(1)

a.

$F = \frac{kq_1q_2}{r^2}$ ... **1**

$q_1 + q_2 = 8\ (\mu C)$ ... **1**

$150 = 9 \times 10^9 \times \frac{q_1q_2}{0.01^2}$

$q_1q_2 = 15\ (\mu C)^2$ ... **1**

$q_1(8 - q_1) = 15$ ... **1**

$q_1 = 3\ (\mu C), q_2 = 5(\mu C)$ ... **1**

b.

$-150 = 9 \times 10^9 \times \frac{q_1q_2}{0.01^2}$

$q_1q_2 = -15\ (\mu C)^2$ ... **1**

$q_1(8 - q_1) = -15$ ... **1**

$q_1 = 4 + \sqrt{31}\ (\mu C), q_2 = 4 - \sqrt{31}(\mu C)$ ... **1**

(2)

$$E = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{r^2} \ldots \textbf{2}$$

$$dq = \lambda dx \ldots \textbf{2}$$

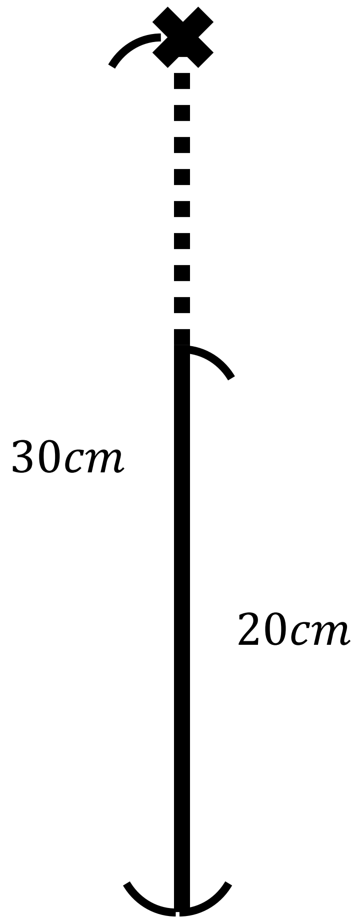
$$E = \frac{1}{4\pi\varepsilon_0}\int_{0.2}^{0.4} \frac{dx}{x^2} \ldots \textbf{2}$$

$$= \frac{1}{4\pi\varepsilon_0}\left[-\frac{1}{x}\right]\Bigg|_{0.2}^{0.4}$$

$$= \frac{1}{4\pi\varepsilon_0}\left[-\frac{1}{0.4}+\frac{1}{0.2}\right]$$

$$= 9 \times 10^9 \times 4 \times 10^{-6} \times 2.5$$

$$E = 9 \times 10^4\ (V/m) \ldots \textbf{2}$$



(3)

$$\varepsilon_0 \Phi = q_{enc} \; ... \; \mathbf{1}$$

$$\varepsilon_0 E 4\pi r^2 = \frac{4}{3}\pi r^3 \rho \; ... \; \mathbf{1}$$

$$E_{10} = \frac{\frac{4}{3}\pi(0.1)^3\rho}{4\pi(0.1)^2} = 2 \times \frac{\frac{4}{3}\pi(0.05)^3\rho}{4\pi(0.05)^2} = 2\mathrm{E}_5 ... \; \mathbf{2}$$

$$E_{20} = \frac{\frac{4}{3}\pi(0.1)^3\rho}{4\pi(0.2)^2} = \frac{1}{4} \times \frac{\frac{4}{3}\pi(0.1)^3\rho}{4\pi(0.1)^2} = \frac{1}{4}\mathrm{E}_{10} ... \; \mathbf{2}$$

$$E_{20} = \frac{1}{2} E_5 = 500 \; (N/C) \; ... \; \mathbf{2}$$

(4)

$E = 0, (\mathrm{r} < \mathrm{a})$ ... **3**

$\varepsilon_0(E \cdot 4\pi r^2) = \int_a^r 4\pi r'^2 dr' q$ ... **2**

$$\varepsilon_0(E \cdot 4\pi r^2) = \int_a^r 4\pi A r' dr'$$

$\varepsilon_0(E \cdot 4\pi r^2) = 2\pi A(r^2 - a^2)$ ... **2**

$E = \frac{\mathrm{A}}{2\varepsilon_0}\left(1 - \frac{a^2}{r^2}\right), (\mathrm{a} < \mathrm{r} < \mathrm{R})$ ... **1**

$E = \frac{\mathrm{A}}{2\varepsilon_0}\left(\frac{R^2 - a^2}{r^2}\right), (\mathrm{R} < \mathrm{r})$ ... **1**

$V = \frac{\mathrm{A}}{2\varepsilon_0}\left(\frac{R^2 - a^2}{r}\right), (\mathrm{R} < \mathrm{r})$ ... **3**

$V(r) = -\int_R^r E\, dr + V(R)$ ... **2**

$V(r) = -\int_R^r \frac{\mathrm{A}}{2\varepsilon_0}\left(1 - \frac{a^2}{r^2}\right) dr + V(R)$ ... **2**

$$= -\frac{\mathrm{A}}{2\varepsilon_0}\left((r - R) - (\frac{a^2}{R} - \frac{a^2}{r})\right) + \frac{\mathrm{A}}{2\varepsilon_0}\left(\frac{R^2 - a^2}{R}\right)$$

$= \frac{\mathrm{A}}{2\varepsilon_0}(2R - r - \frac{a^2}{r}), (\mathrm{a} < \mathrm{r} < \mathrm{R})$ ... **1**

$= \frac{\mathrm{A}}{\varepsilon_0}(R - a), (\mathrm{r} < \mathrm{a})$ ... **1**

(5)

LC oscillation

a.

$q = Q \times cos(\omega t + \phi)$ ... **1**

$i = -Q\omega \times sin(\omega t + \phi)$ ... **2**

$\omega = 1/\sqrt{LC}$ ... **1**

$i_{max} = \omega Q = \frac{\omega}{\sqrt{LC}} = 0.15\ (A)$... **2**

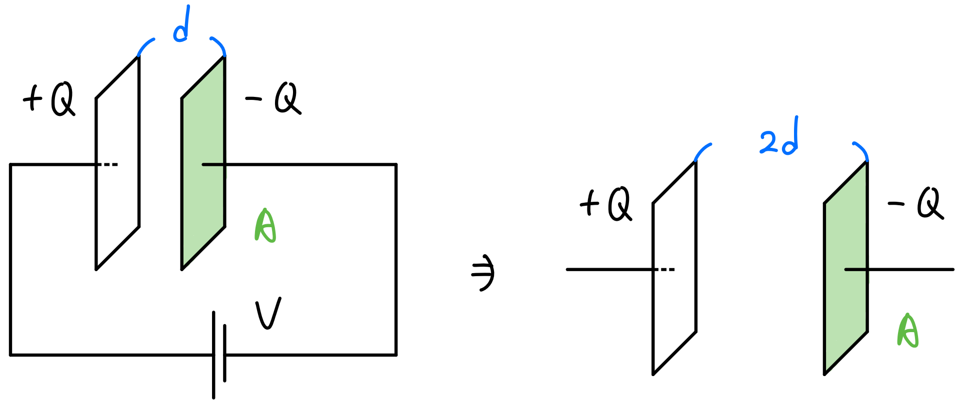
b.

$U_L = U_C$, $when\ \cos(\omega t) = \frac{1}{\sqrt{2}}$ ... **1**

$\omega t = \frac{\pi}{4}$ ... **1**

t= $\frac{\pi}{4}\sqrt{LC} = \pi \times 10^{-4}\ (sec)$ ... **2**

6. A parallel-plate capacitor has plates of area A and separation d and is charged to a potential difference V. The charging battery is then disconnected and the plates are pulled apart until their separation is $2d$. Find the new potential difference, the initial and final stored energy, and the work required to separate the plates. (8%)



Original $C = \frac{\varepsilon_0 A}{d}$ ① $\Rightarrow$ Original $V = \frac{Q}{C}$ ①

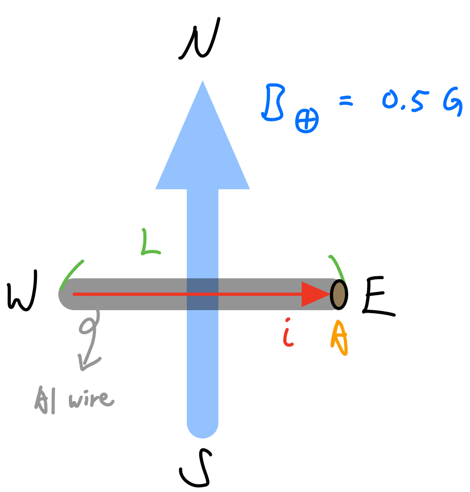
New $C' = \frac{\varepsilon_0 A}{2d}$ $\Rightarrow$ New $V' = \frac{Q}{C'}$ $\Rightarrow$ $2V$ ①

Original $U = \frac{Q^2}{2C} = \frac{1}{2}CV^2$ ① $= \frac{1}{2}\frac{\varepsilon_0 A}{d}V^2$ ①

New $U' = \frac{Q^2}{2C'} = \frac{1}{2}C'(V')^2 = \frac{\varepsilon_0 A}{d}V^2$ ①

$\Rightarrow W_{require} = U' - U$ ① $= \frac{1}{2}\frac{\varepsilon_0 A}{d}V^2$ ① ※

7. What is the required current density through an aluminum wire for the wire to float in the air? Assume the magnitude of earth's magnetic field is 0.5 gauss in the north-south direction, and the wire lies horizontally in the east-west direction. The density of aluminum is 2.7 g/cm^3. The current density is uniformly distributed. (8%)



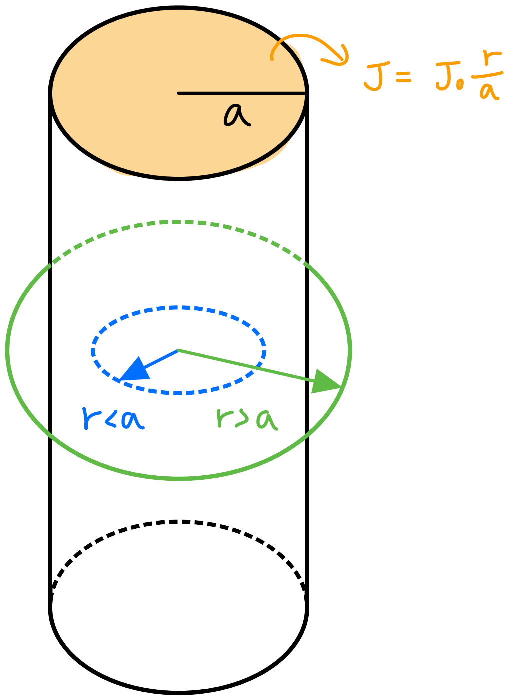
$\vec{F}_B = i\vec{L}\times\vec{B}$ ②

$|\vec{F}_B| = |\vec{F}_g| \Rightarrow i\cdot L\cdot B = (L\cdot A)\cdot\rho\cdot g$ ①

$\Rightarrow |\vec{J}| = \frac{i}{A} = \frac{\rho\cdot g}{B} = \frac{(2.7\times10^{3}\ kg/m^3)\cdot(9.8\ m/s)}{(0.5\times10^{-4}\ T)}$ ③

$\approx 5.3\times10^{8}\ A/m^2$ ② ※

8. The current density inside a long, solid, cylindrical wire of radius a is in the direction of its central axis and varies linearly with distance r from the axis as $J = J_0 r/a$. Find the magnetic field strength inside and outside the wire. (10%)



$\oint \vec{B} \cdot d\vec{s} = \mu_0 i$, $\mu_0 = 4\pi \times 10^{-7}\ T\cdot m/A$ ②

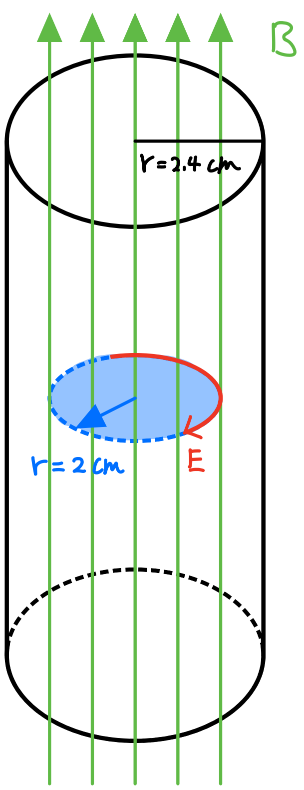
$\Rightarrow B \cdot (2\pi r) = \mu_0 \int_0^{2\pi} \int_0^{r} J_0 \frac{r'}{a} \cdot dr' r' d\theta$ ②

$= \frac{\mu_0 J_0}{a} 2\pi \int_0^{r} (r')^2 dr = \frac{\mu_0 J_0}{a} 2\pi \frac{r^3}{3}$ ②

for $r < a$: using $\int_0^r$ $\Rightarrow$ $B = \frac{\mu_0 J_0}{3a} r^2$ ②

for $r \geq a$: using $\int_0^a$ $\Rightarrow$ $B = \frac{\mu_0 J_0}{3r} a^2$ ② ※

9. A long solenoid with 20 turns/cm has a radius of 2.4 cm. The magnitude of the induced electric field 2 cm from the axis is 5×10^{-3} V/m. At what rate is the current in the solenoid changing? (10%)



$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$ ②

$= -\frac{d}{dt}(BA)$ ①

$= -\frac{d}{dt}\left[(\mu_0 n i) \cdot (\pi r^2)\right]$ ② ①

$= -\mu_0 n \pi r^2 \frac{di}{dt}$

also ok ! ③

$-N \frac{d\Phi_B}{dt} = -L \frac{di}{dt}$

$\Rightarrow -n\ell \frac{d\Phi_B}{dt} = -\mu_0 n^2 (\pi r^2) \ell \frac{di}{dt}$ ①

$\Rightarrow \left|\frac{di}{dt}\right| = \frac{E \cdot (2\pi r)}{\mu_0 n \pi r^2} = \frac{2 \cdot (5 \times 10^{-3}\ V/m)}{(4\pi \times 10^{-7}\ T\cdot m/A) \cdot (2000\ \#/m) \cdot (0.02\ m)}$ ① ①

$\approx 199\ A/s$ ② ※

10. Consider an RLC circuit with an AC emf amplitude $v_0 = 10$ V, $R = 10\,\Omega$, $L = 1.0$ H, and $C = 1.0\,\mu$F. (a) Find the amplitude of the voltage across the inductor at resonance. (b) Find the frequency deviation from the resonance of the AC emf frequency at which the amplitude of the charge accumulated on the capacitor is the maximum. (12%)

$$AC \Rightarrow i = I \sin(\omega_d t - \phi), \quad I = \frac{V_0}{Z} = \frac{V_0}{\sqrt{R^2 + (\omega_d L - \frac{1}{\omega_d C})^2}}$$

Z impedance

R resistance

X_L reactane of inductor

X_C reactane of capacitor

(a) at resonance : $\omega_d = \omega_0 = \frac{1}{\sqrt{LC}}$ ②

$$V_L = -L \frac{di}{dt}$$

$$= -\omega_d I L \cos(\omega_d t - \phi)$$ ① or $V_L = I \cdot X_L$

$$\Rightarrow \text{Amplitude} = |-\omega_d I L| = \frac{1}{\sqrt{LC}} \frac{V_0}{R} L = 10^3\ V$$ ① ① ※

(b) $i = \frac{dq}{dt} \Rightarrow q = -\frac{I}{\omega_d} \cos(\omega_d t - \phi) + \text{Constant}$

$$\Rightarrow \text{Amplitude} = \left| -\frac{I}{\omega_d} \right| = \frac{V_0}{\omega_d \sqrt{R^2 + (\omega_d L - \frac{1}{\omega_d C})^2}}$$ ② or $V_C = I \cdot X_C$

$$= \frac{V_0 / L}{\sqrt{\frac{R^2}{L^2}\omega_d^2 + (\omega_d^2 - \omega_0^2)^2}}$$

min

when $\omega_d = \sqrt{\omega_0^2 - \frac{1}{2}\frac{R^2}{L^2}}$ ② $\Rightarrow q$ has Max Amplitude

$$\Rightarrow |\Delta\omega| = |\omega_d - \omega_0| \approx 0.025\ Hz$$ ② ① ※