(a)

$$\oint \vec{\mathrm{E}} \cdot d\vec{A} = \int \nabla \cdot \vec{\mathrm{E}}\, dV \quad \ldots \mathbf{2}$$

$$q_{enc} = \int \rho\, dV \ldots \mathbf{2}$$

$$\int \nabla \cdot \vec{\mathrm{E}}\, dV = \frac{q_{enc}}{\varepsilon_0} = \int \frac{\rho}{\varepsilon_0}\, dV \ldots \mathbf{1}$$

$$\nabla \cdot \vec{\mathrm{E}} = \frac{\rho}{\varepsilon_0} \ldots \mathbf{1}$$

(b)

$$\nabla \cdot \vec{\mathrm{E}} = \frac{\rho}{\varepsilon_0} \quad \ldots \mathbf{1}$$

$$\nabla \cdot \vec{\mathrm{E}} = \frac{d}{dr}\left(\frac{24\, r}{\varepsilon_0}\right) = \frac{24}{\varepsilon_0} = \frac{\rho}{\varepsilon_0} \quad \ldots \mathbf{2}$$

$$\rho = 24\ [\mathrm{Q/m^3}] \quad \ldots \mathbf{1}$$

Q.2 There is a radio station (regard it as a point source) transmits a FM radio signal ($\sim 100\ MHz$). The average power is 10 kW. **(a)** Please derive the average intensity of Poynting vector is $\frac{E_0 B_0}{2\mu_0}$. Where E_0 and B_0 are the amplitude of the electric and magnetic field, respectively. (5 points) **(b)** What is the amplitude of the electric field strength at a distance of 1 km from the antenna of that radio station. (5 points)

(a)

$$S_{av} = \frac{1}{T}\int_0^T \frac{1}{\mu_0} E_0 \sin(kx_0 - \omega t)\, B_0 \sin(kx_0 - \omega t)\, dt \quad ②$$

$$= \frac{E_0 B_0}{\mu_0 T}\int_0^T \sin^2(kx_0 - \omega t)\, dt \quad ②$$

$$= \frac{E_0 B_0}{\mu_0 T}\int_0^T \frac{1 - \cancel{\cos(2kx_0 - 2\omega t)}}{2}\, dt \quad \because \int_0^T \cos(2kx_0 - 2\omega t)\, dt = 0 \,! \quad ①$$

$$= \frac{E_0 B_0}{\mu_0 T}\frac{T}{2} = \frac{E_0 B_0}{2\mu_0} \quad \#$$

(b)

$$S_{av} \equiv \frac{\text{avg Power}}{\text{Area}} \; ① = \frac{E_0 B_0}{2\mu_0} = \frac{E_0^2}{2c\mu_0} \; ①$$

$$= \frac{10^4\ W}{4\pi (10^3)^2\ m^2} \; ①$$

$$\Rightarrow E_0 = \sqrt{0.6} \approx 0.775\ V/m \quad \# \quad ②$$