(a)

$$u = \frac{u' + v}{1 + u'v/c^2} \quad \ldots \ \mathbf{2}$$

$$u = \frac{c/2 + c/7}{1 + (1/2)\cdot(1/7)} \ldots \ \mathbf{2}$$

$$u = 3c/5 \quad \ldots \ \mathbf{1}$$

(b)

$$L_0 = 100\ [m] \quad \ldots \ \mathbf{1}$$

$$L = L_0/\gamma \quad \ldots \ \mathbf{1}$$

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} = \frac{1}{\sqrt{1 - (3/5)^2}} = \frac{5}{4} \quad \ldots \ \mathbf{2}$$

$$L = 100 \times \frac{4}{5} = 80\ [m] \ldots \ \mathbf{1}$$

Q.2 Two clocks A and B are at rest and 100 m apart in frame S. Clock C, at rest in frame S', moves with velocity 0.9 c along the line joining clock A to clock B. According to the observer in frame S', how long and how far does clock C take to get from clock A to clock B? (10 points)

in frame S, $\Delta t = \dfrac{100\ m}{0.9 \times 3\times 10^{8}\ m/s} \sim 3.7 \times 10^{-7}\ s$

$\Rightarrow$ in frame S', $\Delta t'$ (proper time) $= \dfrac{1}{\gamma}\Delta t$ 2

2 $= \sqrt{1-(0.9)^2} \times 3.7 \times 10^{-7}\ s$

$\sim 1.61 \times 10^{-7}\ s$ 2

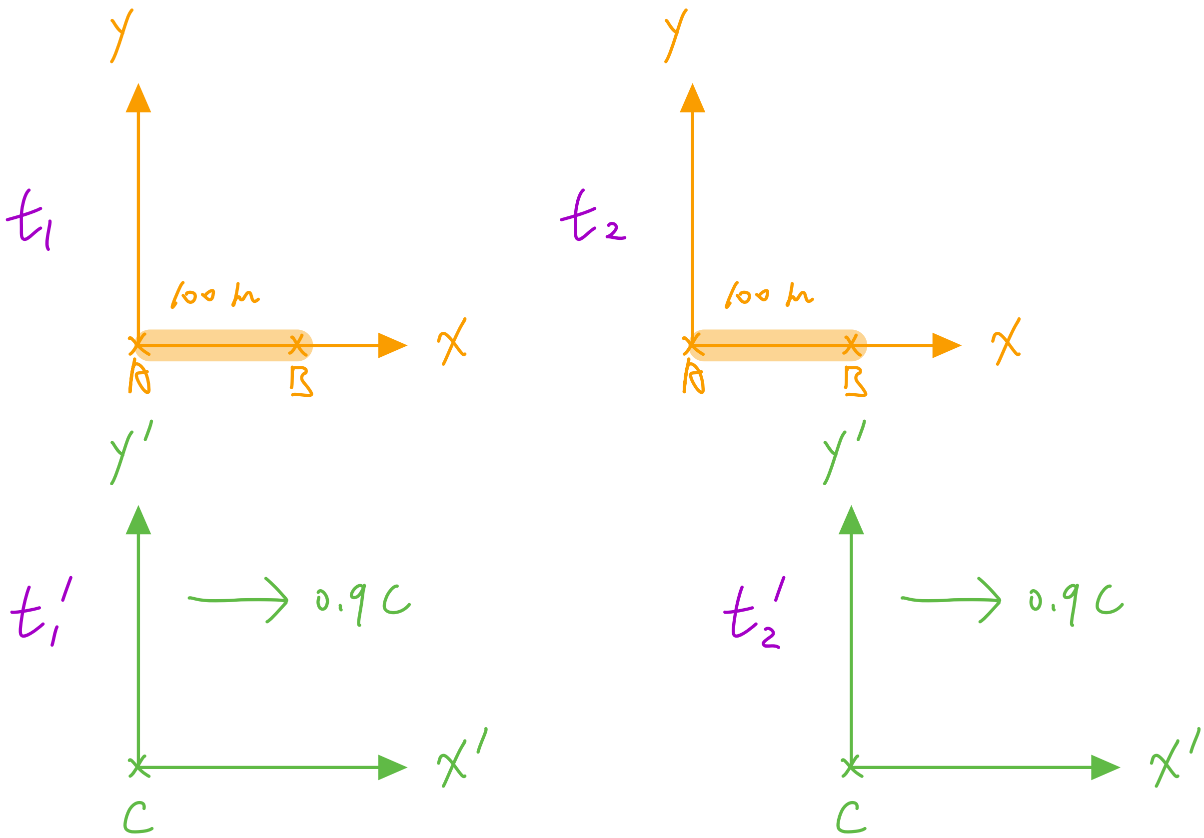
time dilation

$\Rightarrow$ in frame S', $\Delta L' = 0.9\,c \cdot \Delta t' = \dfrac{1}{\gamma}\Delta L$ 2 proper length in frame S

$= \sqrt{1-(0.9)^2} \times 100\ m$

length contraction

$\sim 43.59\ m$ 2



How long in S' ? $\Rightarrow$ $\Delta t'$?

$$\begin{cases} \Delta t' = t_2' - t_1' \\ \text{we know } \Delta t = t_2 - t_1 \end{cases}$$

$x'_{c_2} = x'_{c_1} = 0$!

$$= \gamma\left(t_2' + \frac{v \cdot x'_{c_2}}{c^2}\right) - \gamma\left(t_1' + \frac{v \cdot x'_{c_1}}{c^2}\right)$$

$$= \gamma\,(t_2' - t_1') = \gamma\,\Delta t' = \frac{100\text{ m}}{0.9c}$$

$$\Rightarrow \Delta t' = \frac{1}{\gamma}\Delta t = \sqrt{1-(0.9)^2}\,\frac{100\text{ m}}{0.9c} \checkmark$$

How far in S' ? $\Delta x' = $?

$$\Rightarrow \Delta x' = 0.9c\,\Delta t' = \frac{1}{\gamma}\Delta x = \sqrt{1-(0.9)^2}\;100\text{ m} \checkmark$$