Course announcement

- The total number of enrolled people in this course reached 190 people. This is the maximum capacity of classroom. No more extra enrollment will be approved.

Week	Date	Content
1	9/13(Tue.)	**Course Information** **Fundamental Tools**: measurement & unit
1	9/16(Fri.)	**Fundamental Tool:** vector & basic calculus
2	9/20(Tue.)	**Kinetics:** motion in 1D
2	9/23(Fri.)	**Kinetics:** motion in 2D and 3D
3	9/27(Tue.)	**Newton's law:** Newton's first and second law I
3	9/30(Fri.)	**Newton's law:** Newton's first and second law II (Homework 1)

GENERAL PHYSICS 1
KINETICS

Dynamics:

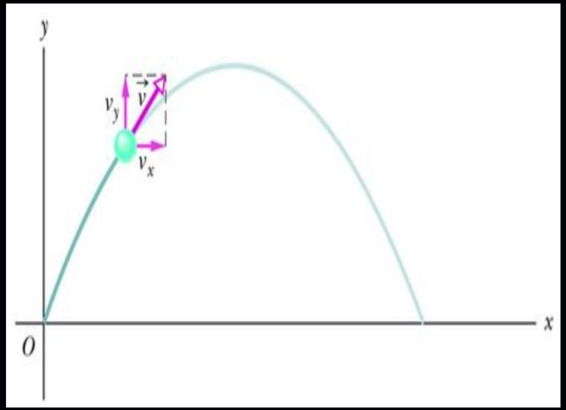
Motion in two and three dimension

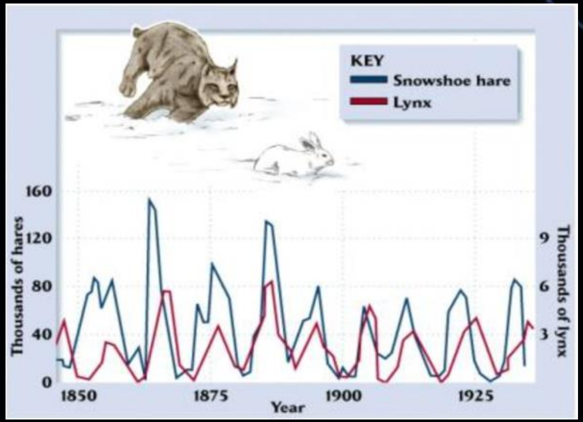
2022/09/23

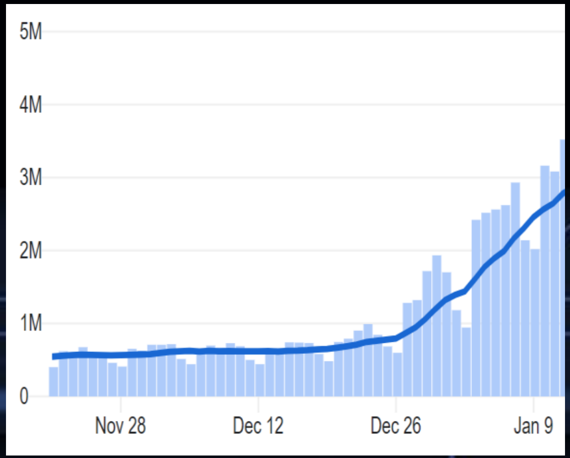
Dynamics: physical quantities evolving with time

- If the physical quantities that we are interested in are a function of time. The study of how these physical quantities change with time is called dynamics.

Examples of dynamical systems







Motion in one dimension

- Physical quantities that we are interested in to describe the motion of an object along a straight line:

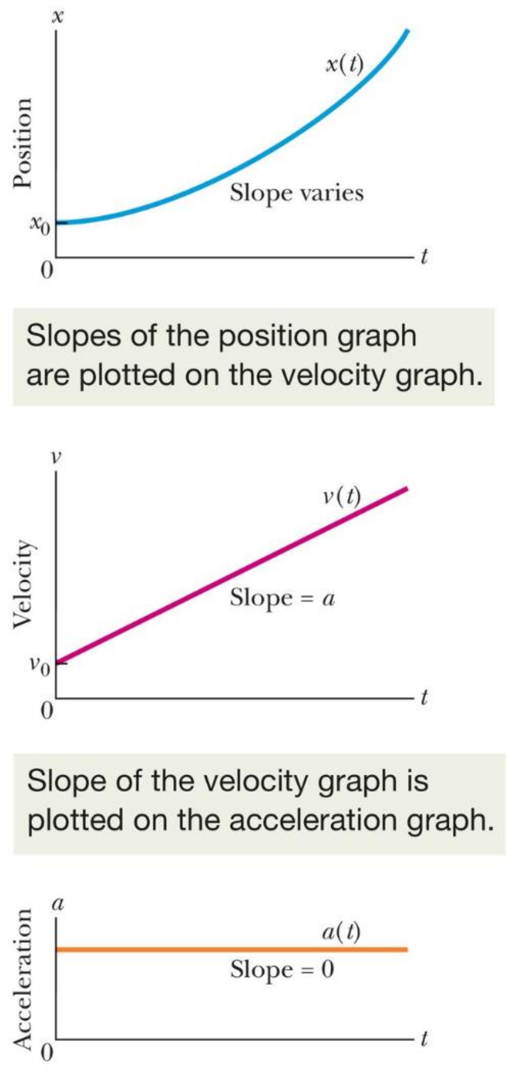
position, velocity, and acceleration.

- Position: x_1
- Displacement: $\Delta x = x_2 - x_1$ (a vector)
- Average velocity: $v_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$ (a vector)
- Average speed: $S_{\text{avg}} = \frac{\text{total distance}}{\Delta t}$ (a scalar)
- Instantaneous velocity: $v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$ (a vector)
- Average acceleration: $a_{\text{avg}} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}$ (a vector)
- Instantaneous acceleration : $a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2 x}{dt^2}$ (a vector)

Constant acceleration motion

- When acceleration is constant, we have:

Equation	Missing Quantity
$v = v_0 + at$	$x - x_0$
$x - x_0 = v_0 t + \frac{1}{2} a t^2$	v
$v^2 = v_0^2 + 2a(x - x_0)$	t
$x - x_0 = \frac{1}{2}(v_0 + v)t$	a
$x - x_0 = vt - \frac{1}{2} a t^2$	v_0



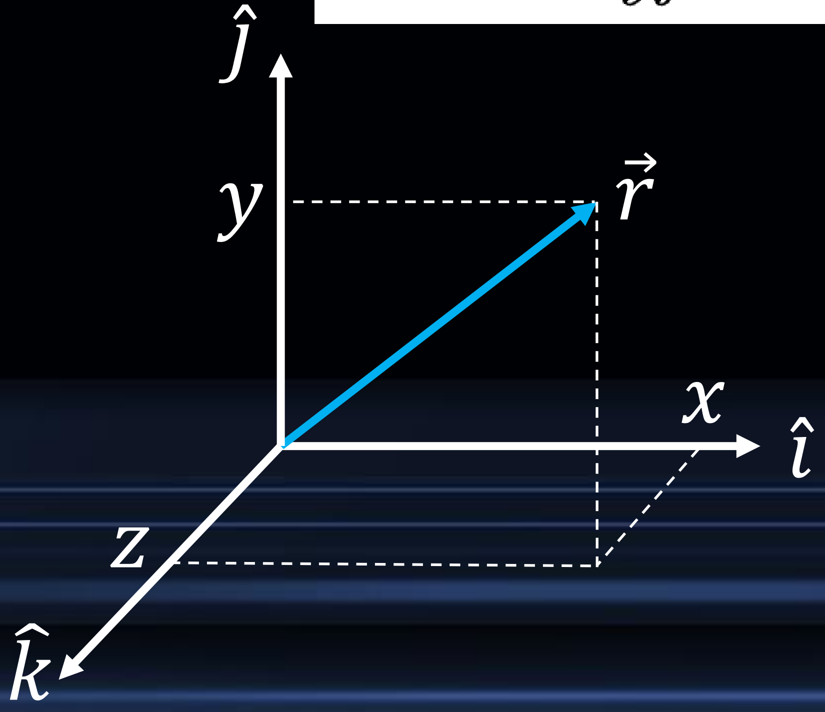
Motion in two and three dimension

- Position, velocity, and acceleration in 2D and 3D
- Relative motion
- Projectile motion
- Circular motion

Position in two and three dimension

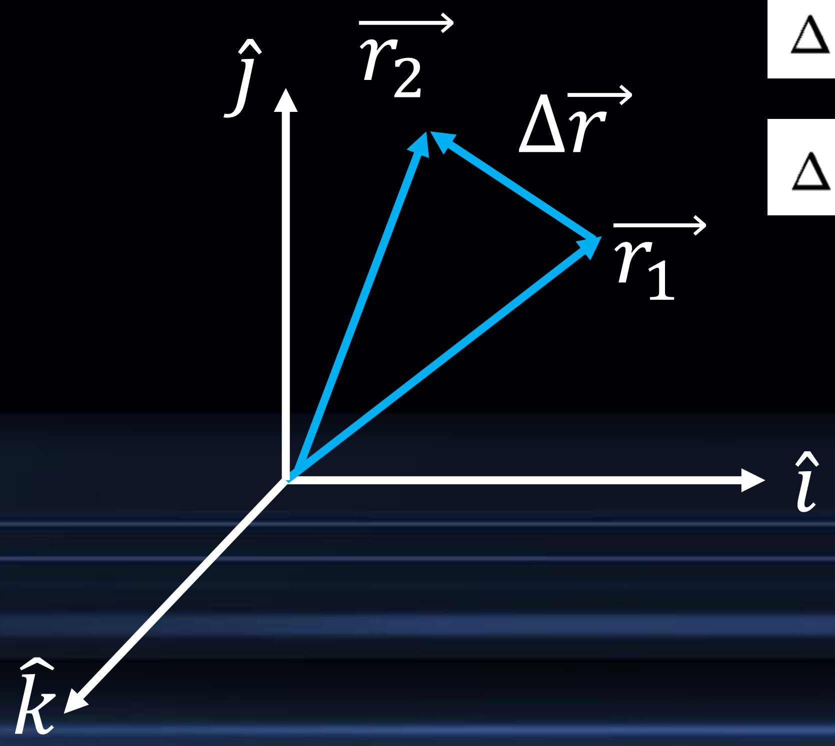
- Position: a vector that extends from a reference point (usually the origin) to the point of interest.

$$\vec{r} = x\hat{\mathrm{i}} + y\hat{\mathrm{j}} + z\hat{\mathrm{k}}$$



Position in two and three dimension

- Displacement: a vector represents the change of position during a certain time interval.



$$\Delta\overrightarrow{r} = \overrightarrow{r}_2 - \overrightarrow{r}_1$$

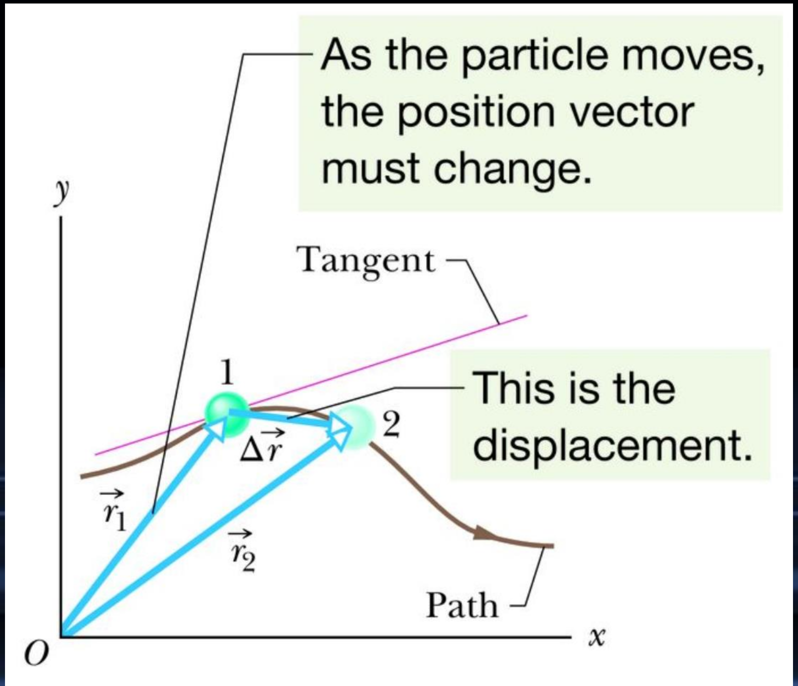
$$\Delta\overrightarrow{r} = (x_2 - x_1)\,\hat{\mathrm{i}} + (y_2 - y_1)\,\hat{\mathrm{j}} + (z_2 - z_1)\,\widehat{\mathrm{k}}$$

Average velocity and instantaneous velocity

- Average velocity

$$\vec{v}_{avg} = \frac{\Delta \vec{r}}{\Delta t}$$

$$\vec{v}_{avg} = \frac{\Delta x\hat{i} + \Delta y\hat{j} + \Delta z\hat{k}}{\Delta t} = \frac{\Delta x}{\Delta t}\hat{i} + \frac{\Delta y}{\Delta t}\hat{j} + \frac{\Delta z}{\Delta t}\hat{k}.$$

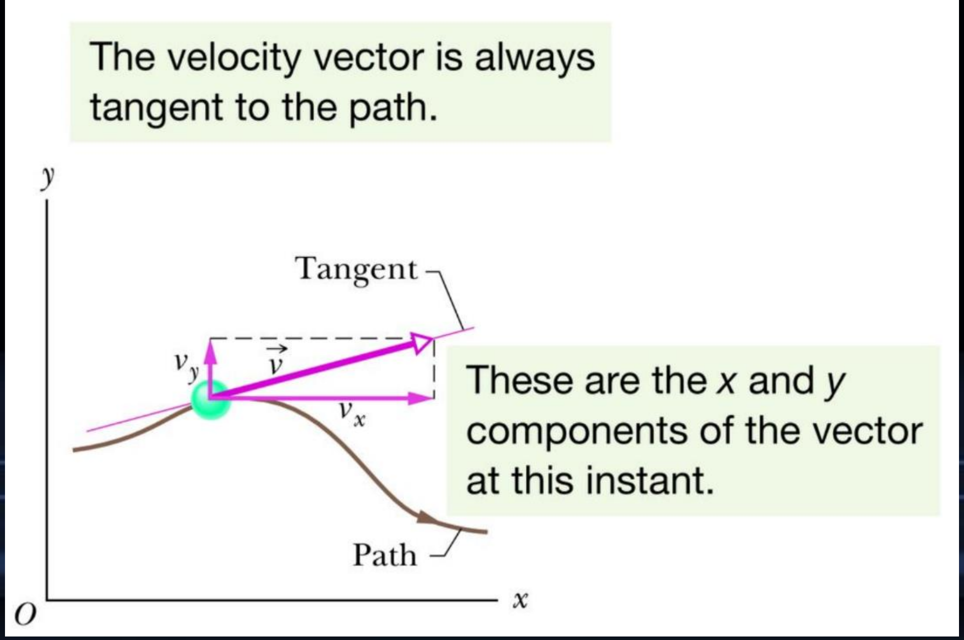


Average velocity and instantaneous velocity

- Instantaneous velocity

$$\vec{v} = \frac{d\vec{r}}{dt}$$

$$\vec{v} = \frac{d}{dt}\left(x\hat{\imath} + y\hat{\jmath} + z\hat{k}\right) = \frac{dx}{dt}\hat{\imath} + \frac{dx}{dt}\hat{\jmath} + \frac{dz}{dt}\hat{k}$$



Average acceleration and instantaneous acceleration

- Average acceleration

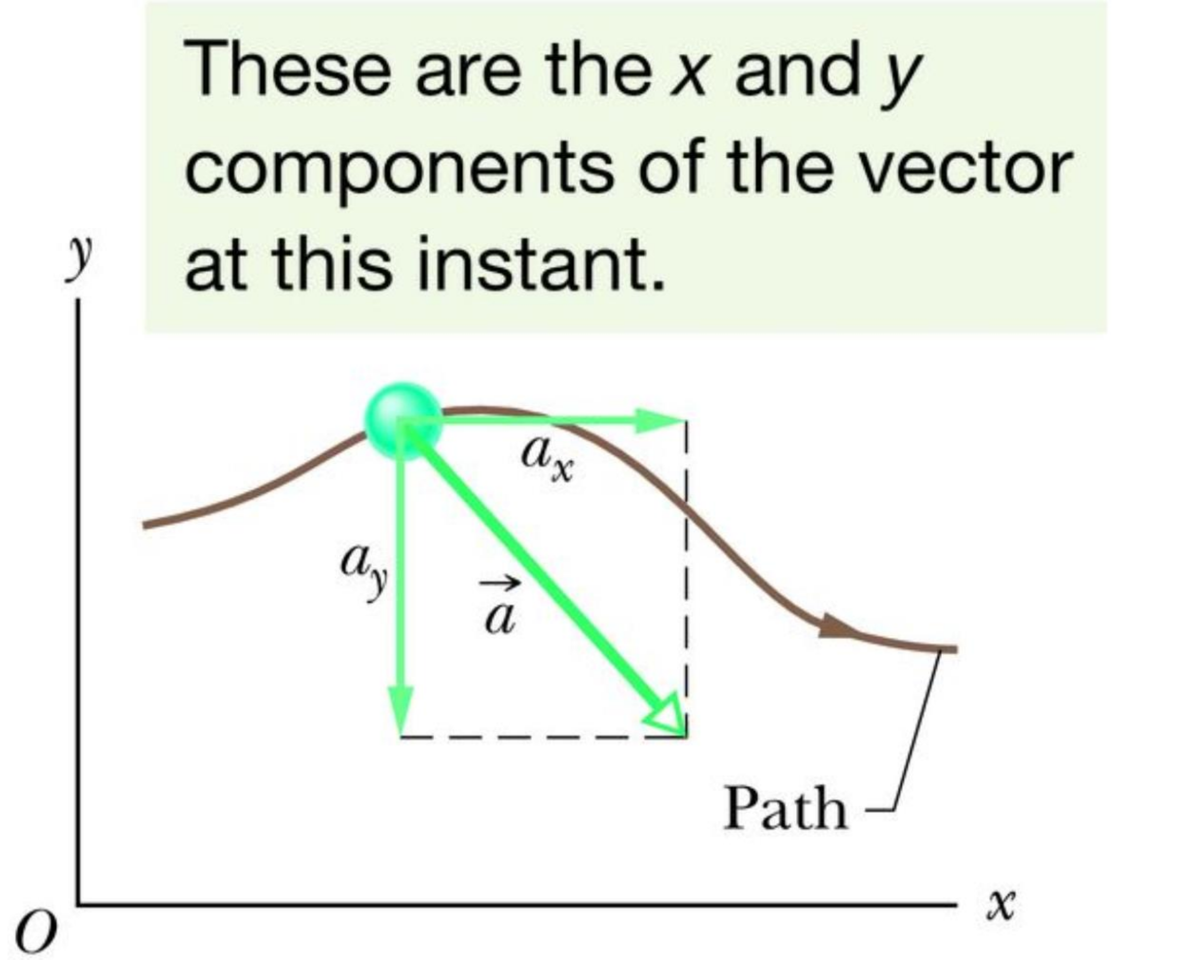
$$\vec{a}_{\text{avg}} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t} = \frac{\Delta \vec{v}}{\Delta t}$$

- Instantaneous acceleration

$$\vec{a} = \frac{d\vec{v}}{dt}$$

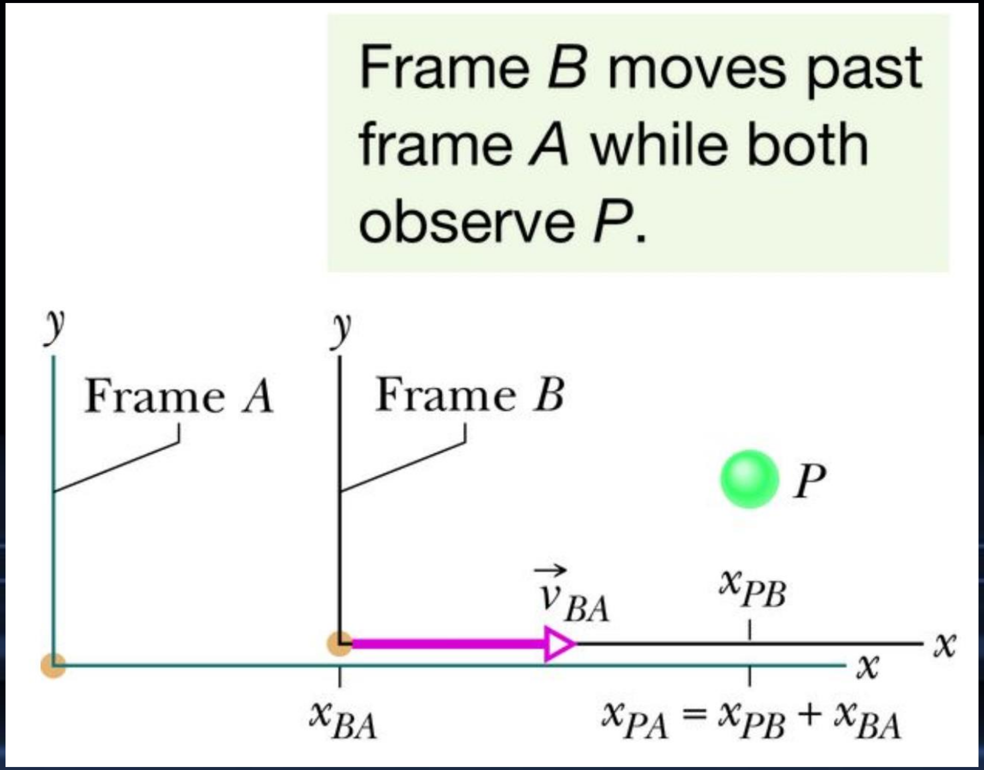
$$\begin{aligned}\vec{a} &= \frac{d}{dt}\left(v_x \hat{\mathrm{i}} + v_y \hat{\mathrm{j}} + v_z \hat{\mathrm{k}}\right) \\ &= \frac{dv_x}{dt}\hat{\mathrm{i}} + \frac{dv_y}{dt}\hat{\mathrm{j}} + \frac{dv_z}{dt}\hat{\mathrm{k}}.\end{aligned}$$

Average acceleration and instantaneous acceleration



Relative motion in one dimension

- If the observer have a relative constant velocity to the observed object, then:



Relative motion in one dimension (2)

- We can find that:

$$x_{PA} = x_{PB} + x_{BA}$$

- Thus, the conversion of velocity between frames:

$$\frac{d}{dt}(x_{PA}) = \frac{d}{dt}(x_{PB}) + \frac{d}{dt}(x_{BA})$$

$$v_{PA} = v_{PB} + v_{BA}$$

Relative motion in one dimension (3)

- For the acceleration:

$$\frac{d}{dt}(v_{PA}) = \frac{d}{dt}(v_{PB}) + \frac{d}{dt}(v_{BA})$$

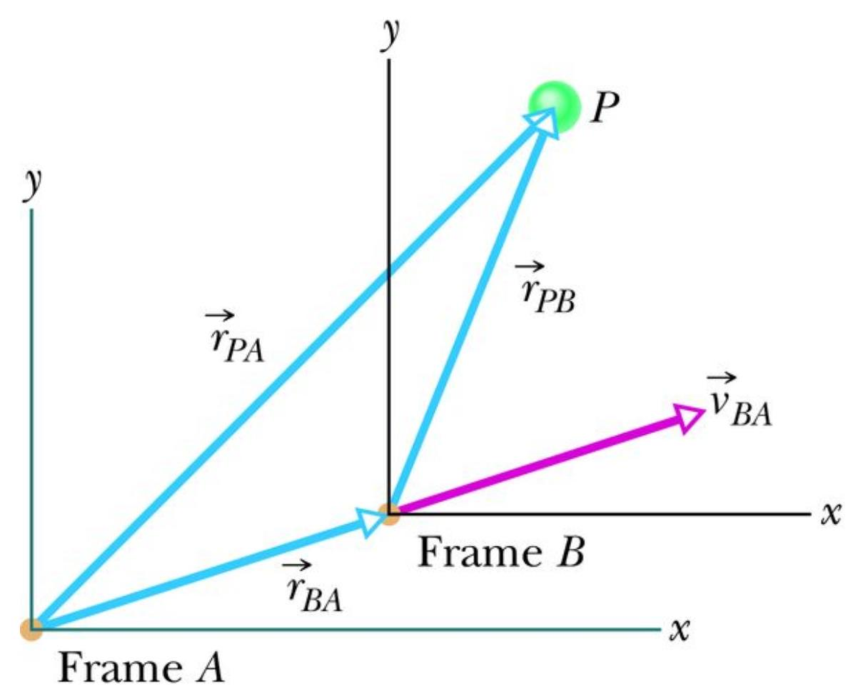
- Frame B moves with constant velocity, we have:

$$a_{PA} = a_{PB}$$

- The velocity changes on different frames but acceleration is a constant if the frame moves with constant velocity.

Relative motion in two or three dimension

- The relative motion can be generalized in two or even three dimension:



$$\vec{r}_{PA} = \vec{r}_{PB} + \vec{r}_{BA}$$

$$\vec{v}_{PA} = \vec{v}_{PB} + \vec{v}_{BA}$$

$$\vec{a}_{PA} = \vec{a}_{PB}$$

Example of Relative Motion

- An airplane flies at 960 km/h relative to the air in a wind blowing eastward at 190 km/h. In what direction should the plane point to track exactly northward?

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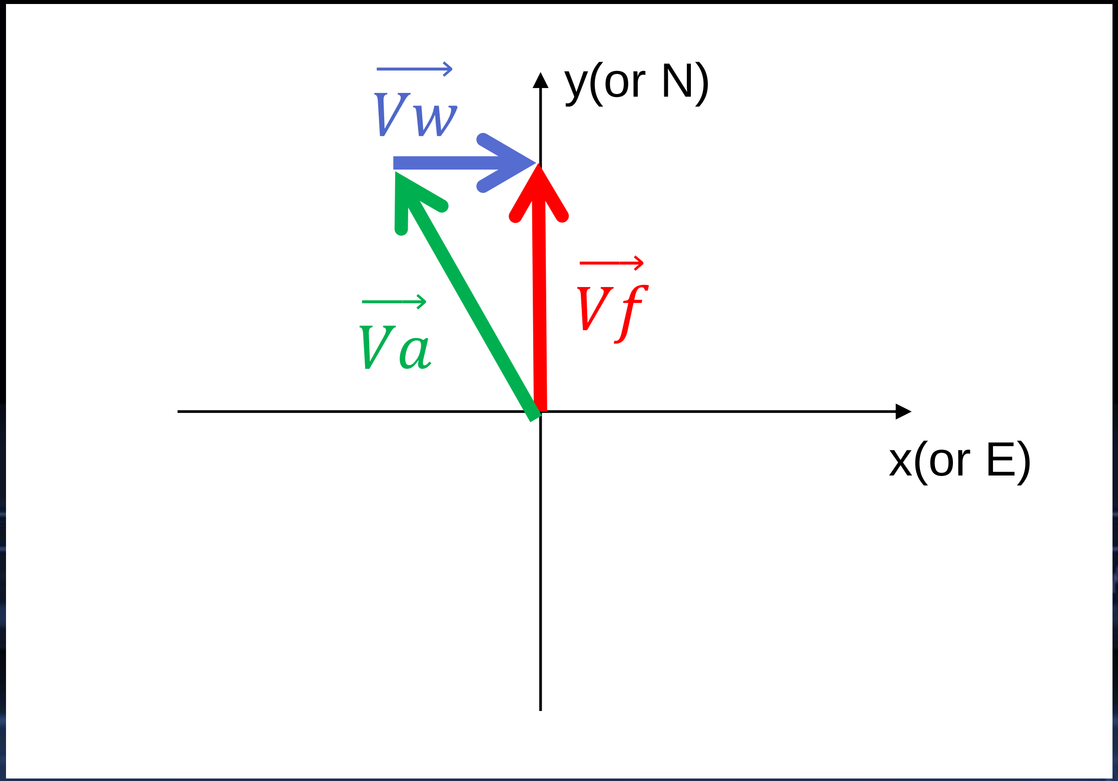

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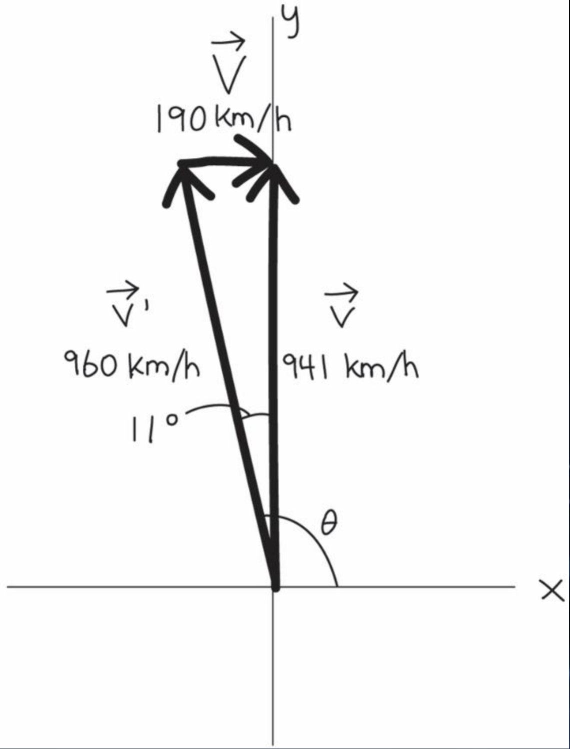
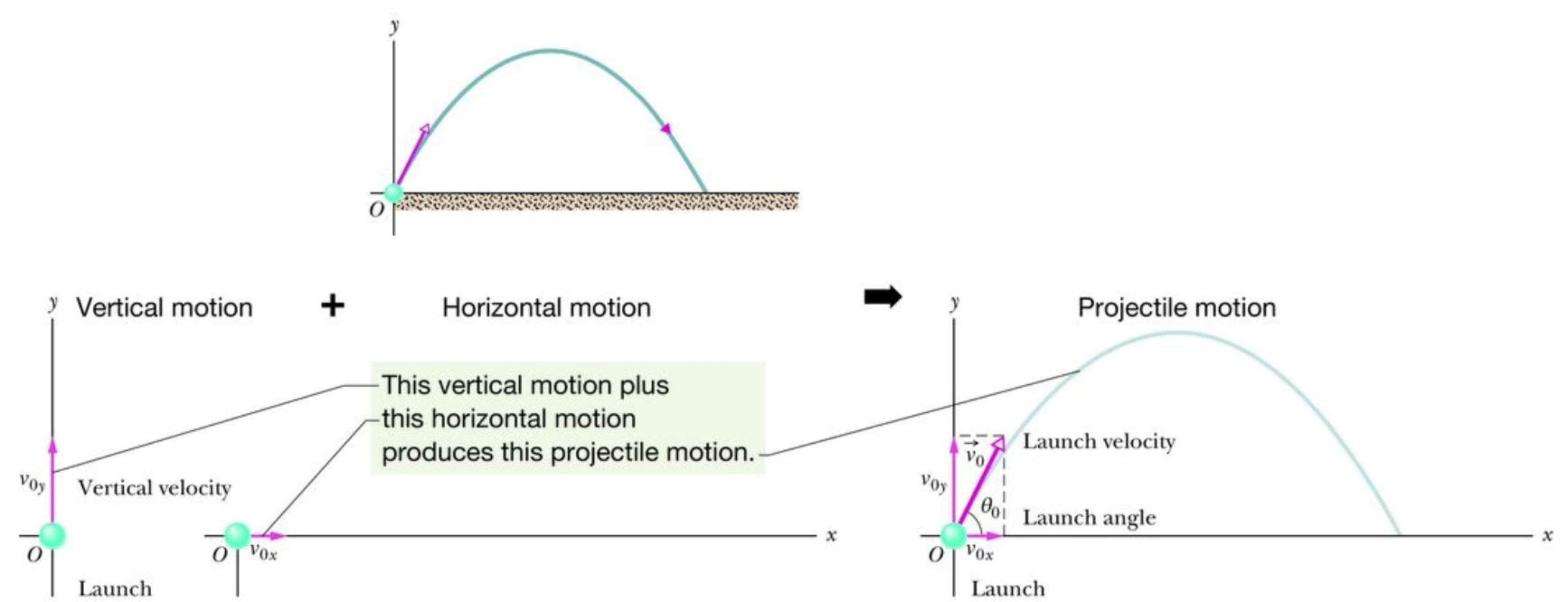


Projectile motion



Projectile motion

- Horizontal motion:

$$x - x_0 = v_{0x} t = (v_0 \cos \theta_0) t$$

- Vertical motion:

$$y - y_0 = v_{0y} t - \frac{1}{2} g t^2 = (v_0 \sin \theta_0) t - \frac{1}{2} g t^2$$

- Equation of the path:

$$y = (\tan \theta_0) x - \frac{g x^2}{2 (v_0 \cos \theta_0)^2}$$

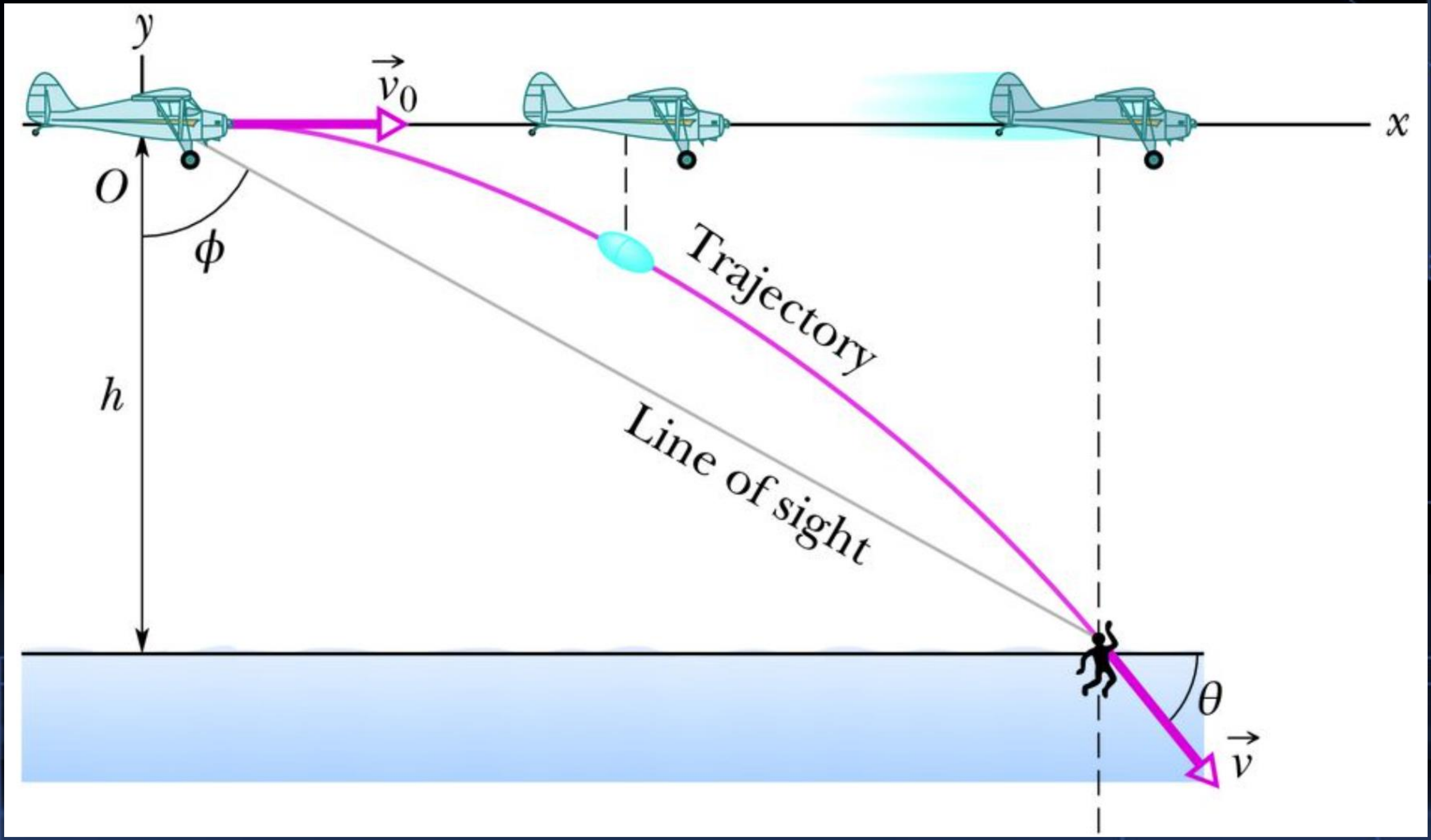
Example of projectile motion

a rescue plane flies at 198 km/h (= 55.0 m/s) and constant height h = 500 m toward a point directly over a victim, where a rescue capsule is to land.

(a) What should be the angle ϕ of the pilot's line of sight to the victim when the capsule release is made?

(b) As the capsule reaches the water, what is its velocity ?

Example of projectile motion



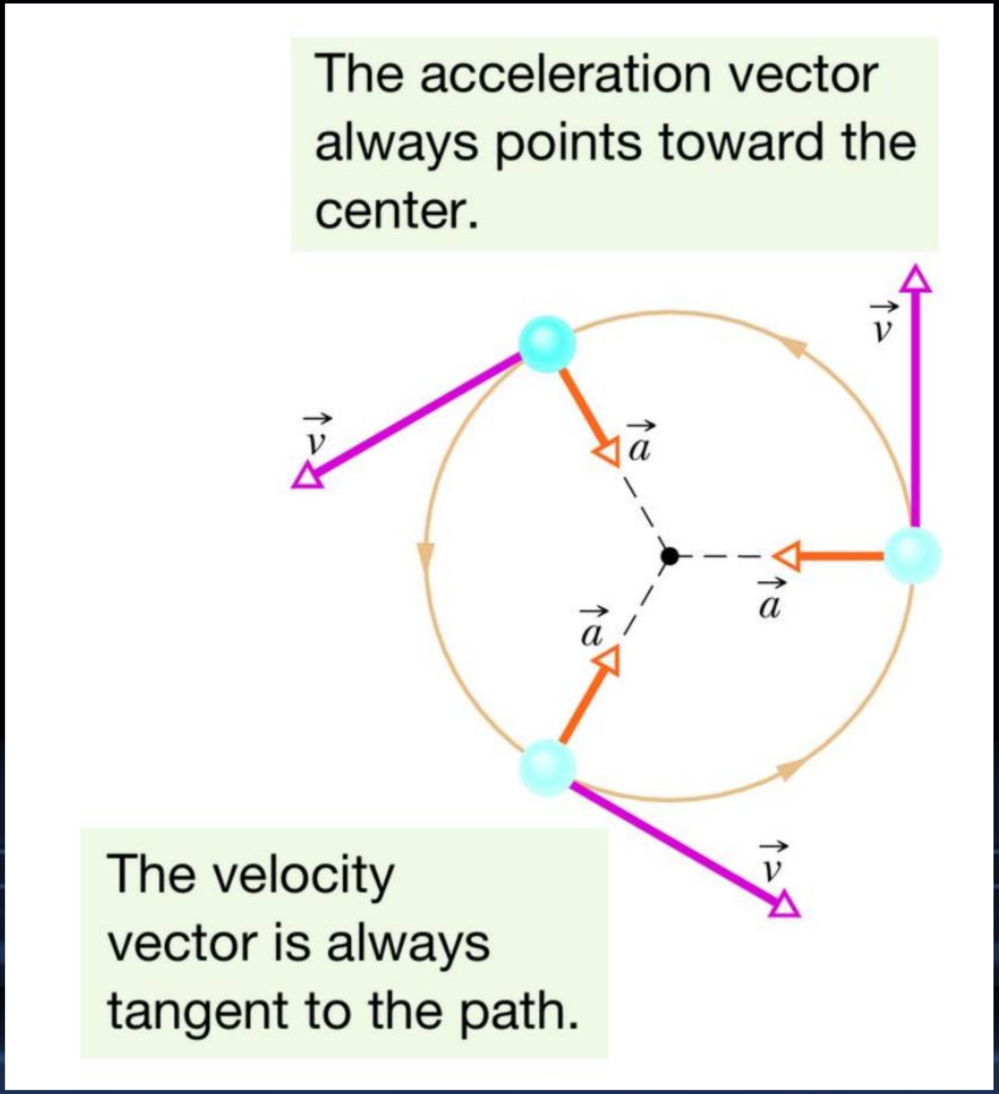
Key idea for (a)

- To find ϕ, we will use $\phi = tan^{-1}\frac{x}{h}$, therefore, we want to find x .
- To find x, we can use $x = v_x t$, where $v_x = 55m/s$. therefore, we want to find t, the time of the whole process.
- To find t, we can use $h = \frac{1}{2}gt^2$, where $h = 500\ m$ and $g = 9.8m/s^2$. Therefore, we can find $t = 10.1s$ and
- Therefore, we can find $t = 10.1s$ and $x = 555m$. Thus we can get $\phi = 48°$.

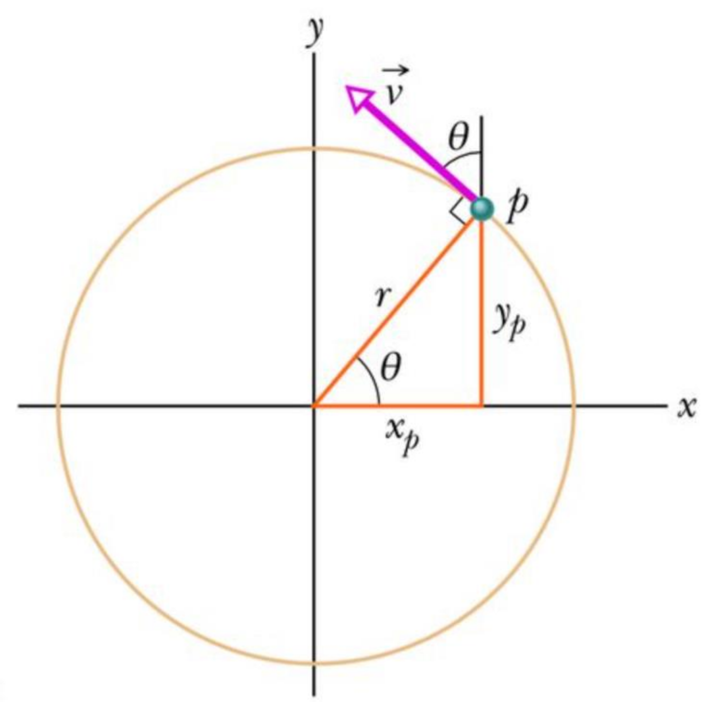
Key idea for (b)

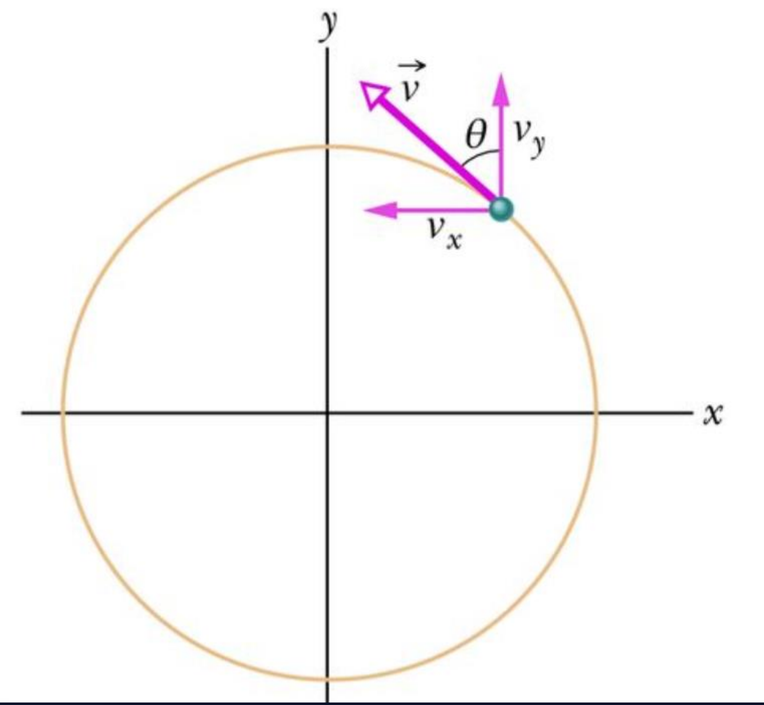
- To find v, we will need to know v_x and v_y.
- $v_x = 55m/s$ since it is constant along x-direction.
- To find v_y, we can use $v_y = gt$, $g = \frac{9.8m}{s^2}$ and $t = 10.1s$
- Therefore, we can find $v = \frac{55m}{s}\hat{\imath} - \frac{99m}{s}\hat{\jmath}$

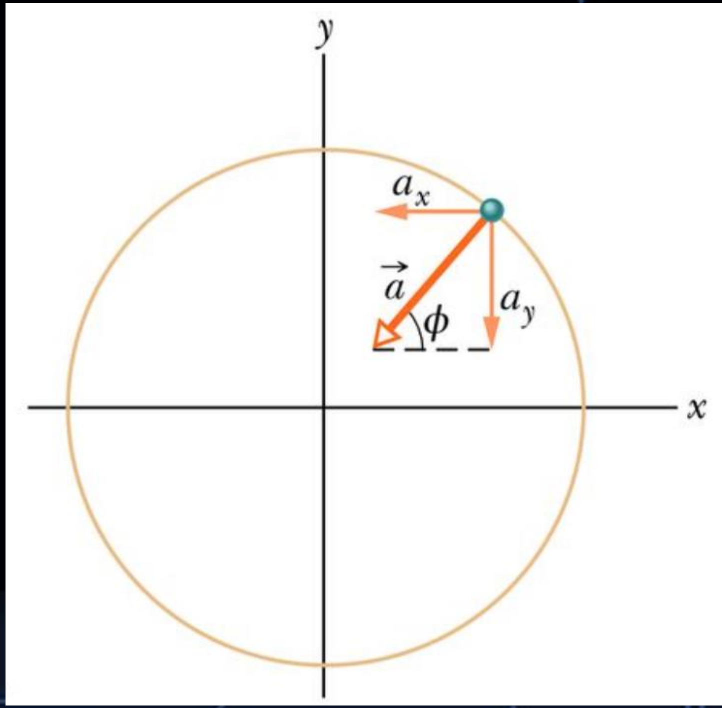
Uniform Circular Motion



Analysis of position, velocity and acceleration







Think about it…

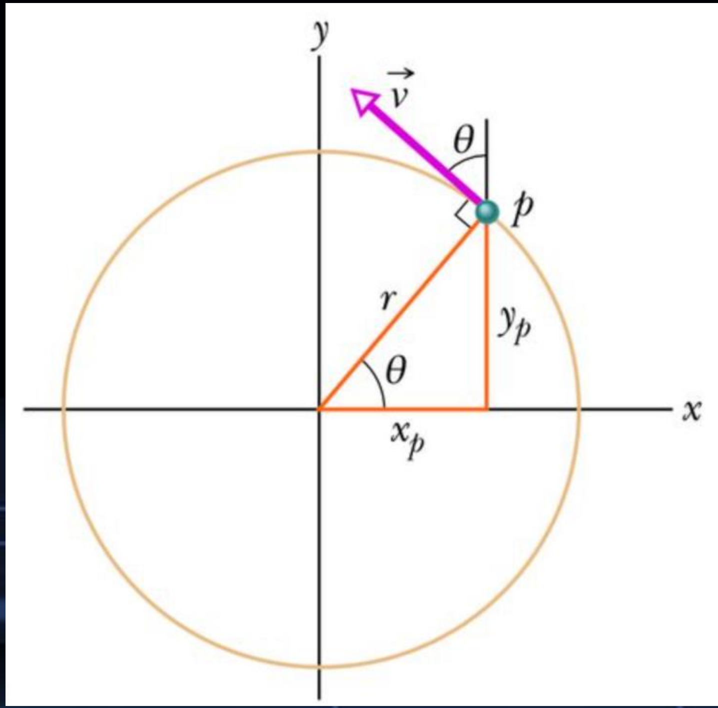
- Which set of equation is the best for describing position of a particle in uniform circular motion given (x,y)=(r,0) when t=0?

a. $x = rsin\theta, y = rsin\theta, \theta = \omega t$

b. $x = rcos\theta, y = rsin\theta, \theta = \omega t$

c. $x = rsin\theta, y = rcos\theta, \theta = \omega t$

d. $x = rcos\theta, y = rcos\theta, \theta = \omega t$



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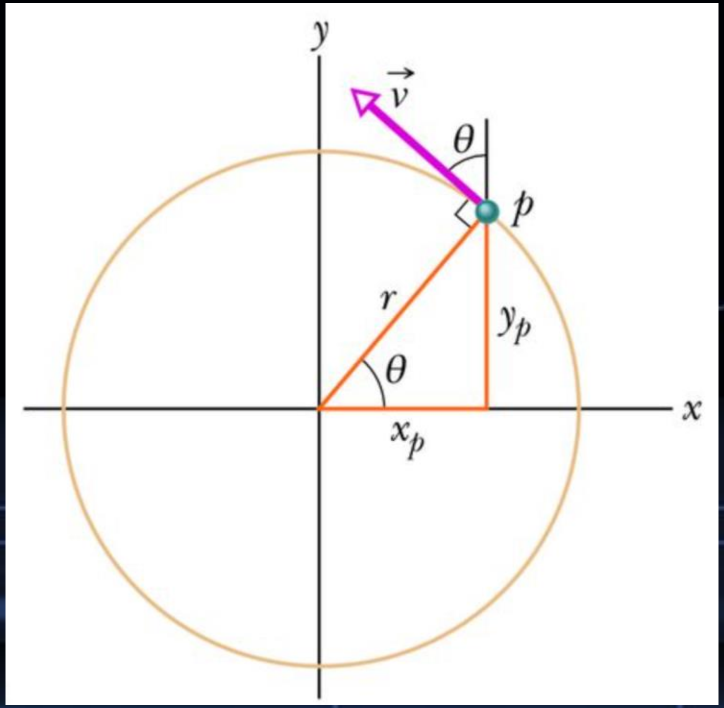

The condition for uniform circular motion

- The velocity of the particle is:

$$\overrightarrow{v} = v_x \hat{\mathrm{i}} + v_y \hat{\mathrm{j}} = (-v \sin \theta) \hat{\mathrm{i}} + (v \cos \theta) \hat{\mathrm{j}}$$

- Or we can rewrite as:

$$\overrightarrow{v} = \left(-\frac{v y_p}{r}\right) \hat{\mathrm{i}} + \left(\frac{v x_p}{r}\right) \hat{\mathrm{j}}$$

- Thus the acceleration:

$$\overrightarrow{a} = \frac{d\overrightarrow{v}}{dt} = \left(-\frac{v}{r} \frac{dy_p}{dt}\right) \hat{\mathrm{i}} + \left(\frac{v}{r} \frac{dx_p}{dt}\right) \hat{\mathrm{j}}$$

The condition for uniform circular motion (2)

- Thus, we can find:

$$\overrightarrow{a} = \left(-\frac{v^2}{r}\cos\ \theta\right)\hat{\mathrm{i}} + \left(-\frac{v^2}{r}\sin\ \theta\right)\hat{\mathrm{j}}$$

- The condition of having uniform circular motion is:

$$a = \sqrt{a_x^2 + a_y^2} = \frac{v^2}{r}\sqrt{(\cos\ \theta)^2 + (\sin\ \theta)^2} = \frac{v^2}{r}\sqrt{1} = \frac{v^2}{r}$$

Summary

- In two and three dimensions, position, velocity, and acceleration become vector quantities.
  - Velocity is the rate of change of position: $\vec{v} = \frac{d\vec{r}}{dt}$
  - Acceleration is the rate of change of velocity: $\vec{a} = \frac{d\vec{v}}{dt}$
- In general, acceleration changes both the magnitude and direction of the velocity.
- Projectile motion results from the constant acceleration of gravity.
- In uniform circular motion, the acceleration has magnitude v^2 / r and points toward the center of the circular path.