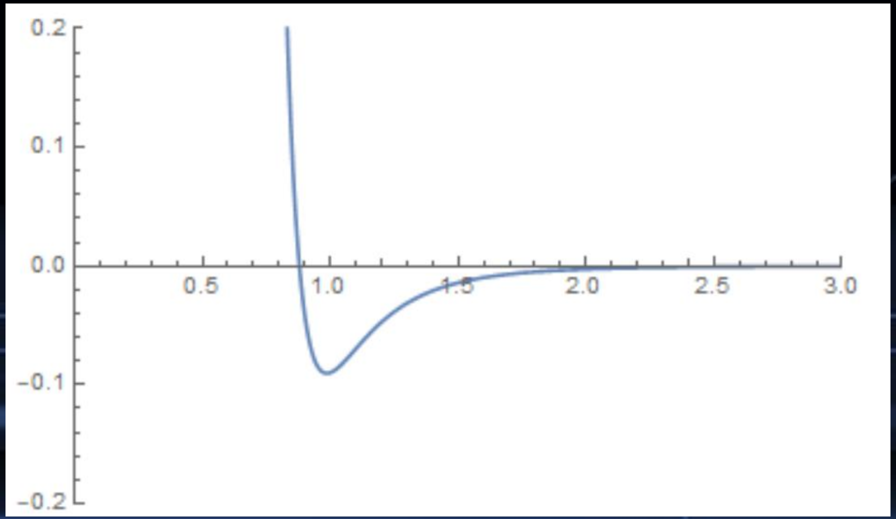
- $\overrightarrow{F(r)} = (\frac{A}{r^{13}} - \frac{B}{r^{7}})\hat{r}$

- $U(R) = \left(\frac{A}{12R^{12}} - \frac{B}{6R^{6}}\right)$

- Notice that:

$$\overrightarrow{F(r)} = -\frac{dU}{dr}\hat{r}(= -\nabla U)$$





Conservation of mechanical energy in 3D

- Considering there is only a conservative force $\vec{F_c}$ doing work W_c on an object, then we have:

$$W_c = \int_1^2 \vec{F_c} \cdot d\vec{r} = \Delta K = K_2 - K_1$$

- With the definition $W_c = -\Delta U$, we have:

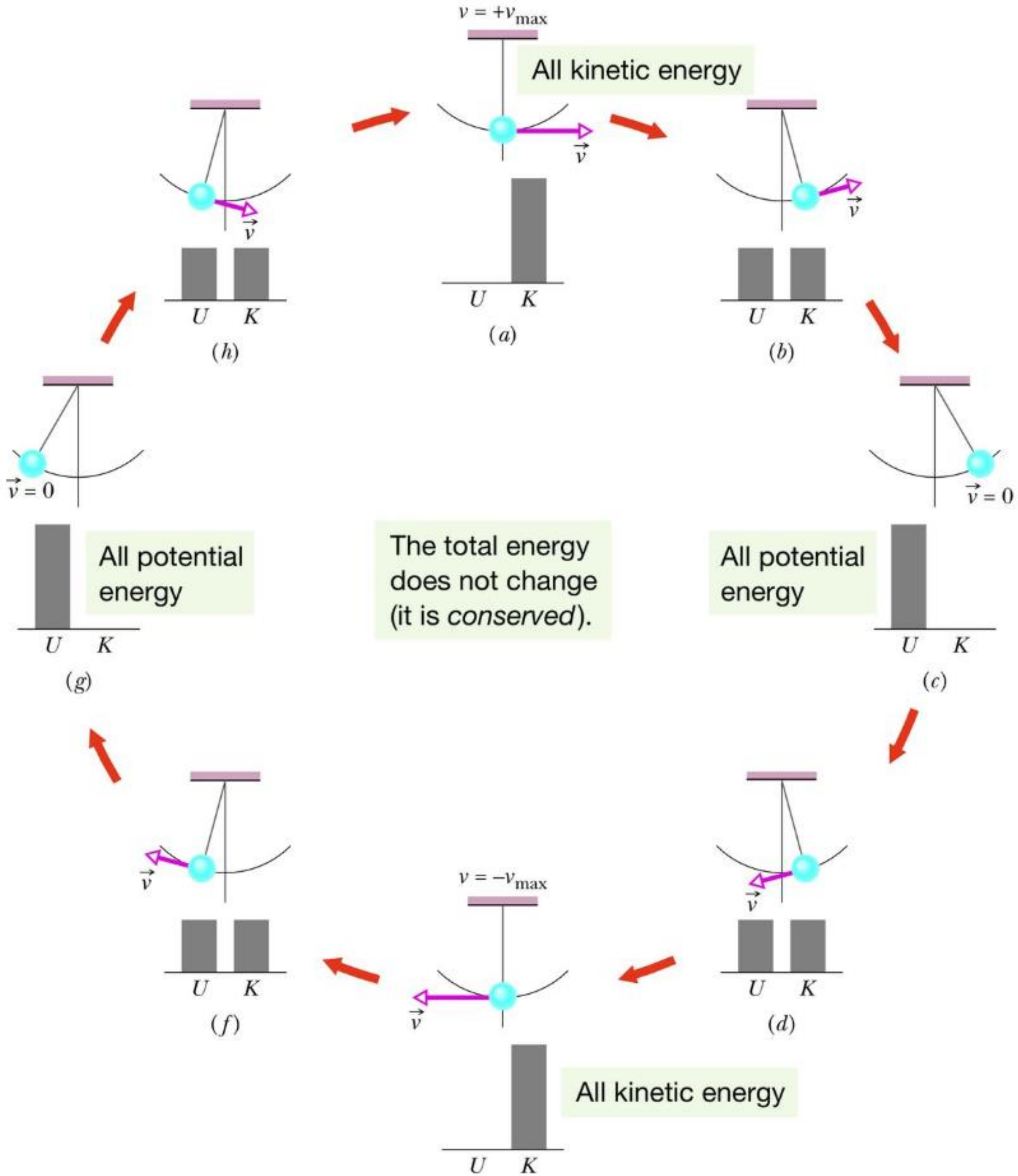
$$W_c = -\Delta U = -(U_2 - U_1) = K_2 - K_1$$

$$K_1 + U_1 = K_2 + U_2 \equiv E_{mec}$$

Conservation of mechanical energy

$$K_1 + U_1 = K_2 + U_2 = E_{mec}$$

In an isolated system where only conservative forces cause energy changes, the kinetic energy and potential energy can change, but their sum, the mechanical energy E_{mec} of the system, cannot change.



$v = +v_{max}$
All kinetic energy
$\vec{v}$
U K
(a)
$\vec{v}$
U K
(b)
$\vec{v} = 0$
All potential energy
U K
(c)
$\vec{v}$
U K
(d)
$v = -v_{max}$
$\vec{v}$
All kinetic energy
U K
$\vec{v}$
U K
(f)
$\vec{v} = 0$
All potential energy
U K
(g)
$\vec{v}$
U K
(h)
The total energy does not change (it is *conserved*).

Conservation of mechanical energy in 3D

- Considering there is only a conservative force $\vec{F_c}$ doing work W_c on an object, then we have:

$$W_c = \int_1^2 \vec{F_c} \cdot d\vec{r} = \Delta K = K_2 - K_1$$

- With the definition $W_c = -\Delta U$, we have:

$$W_c = -\Delta U = -(U_2 - U_1) = K_2 - K_1$$

$$K_1 + U_1 = K_2 + U_2 \equiv E_{mec}$$

Conservation of energy

- Now we consider the case that there are both conservative forces $\overrightarrow{F_c}$ and by non-conservative forces $\overrightarrow{F_{nc}}$ on an object. Then we have equation of motion:

$$\overrightarrow{F_c} + \overrightarrow{F_{nc}} = m\vec{a}$$

- Let's integrate equation of motion respect to displacement:

$$\int_1^2 (\overrightarrow{F_c} + \overrightarrow{F_{nc}}) \cdot d\vec{r} = \int_1^2 m\frac{d\vec{v}}{dt} \cdot d\vec{r}$$

Conservation of energy

$$\underbrace{\int_1^2 \overrightarrow{F_c} \cdot \mathrm{d}\vec{r}}_{W_c = -\Delta U} + \underbrace{\int_2^2 \overrightarrow{F_{nc}} \cdot d\vec{r}}_{W_{nc}} = \underbrace{\int_1^2 m \frac{d\vec{v}}{dt} \cdot d\vec{r}}_{\Delta K}$$

Work done by non-conservative force = change of mechanical energy

$$W_{nc} = \Delta K + \Delta U$$

This is conservation of energy

Summary

- Conservation of mechanical energy: Considering there is only a conservative force $\vec{F_c}$ doing work W_c on an object, then we have:

$$W_c = -\Delta U = -(U_2 - U_1) = K_2 - K_1$$

$$K_1 + U_1 = K_2 + U_2 \equiv E_{mec}$$

- Conservation of energy:

Work done by non-conservative force = change of mechanical energy

- $W_{nc} = \Delta K + \Delta U$