Course announcement

- Homework set 1 has been posted on eLearn. It will be due on Friday(10/7) at 5PM. Please submit your homework via eLearn. No late homework will be accepted.

4	10/4(Tue.)	**Newton's law:** Newton's third law and Using Newton's law
4	10/7(Fri.)	**Energy:** kinetic energy and work
5	10/11(Tue.)	**Energy:** potential energy and conservation of energy
5	10/14(Fri.)	**Gravity:** Law of gravity (Homework2)

GENERAL PHYSICS B1
ENERGY

Work and Kinetic Energy
2022/10/07

Concept of conservation law

- You have a box of Lego with 100 pieces in there. There are kids playing them in a room. These kids never go out of this room. After they played, how many pieces left?

https://www.blueistyleblog.com/10tipscleanupkidstoysfaster/

Concept of conservation law

- You have a box of Lego with 100 pieces in there. There are kids playing them in a room. These kids never go out of this room. After they played, how many pieces left?

100 pieces!

https://www.blueistyleblog.com/10tipscleanupkidstoysfaster/

Concept of conservation law

- The number of Lego blocks is a constant all the time! (You know when to stop finding them when you clean the room!)

https://www.blueistyleblog.com/10tipscleanupkidstoysfaster/

Concept of conservation law

- In the physical world, there are also some “numbers” or “physical quantities” of a system that don’t change when the system evolving with time. These are conservation laws.
- Studying these conservation laws is one of the fundamental key questions of physics.
- Examples of conversation law: conservation of energy, conservation of linear momentum, conservation of angular momentum…

Energy

- Energy is one of these quantities having conservation law. Energy can be transformed from one type to another and transferred from one object to another, but the total amount is always the same (energy is conserved)
- Form of energy: kinetic energy, potential energy, thermal energy,…

https://i.pinimg.com/originals/7b/e8/f7/7be8f7460be272978e0cac84710e982f.gif

Topics

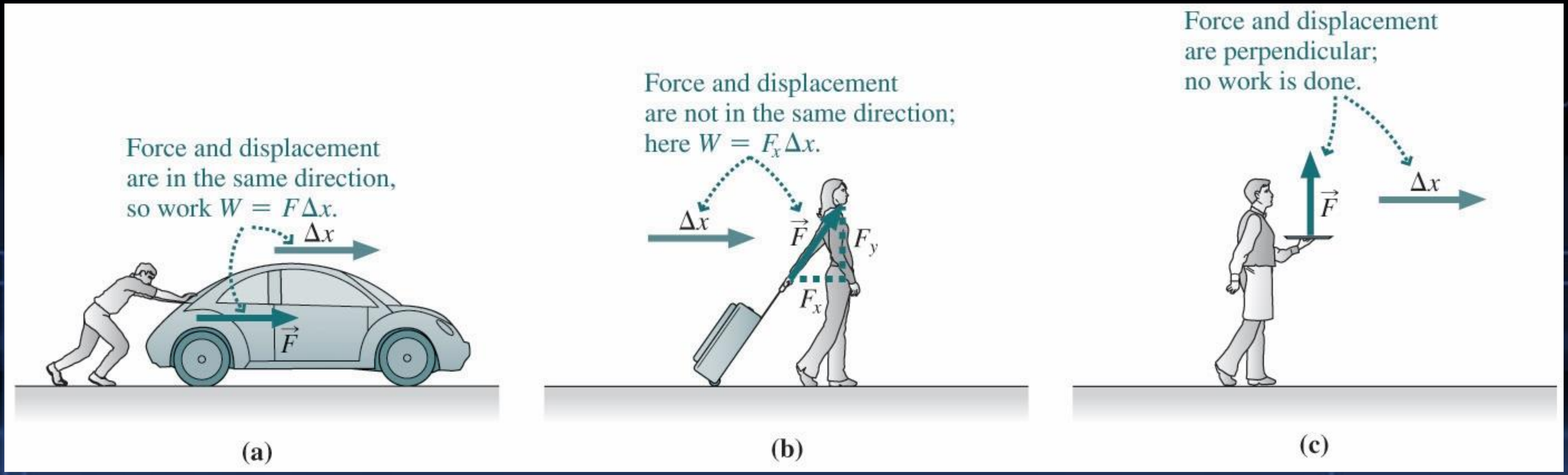
- Work and Kinetic energy
- Work-Kinetic energy theorem
- Work done by a spring force

Work

- Work W is energy transferred to or from an object by means of a force acting on the object. Energy transferred to the object is positive work, and energy transferred from the object is negative work.
- The work W done by a constant force $\vec{F}$ when its point of application undergoes a displacement $\vec{s}$ is defined to be $W = \vec{F} \cdot \vec{s}$.
- Work is a scalar and has the same unit of kinetic energy

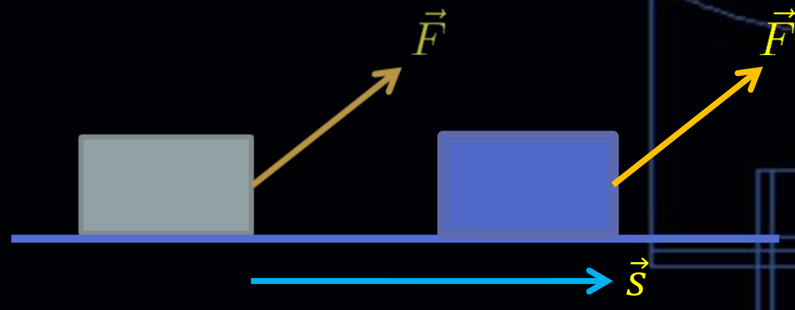
Concept of Work

- Only the component of force in the direction of the displacement contributes to work. Special cases:
  - A force applied to a stationary rigid object does no work on the object.
  - A force applied perpendicular to the displacement of an object does no work on the object.



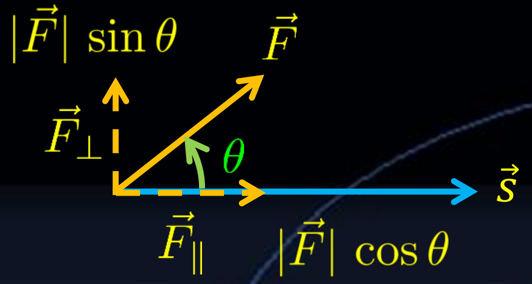
Math description of work

$$W = \vec{F} \cdot \vec{s}$$
$$= (F_x\hat{\imath} + F_y\hat{\jmath} + F_z\hat{k}) \cdot (s_x\hat{\imath} + s_y\hat{\jmath} + s_z\hat{k})$$
$$= F_x s_x + F_y s_y + F_z s_z$$



It can also be express as:

$$W = \vec{F} \cdot \vec{s} = (\overrightarrow{F_\perp} + \overrightarrow{F_\parallel}) \cdot \vec{s}$$
$$= |\vec{F}||\vec{s}|cos\theta$$

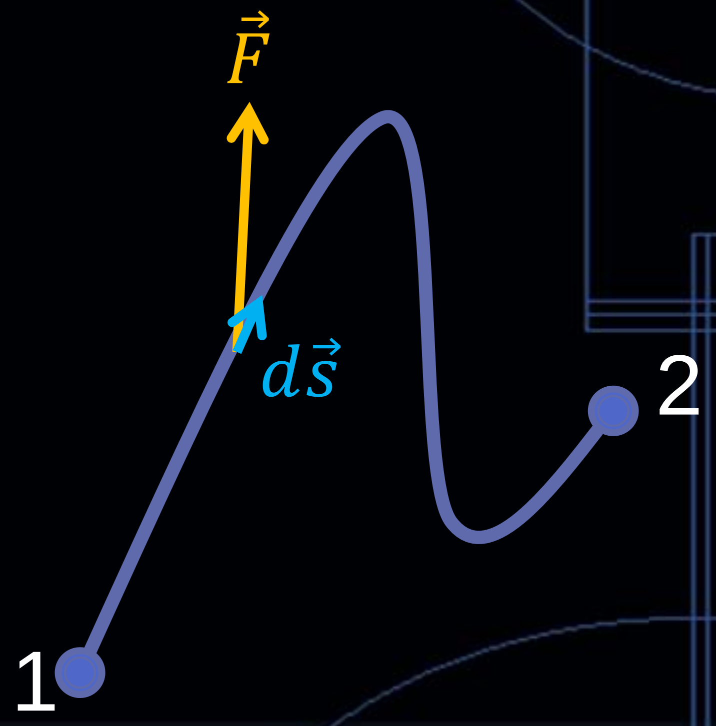


No work is done if the force and the displacement are perpendicular to each other.

Work on an object with a curve trajectory

- If the force acting on an object that moves in a curve. Then, the work done from position 1 to position two can be expressed as:

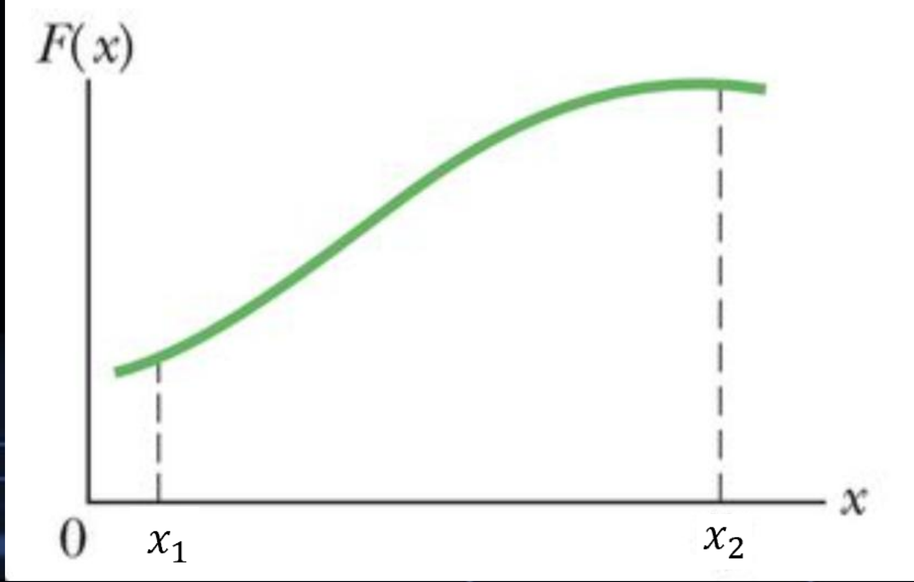
$$W = \int_{1}^{2} \vec{F} \cdot d\vec{s}$$



Work done by general force in 1D

- Consider in 1D, a force F(x) apply on an object is a function of position, as in the following plot. What is the work done by this F(x) from position x_1 to position x_2?

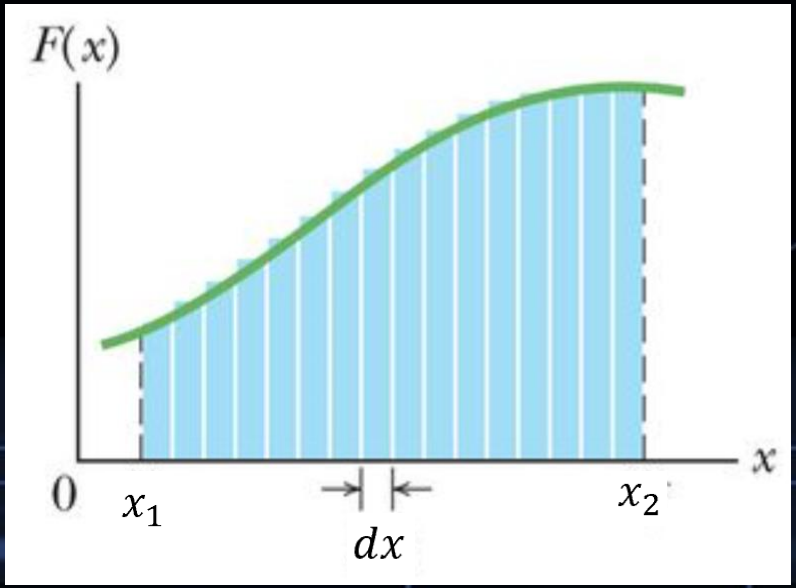
$$W_s = \int_{x=x_1}^{x=x_2} \mathrm{F(x)}dx$$



Work done by general force in 1D

- Consider in 1D, a force F(x) apply on an object is a function of position, as in the following plot. What is the work done by this F(x) from position x_1 to position x_2?

$$W_s = \int_{x=x_1}^{x=x_2} \mathrm{F(x)}dx$$



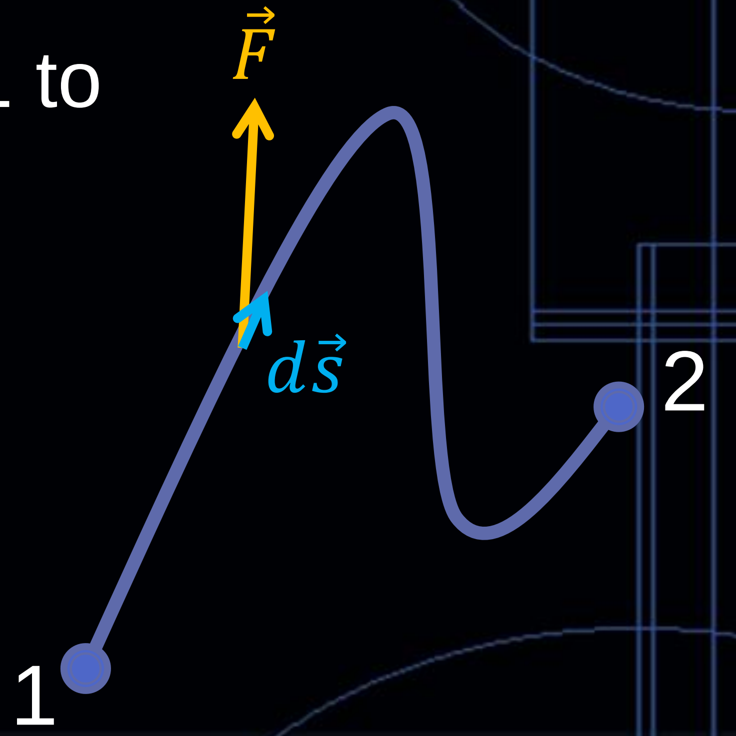
Work done by general force in 3D

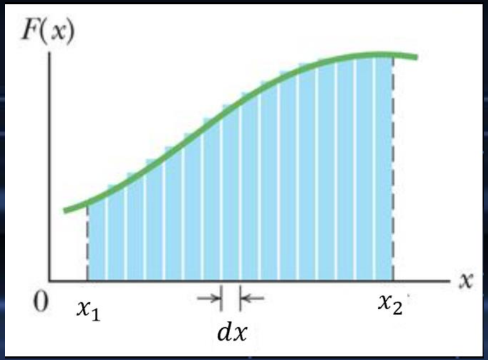
- We defined the work done from position 1 to position two can be expressed as:

$$W = \int_1^2 \vec{F} \cdot d\vec{s}$$

$$= \int_1^2 \left(F_x \hat{\imath} + F_y \hat{\jmath} + F_z \hat{k}\right) \cdot \left(dx\hat{\imath} + dy\hat{\jmath} + dz\hat{k}\right)$$

$$= \int_1^2 \left(F_x dx + F_y dy + F_z dz\right)$$





in x, y, z three different direction!

Relationship between work and kinetic energy

- Since we introduce Newton's second law and work, what is the relationship between them?

Let's start from equation of motion - Newton's second law:

$$\overrightarrow{F_{NET}} = m\vec{a} = m\frac{d\vec{v}}{dt}$$

To link the force with work, let's integral respect displacement

$$\int_1^2 \vec{F} \cdot d\vec{s} = \int_1^2 m\frac{d\vec{v}}{dt} \cdot d\vec{s}$$

Relationship between work and kinetic energy

- Since we introduce Newton's second law and work, what is the relationship between them?

The left hand side is work W done by the force from 1 to 2.

There for we have

$$W = \int_1^2 m\frac{d\vec{v}}{dt} \cdot d\vec{s} = \int_1^2 md\vec{v} \cdot \frac{d\vec{s}}{dt} = \int_1^2 m\vec{v} \cdot d\vec{v}$$

Note that: $d(\vec{v} \cdot \vec{v}) = (d\vec{v}) \cdot \vec{v} + \vec{v} \cdot (d\vec{v}) = 2\vec{v} \cdot (d\vec{v})$

Relationship between work and kinetic energy

- Since we introduce Newton's second law and work, what is the relationship between them?

Thus we have:

$$W = \int_1^2 \frac{1}{2} m d(\vec{v} \cdot \vec{v}) = \int_1^2 d(\frac{1}{2} m v^2) = K_2 - K_1$$

Kinetic energy

- Kinetic energy K is energy associated with the state of motion of an object. For an object of mass m whose speed v is well below the speed of light:

$$\mathrm{K} = \frac{1}{2}\mathrm{mv}^2$$

- The SI unit of kinetic energy (and all types of energy) is the joule (J):

$$1\mathrm{J} = 1\mathrm{kg} \cdot 1m^2/s^2$$

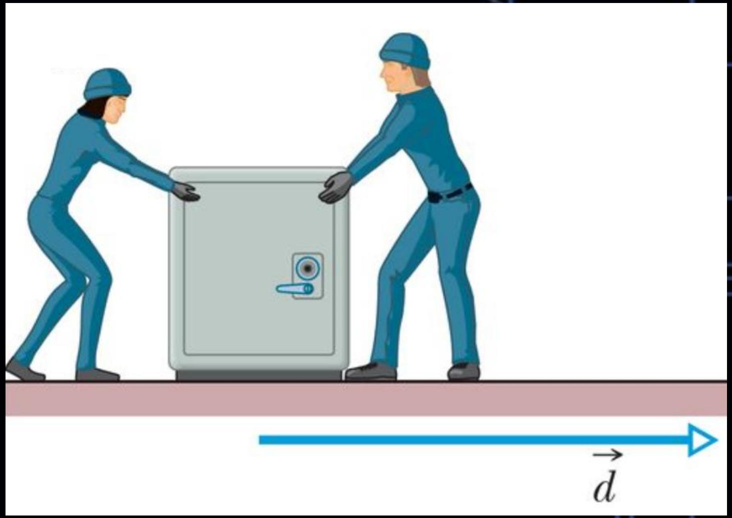
Work-kinetic energy theorem

- We integral respect to displacement on equation of motion, we got relationship between work and kinetic energy.
- Work-kinetic energy theorem: The net work done on an object equal to the change in kinetic energy.

$$\mathrm{W} = K_2 - K_1$$

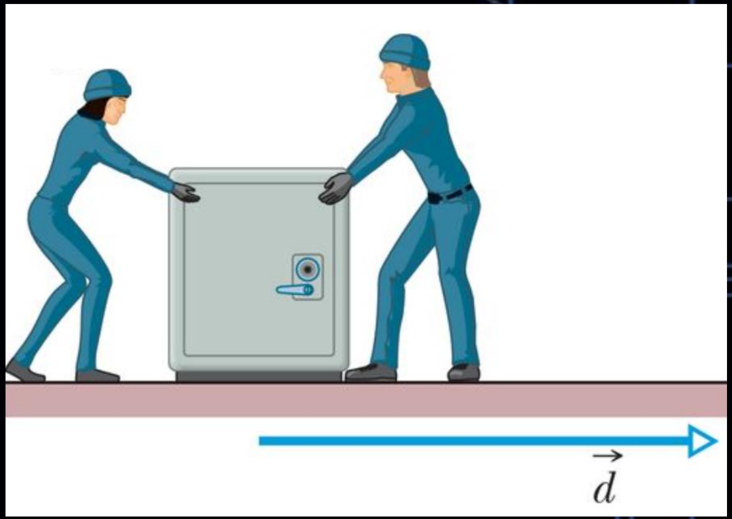
Example of work and kinetic energy

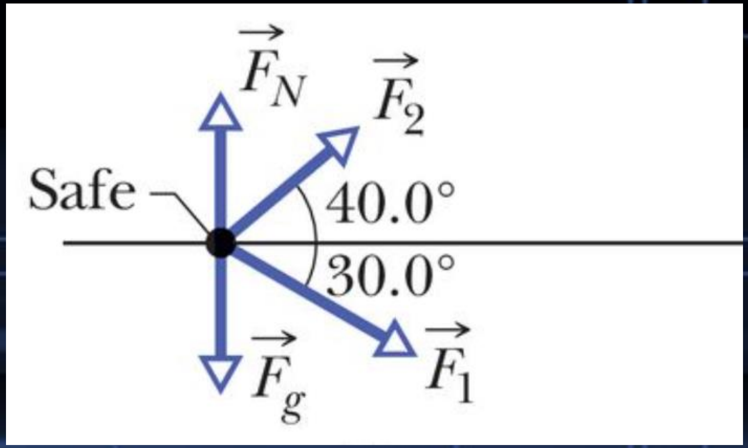
Two people (A and B) sliding an initially stationary 225 kg floor safe a displacement $\vec{d}$ of magnitude 8.50 m. The push $\overrightarrow{F_1}$ of A is 12.0 N at an angle of 30.0° downward from the horizontal; the pull $\overrightarrow{F_2}$ of B is 10.0 N at 40.0° above the horizontal. The magnitudes and directions of these forces do not change as the safe moves, and the floor and safe make frictionless contact.



Example of work and kinetic energy

(a) What is the net work done on the safe by forces $\overrightarrow{F_1}$ and $\overrightarrow{F_2}$ during the displacement ?





The work done by $\overrightarrow{F_1}$ is

$$W_1 = F_1 \cdot d \cdot cos30^{\circ} = 88.3J$$

and the work done by $\overrightarrow{F_2}$ is

$$W_2 = F_2 \cdot d \cdot cos40^{\circ} = 65.1J$$

Thus, the net work W

$$W = W_1 + W_2 = 153.4J$$

Example of work and kinetic energy

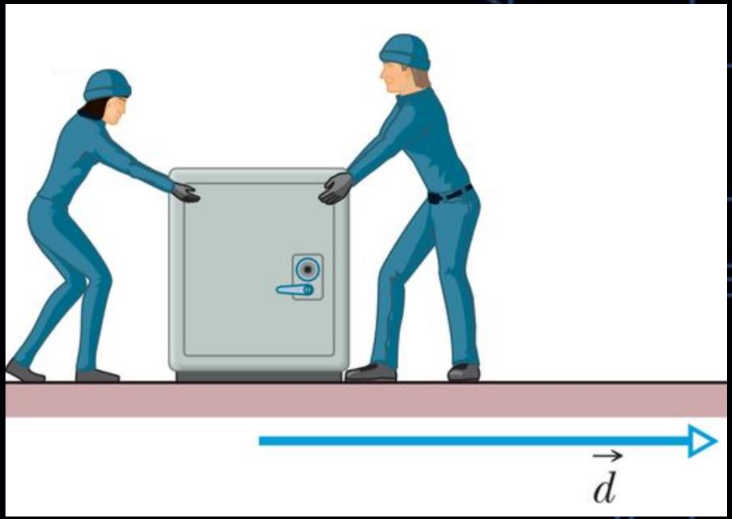
(b) During the displacement, what is the work W_g done on the safe by the gravitational force $\overrightarrow{F_g}$ and what is the work W_N done on the safe by the normal force $\overrightarrow{F_N}$ from the floor?

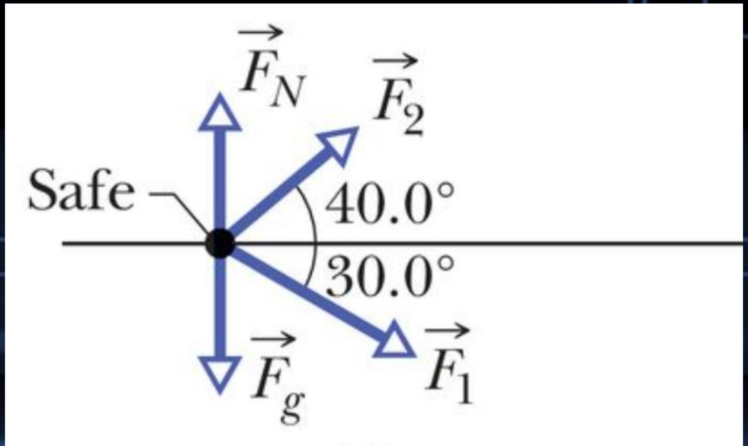
The work done by $\overrightarrow{F_g}$ is

$$W_g = F_g \cdot d \cdot cos90^{\circ} = 0J$$

and the work done by $\overrightarrow{F_N}$ is

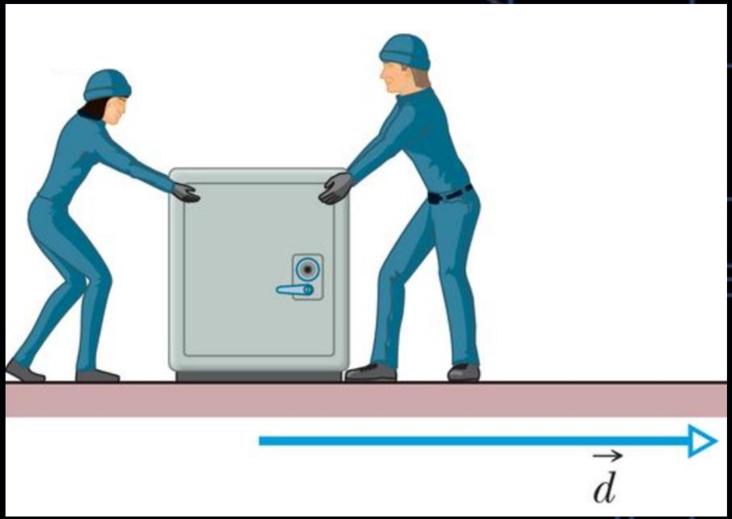
$$W_N = F_N \cdot d \cdot cos90^{\circ} = 0J$$





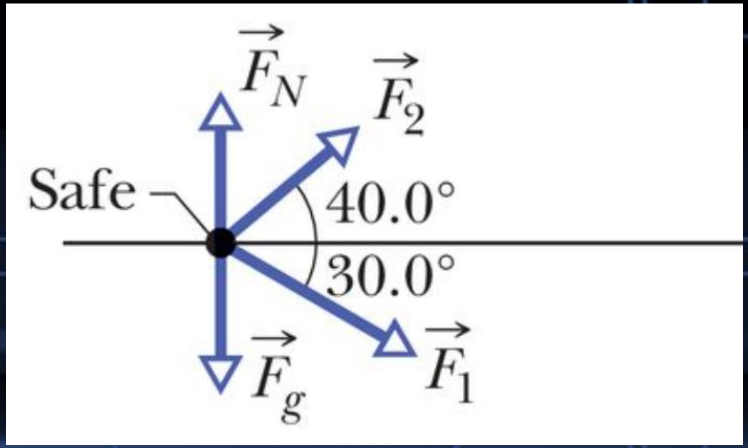
Example of work and kinetic energy

(c) The safe is initially stationary. What is its final speed v_f at the end of the 8.50 m displacement?



We relate the speed to the work done by combining the work–kinetic energy theorem.

$$W = K_f - K_i = \frac{1}{2}m{v_f}^2 - \frac{1}{2}m{v_i}^2$$



The initial speed v_i is zero, and we now know that the work done is 153.4J Thus

$$v_f = \sqrt{\frac{2W}{m}} = 1.17m/s$$

Work done by gravitational force

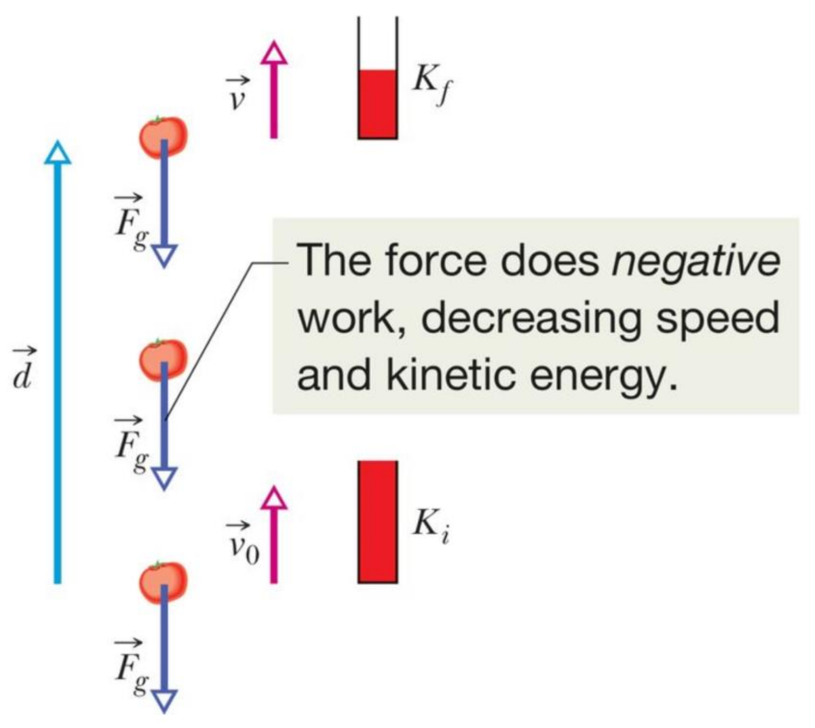
- An object has an initial velocity v_o. As the object rises, it is slowed by a gravitational force $\overrightarrow{F_g}$; that is, the object's kinetic energy decreases because $\overrightarrow{F_g}$ does negative work on the object.

During rising the work done by gravitational force is

$$W_g = F_g dcos\theta = -mgd$$

During dropping the work done by gravitational force is

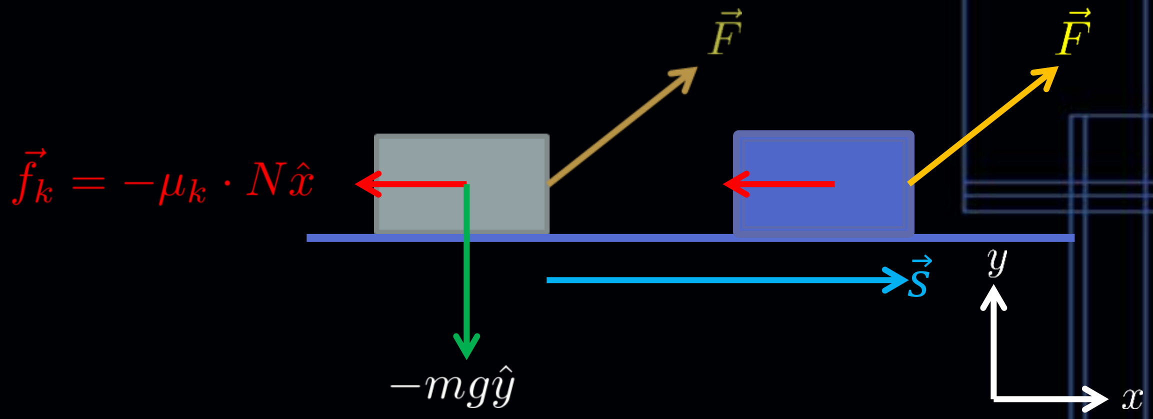
$$W_g = F_g dcos\theta = +mgd$$



Work done by frictional force

If there exists friction in the system, what is the work done by friction?

$$W_f = \overrightarrow{f_k} \cdot \vec{s} = \left|\overrightarrow{f_k}\right| |\vec{s}| cos 180^\circ$$
$$= -|\overrightarrow{f_k}||\vec{s}|$$



The work done by the friction force in this case is negative.

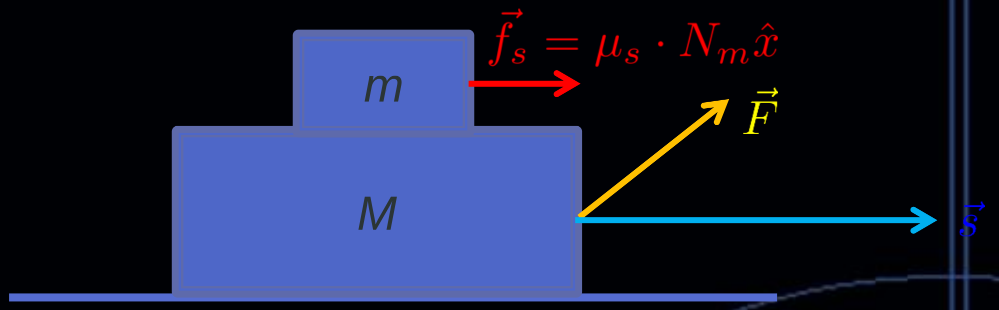
If friction is the only force, the block will move slower and slower.

Work done by frictional force

Can frictional force do positive work? If yes, how?

Let's look at mass m that stays on top of mass M

$$W_f = \overrightarrow{f_s} \cdot \vec{s} = \left|\overrightarrow{f_s}\right| |\vec{s}| cos 0^{\circ}$$
$$= |\overrightarrow{f_s}||\vec{s}|$$

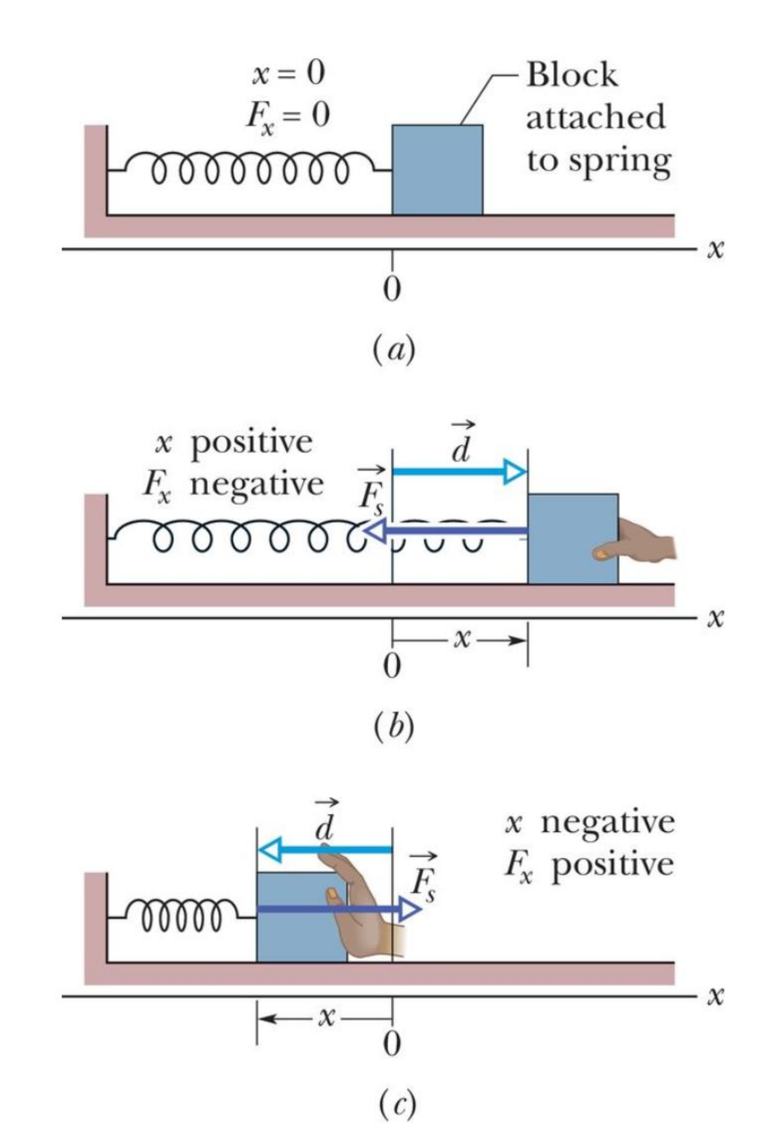


The work done by the friction force in this case is positive.

Review of spring force

Let's review spring force in 1D:

- In plot (a), a spring in its relaxed state—that is, neither compressed nor extended: $\vec{F} = F_x = 0$
- In plot (b), the spring pulls on the block toward the left: $\vec{F} = F_x = -\mathrm{kd}$
- In plot (b), the spring pushes on the block toward the right: $\vec{F} = F_x = \mathrm{kd}$
- We have: $\vec{F} = -\mathrm{k}\vec{d}$ (Hooke's law)

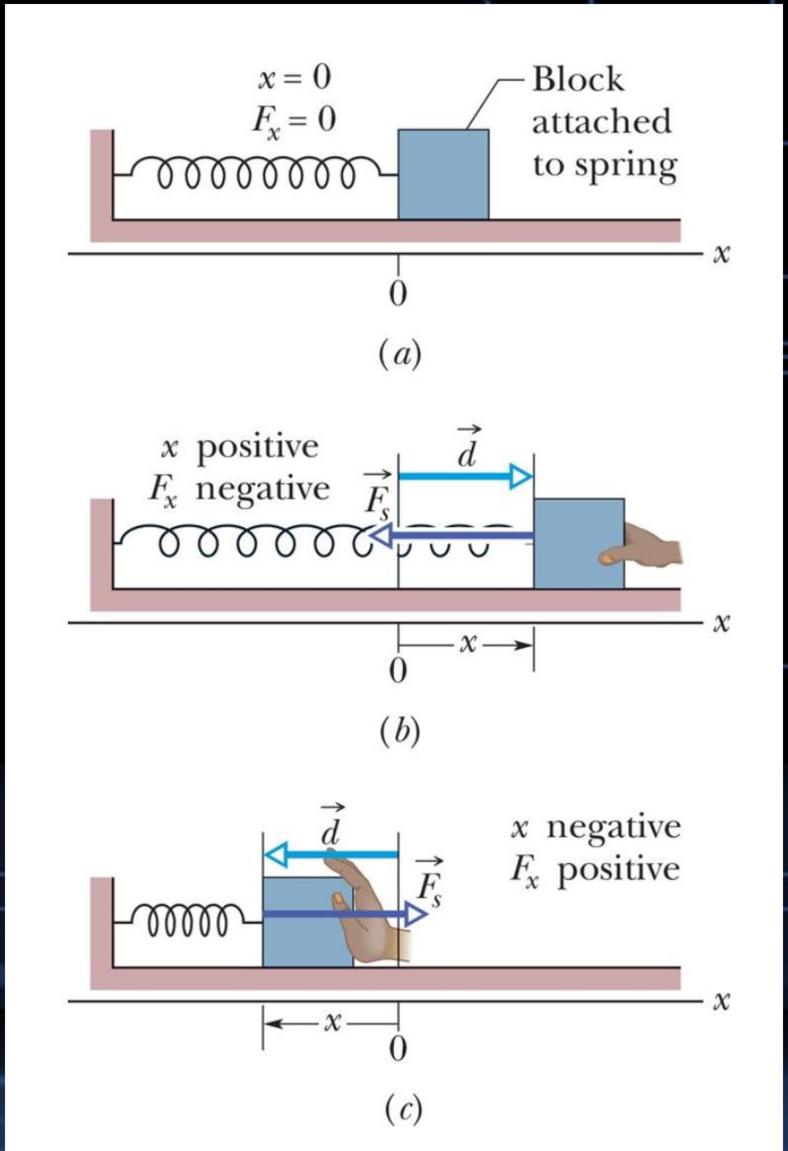


Work done by a spring force

Let's find the work done by a spring force

- Two assumption: (1) the spring is massless (2) the spring is an ideal spring, i.e., Hooke's law is valid at all positions
- Follow the definition of work:

$$W_s = \int_{x=x_1}^{x=x_2} \overrightarrow{F_s} \cdot d\vec{x}$$



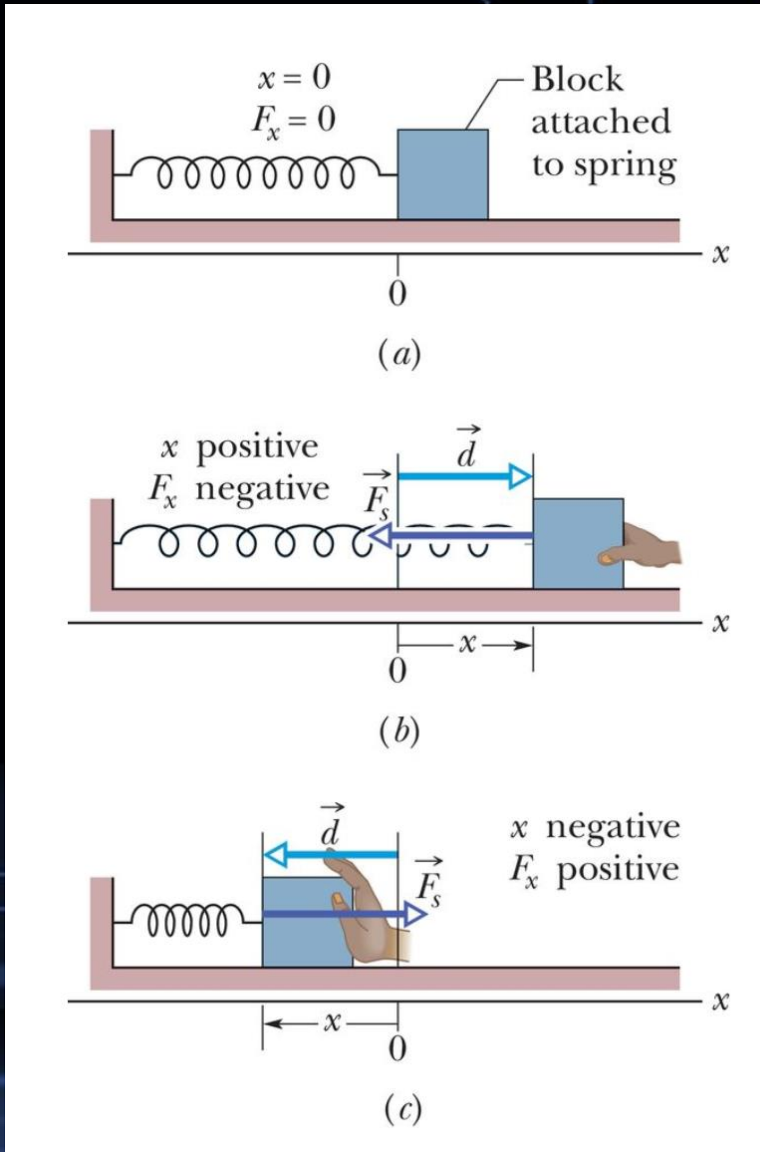
Work done by a spring force

- Following the definition of work:

$$W_s = \int_{x=x_1}^{x=x_2} \vec{F}_s \cdot d\vec{x}$$

$$= \int_{x=x_1}^{x=x_2} (-kx)dx = -k \int_{x=x_1}^{x=x_2} xdx$$

$$= -(\frac{1}{2}kx_2^2 - \frac{1}{2}kx_1^2)$$



Work-kinetic energy theorem of a spring force

- The work done by a spring force from position x_1 to x_2 :

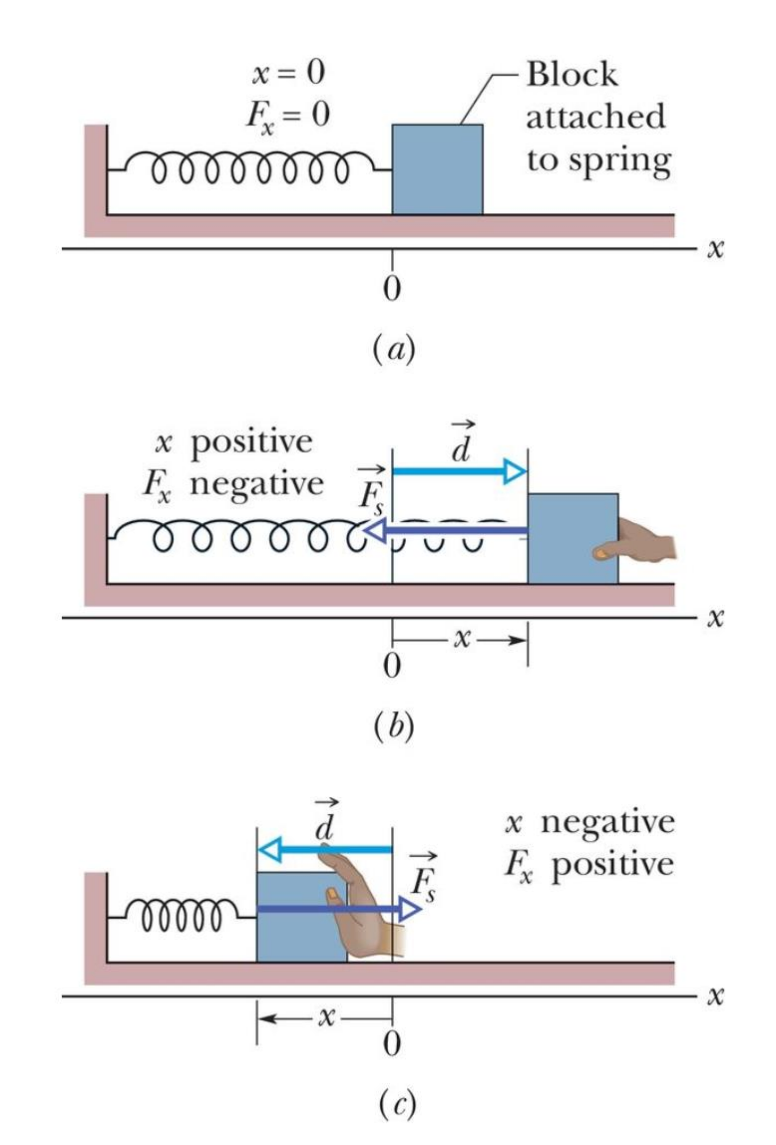
$$W_s = -(\frac{1}{2}kx_2^2 - \frac{1}{2}kx_1^2)$$

- With work-kinetic energy theorem :

$$W_s = K_2 - K_1 = -\left(\frac{1}{2}kx_2^2 - \frac{1}{2}kx_1^2\right)$$

$$K_1 + \left(\frac{1}{2}kx_1^2\right) = K_2 + \left(\frac{1}{2}kx_2^2\right)$$

A constant at any position!



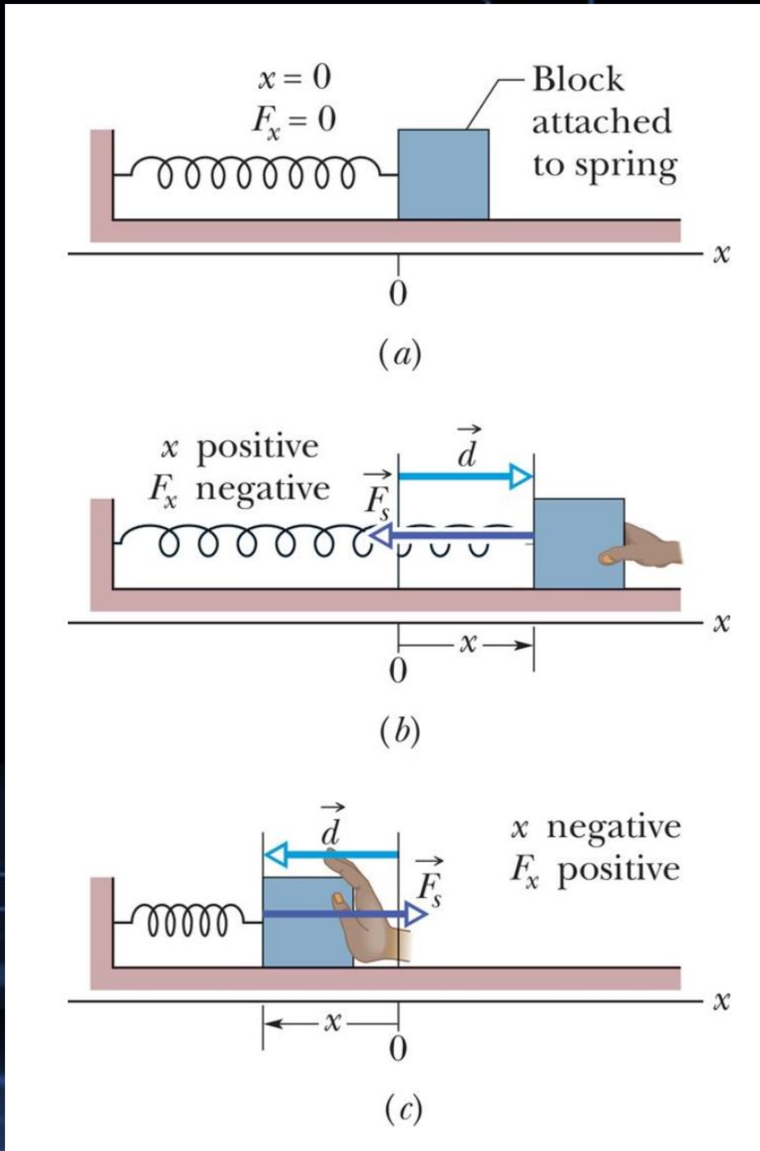
Work of external force and spring force

Now suppose that we displace the block along the x axis while continuing to apply a force $\vec{F_a}$ to it. The work done by $\vec{F_a}$ is W_a and the work done by spring for is W_s from position x_1 to x_2. Then, we will have:

$$W_a + W_s = W_a - \left(\frac{1}{2}kx_2^2 - \frac{1}{2}kx_1^2\right) = K_2 - K_1$$

$$W_a + K_1 + \left(\frac{1}{2}kx_1^2\right) = K_2 + \left(\frac{1}{2}kx_2^2\right)$$

The work done by external force add in to the constant!



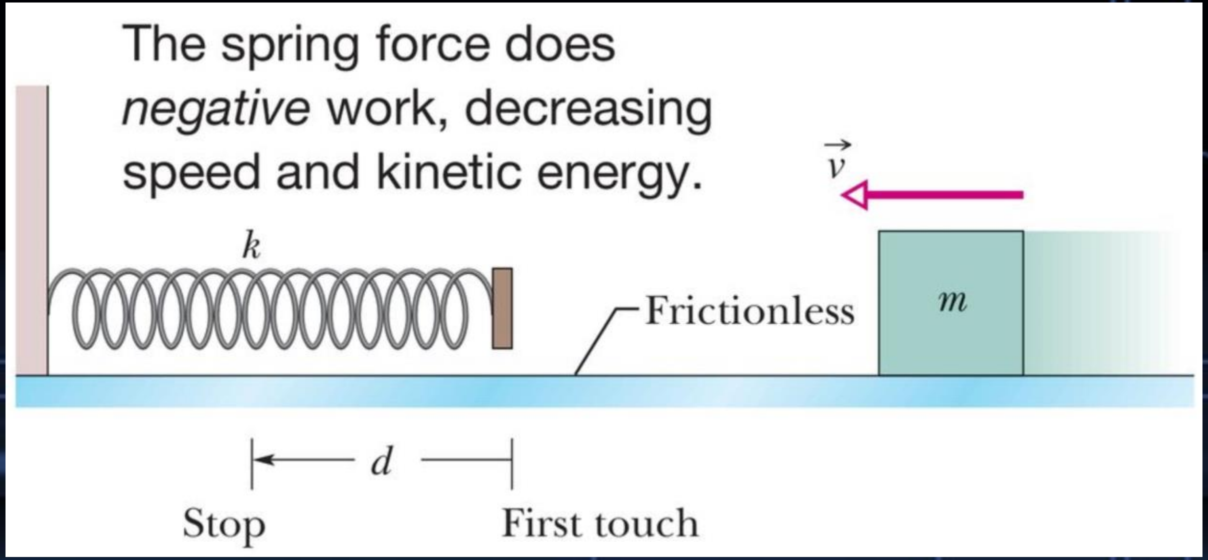
Example of work done by spring force

A mass m = 0.40 kg slides across a horizontal frictionless counter with speed v = 0.50 m/s. It then runs into and compresses a spring of spring constant k = 750 N/m. When the mass is momentarily stopped by the spring, by what distance d is the spring compressed?

$$K_1 + \left(\frac{1}{2}kx_1^2\right) = K_2 + \left(\frac{1}{2}kx_2^2\right)$$

$$\frac{1}{2}mv^2 + 0 = 0 + \left(\frac{1}{2}kx_2^2\right)$$

$$x_2 = v\sqrt{\frac{m}{k}} = 1.2cm$$



Summary

- **Work** involves force applied over distance:

-In one dimension: $W = \vec{F} \cdot \vec{s}$.

-In three dimension: $W = \int_1^2 \vec{F} \cdot d\vec{s}$

- **Work-kinetic energy theorem**: The net work done on an object equal to the change in kinetic energy.

$$\mathrm{W} = K_2 - K_1$$

Next

- Power
- Potential energy and conservative force
- Conservation of mechanical energy
- Conservation of energy