

(1)

$$\Delta x \Delta p_x \geq \hbar \quad \dots \mathbf{1}$$

$$\Delta y \Delta p_y \geq \hbar \quad \dots \mathbf{1}$$

$$p_x = mv_x = 1.67 \cdot 10^6 \times 3 \cdot 10^6 \sim 5.01 \cdot 10^{-21} \text{ kg} \cdot \text{m/s} \quad \dots \mathbf{1}$$

$$p_y = mv_y = 1.67 \cdot 10^6 \times 4 \cdot 10^6 \sim 6.69 \cdot 10^{-21} \text{ kg} \cdot \text{m/s} \quad \dots \mathbf{1}$$

$$\Delta p_x = 0.01 \times p_x \sim 5.01 \cdot 10^{-23} \text{ kg} \cdot \text{m/s} \quad \dots \mathbf{1}$$

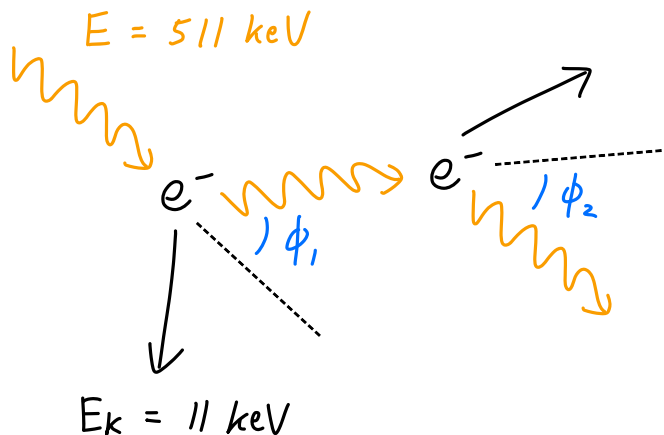
$$\Delta p_y = 0.01 \times p_y \sim 6.69 \cdot 10^{-23} \text{ kg} \cdot \text{m/s} \quad \dots \mathbf{1}$$

$$\Delta x \geq \hbar / \Delta p_x \sim 2.10 \cdot 10^{-12} \text{ m} \quad \dots \mathbf{1}$$

$$\Delta y \geq \hbar / \Delta p_y \sim 1.58 \cdot 10^{-12} \text{ m} \quad \dots \mathbf{1}$$

$$\Delta r = \sqrt{\Delta x^2 + \Delta y^2} \sim 2.63 \cdot 10^{-12} \text{ m} \quad \dots \mathbf{2}$$

Q.2 One γ -ray photon with 511 keV emitted from some specific direction. After the photon undergoes two times Compton scattering with two originally stationary electrons, the original incoming and final outgoing directions of the photon are the same. If we know the kinetic energy of the first recoiling electron is 11 keV, what is the scattering angle of the photon in the second Compton scattering process? (Hint: $m_e c^2 = 511 \text{ keV}$) (10 points)



incoming direction = outgoing direction

$$\Rightarrow \underline{\phi_1 = \phi_2} \quad (2)$$

$$hf + m_e c^2 = hf' + \sqrt{(pc)^2 + (m_e c^2)^2}$$

$$\Rightarrow hf = hf' + E_k$$

$$\Rightarrow hf' = 511 \text{ keV} - 11 \text{ keV} = \underline{500 \text{ keV}} \quad (2)$$

$$\Delta\lambda = \lambda' - \lambda = \underline{\frac{2}{500 \text{ keV}} \frac{hc}{511 \text{ keV}} = \frac{h}{m_e c} (1 - \cos \phi_1)} \quad (2)$$

$$\Rightarrow 1 - \cos \phi_1 = \frac{511}{500} - 1$$

$$\Rightarrow \phi_1 = \cos^{-1} \left(2 - \frac{511}{500} \right) \approx \underline{12^\circ} \quad (2)$$