General Physics B1 - Midterm Exam 1

Total:100 points

You may answer in English or Chinese. Please use SI units and take significant figure to the second decimal place for the answers. The graviational acceleration g$=9.8m/s^2$**.**

1.Safety distance on a highway

On a dry road, a car with good tires may be able to brake with a constant deceleration of $5.00m/s^2$. (a) How many seconds does such a car, initially traveling at $90.00km/hr$, take to stop? (5points) (b) How many meters does it travel in this time? (10points)

Solution

(a)The initial velocity $v_0 = 90.00km/hr = 25.00m/s$,The motion of the car is constant acceleration with initial velocity. For the time to stop the car, $t = v_0/a = 5.00sec$.

(b) We want to find out travel distance without knowing time. Thus we should use:

$$v^2 = v_0^2 - 2a(x - x_0)$$

where $v = 0m/s$. Thus we can found the distance for the car to stop is $(x - x_0) = \frac{v_0^2}{2a} = 62.50m$.

2. Relative motion

Ship A is located 40.00m north and 25.00m east of ship B. Ship A has a velocity of 3.00 m/sec toward the south, and ship B has a velocity of 2.00 m/s in a direction 45° north of east. (a) Write an expression (in terms of $\hat{\imath}$ and $\hat{\jmath}$) for the position of A relative to B as a function of t, where t = 0 when the ships are in the positions described above. (5points) (b) At what time is the separation between the ships least? (10points)

maybe useful: sin45°=0.7071, cos45°=0.7071

Solution:

The velocity A respect to rest frame:$\overrightarrow{V_A} = (0km/h)\widehat{i} - (3.00m/s)\widehat{j}$, The velocity B respect to rest frame:$\overrightarrow{V_B} = 2.00cos45\widehat{i} + 2.00sin45\widehat{j} = (1.41m/s)\widehat{i} + (1.41m/s)\widehat{j}$

(a)Assuming the ship B's position at t=0 is the origin of the cooridnate.

The position of ship A $\overrightarrow{r_A} = \overrightarrow{r_A}(t=0) + \overrightarrow{V_A}t = [(25.00m)\widehat{i} + [40.00m - (3.00m/s)t]\widehat{j}]$.

The position of ship B $\overrightarrow{r_B} = \overrightarrow{r_B}(t=0) + \overrightarrow{V_B}t = [(1.41m/s)t\widehat{i} + (1.41m/s)t\widehat{j}]$

The relative position of A relavtive to B is

$\overrightarrow{r_{AB}} = \overrightarrow{r_A} - \overrightarrow{r_B} = [(25.00m)\widehat{i} + [40.00m - (3.00m/s)t]\widehat{j}] - [(1.41m/s)t\widehat{i} + (1.41m/s)t\widehat{j}] = [(25.00m) - (1.41m/s)t]\widehat{i} + [(40.00m) - (4.41m/s)t]\widehat{j}$

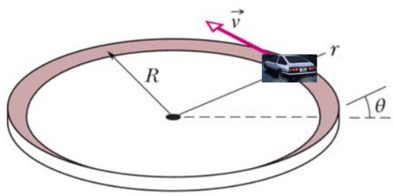
(b)The separation between the ships is $|\overrightarrow{r_{AB}}| = \{[(25.00m) - (1.41m/s)t]^2 + [(40.00m) - (4.41m/s)t]^2\}^{\frac{1}{2}} = [(625 - 70.5t + 1.99t^2) + (1600 - 352.8t + 19.45t^2)]^{\frac{1}{2}} = [2225.00 - 423.3t + 21.44t^2]^{\frac{1}{2}}$

The least seperation happen when the derivative of seperation respect to t is 0. Therefore, it is $t = 423.3/(2 \cdot 21.44) = 9.87sec$.

3. Car on a tilted curve with friction

Assuming a car with mass $m = 1000kg$ turn on a tilted curve. The tilted angle is $\theta = 15°$. The curve can be approximate as a part of a circle with radius $R = 60m$. If the static frictional coefficient $\mu_s = 0.8$. What is the maximum speed of this car can make the turn? (20points)

maybe useful: sin15°=0.2588, cos15°=0.9659



Solution:

The vertical component of normal force cancels out the gravitational force: The centripetal acceleration is provided by the horizontal components of normal force.

$$F_N cos\theta = mg + \mu_s F_N sin\theta$$

Thus, we can calculate out $F_N = \frac{mg}{cos\theta - \mu_s sin\theta} = 12913N$

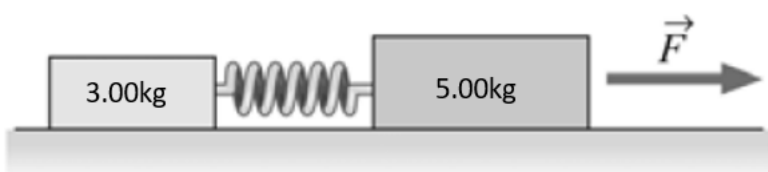
Finally, the centripetal acceleration is provided by the horizontal components of normal force.

$$F_N sin\theta + \mu_s F_N cos\theta = \frac{mv_{max}^2}{R}$$

Thus, we can calculate out $v_{max} = 28.27m/s$

4.Spring Force Between Blocks

A 3.00kg mass and a 5.00kg mass are on a horizontal frictionless surface connected by a massless spring with spring constant $k = 90N/m$. A 24.00N force is applied to the larger mass, as shown in the following figure. How much does the spring stretch from its equilibrium length? (10point)

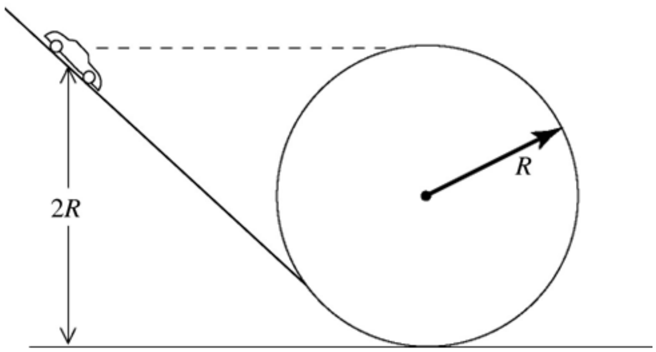


Solution We first treat the two block as one system. Therefore, the acceleration of this system due to force F is $a_{tot} = \frac{F}{m_1+m_2} = 3m/s^2$

Since the two blocks will move together and thus $a_{tot} = a_1 = a_2$. Now if we focus on the second block with 3.00kg, the net force on this block is to the right due to spring force. This results in an acceleration $a_2 = 3m/s^2$. Thus, the spring force exerting on the second block with 3.00kg is $m_2a_2 = 9N$. Following with Hooke's law, we kno the spring stretch from its equilibrium length with $\Delta x = m_2a_2/k = 0.10m$.

5. Conservation of energy in a Looping-Loop

In the figure, a toy race car of mass m=0.50kg is released from rest on the loop-the-loop track. If it is released at a height 2R=2.00m above the floor, (a)What is the velcoity of the toy car at the very bottom of the loop? (10points) (b)how high is it above the floor when it leaves the track, neglecting friction? (10point)



Solution Let the height from the lowest point to the position of the car is h. During the whole process, only graviational force is doing work. From conservation of mechanical energy, assuming the magnitude of toy car's velocity at positon h is v, thus we have:

$$mg2R = mgh + \frac{1}{2}mv^2 \tag{1}$$

(a)At the very bottom of the loop, $h = 0$ and we have $v = \sqrt{4gR} = 6.26(m/s)$

(b)Note that the velocity on the loop-the-loop track is in the tangential direction of the loop. During the toy car traveling on looping track, the centripical acceleration is provided by net force of normal force and gravitational force. The toy car leaves the track when the centripal acceleration is only provide by the components of gravitational force and the normal force is zero. The centripical acceleration is provided by a component of graviational force pointing to the ceter of the circle and the magnitude of this force is:

$$mg\frac{h-R}{R} = m\frac{v^2}{R} \tag{2}$$

From (1), we have $v^2 = 2g(2R-h)$. Plug into (2) and we have $h - R = 2(2R - h)$. Thus, we can get $h = \frac{5}{3}R = 1.67(m)$.

6.Circular Motion around the Moon

During the Apollo Moon landings, one astronaut remained with the command module in lunar orbit, about 130km above the moon survace. For half of each orbit, this astronaut was completely cut off from the rest of humanity as the spacecraft rounded the far side of the Moon. (a)How long did this period last? (10point) (b)What is the velocity of this spacecraft should have in order to completely escape from the bound of this lunar orbit (130km above the moon survace)? (10points)Given the radius of the Moon is $R_M = 1.74 \times 10^6 m$ and the mass of the Moon is $M = 7.35 \times 10^{22} kg$. The gravitational constant $G = 6.67 \times 10^{-11} N \cdot m^2/kg^2$.

Solution (a)Since the spacecraft is doing a circular motion surroundin the moon with a radius $d + R_M = 1.30 \times 10^5 + 1.74 \times 10^6 = 1.87 \times 10^6(m)$. The time for the spacecraft to surround half of the orbit is:

$$\frac{T}{2} = \frac{2\pi(d+R_M)}{2(v)} = \frac{2\pi(d+R_M)}{2(\sqrt{\frac{GM}{d+R_M}})} = \pi\sqrt{\frac{(d+R_M)^3}{GM}} = 3628.32 = 3.63 \times 10^3(s)$$

So the period of totally silence is $3.63 \times 10^3 sec.$

(b)The escape velocity is $v = \sqrt{\frac{2GM}{(d+R_M)}} = 2289.81(m/s)$