

$$V = \frac{1}{4\pi\epsilon_0} \frac{q}{r} \quad \dots \mathbf{1}$$

$$V_1 = \frac{1}{4\pi\epsilon_0} \frac{10}{\sqrt{(-4)^2 + (-4)^2 + 2^2}} = \frac{10}{24\pi\epsilon_0} \quad \dots \mathbf{1}$$

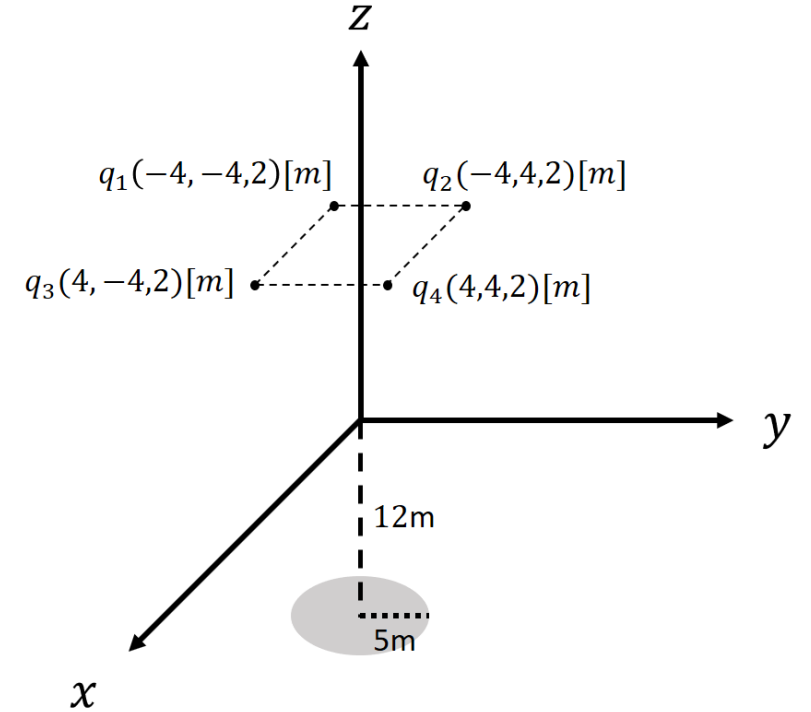
$$V_2 = \frac{1}{4\pi\epsilon_0} \frac{27}{\sqrt{(-4)^2 + 4^2 + 2^2}} = \frac{27}{24\pi\epsilon_0} \quad \dots \mathbf{1}$$

$$V_3 = \frac{1}{4\pi\epsilon_0} \frac{9}{\sqrt{4^2 + (-4)^2 + 2^2}} = \frac{9}{24\pi\epsilon_0} \quad \dots \mathbf{1}$$

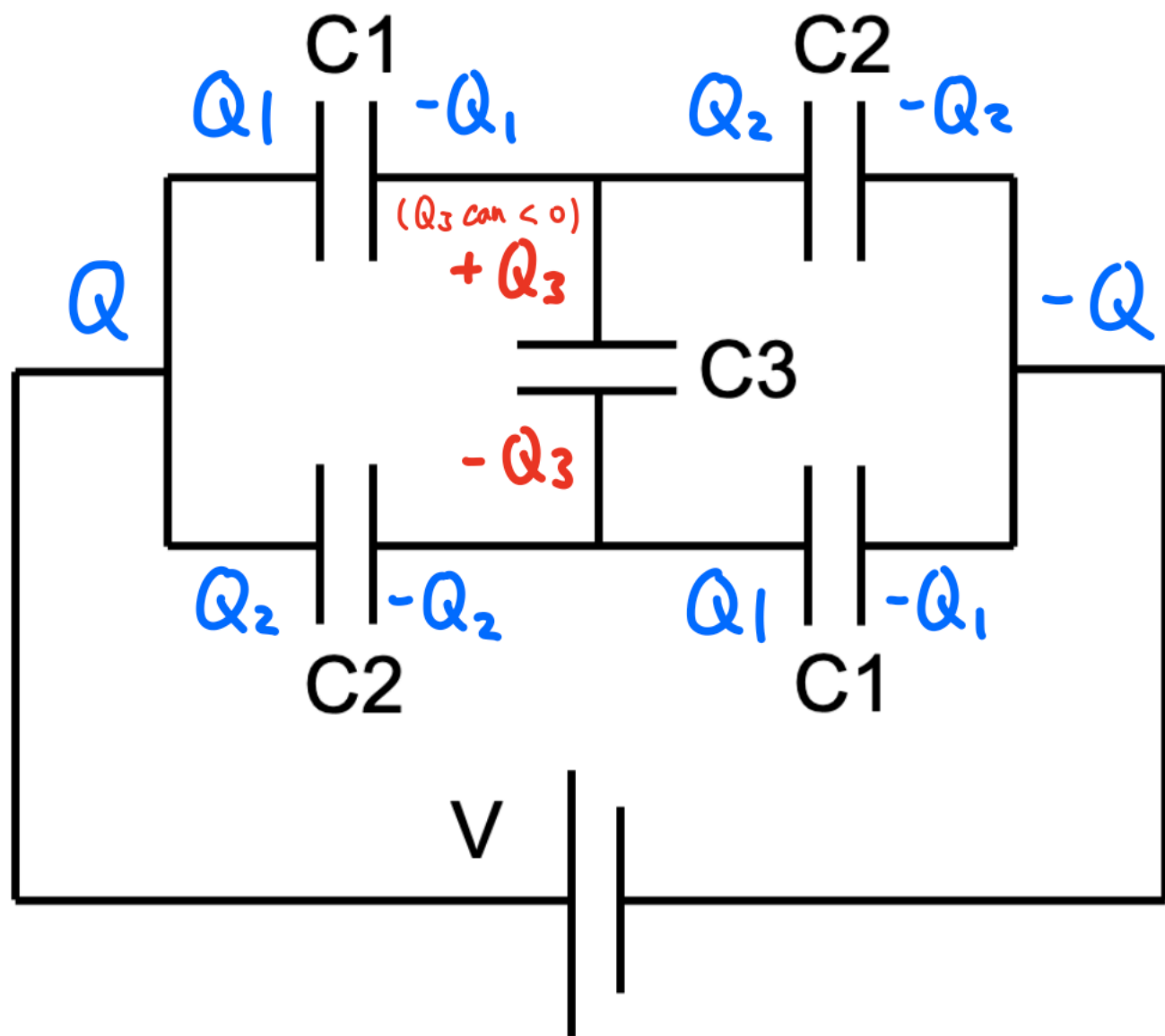
$$V_4 = \frac{1}{4\pi\epsilon_0} \frac{-6}{\sqrt{4^2 + 4^2 + 2^2}} = \frac{-6}{24\pi\epsilon_0} \quad \dots \mathbf{1}$$

$$V_D = \frac{\sigma}{2\epsilon_0} (\sqrt{z^2 + R^2} - z) = \frac{10}{6\pi\epsilon_0} \quad \dots \mathbf{3}$$

$$V_{tot} = \frac{10+27+9+(-6)}{24\pi\epsilon_0} + \frac{10}{6\pi\epsilon_0} = \frac{10}{3\pi\epsilon_0} \text{ (V)} \quad \dots \mathbf{2}$$



**Q.2** Figure 2. shows a symmetrical Wheatstone Bridge capacitor. The capacitance of  $C_1 = 3 F$ ,  $C_2 = 5 F$  and  $C_3 = 15 F$ . What is the equivalent capacitance  $C_{eq}$  of this circuit when system reaches equilibrium? (10 points)



purpose :  $C_{eq} = \frac{Q}{V}$  ①

about  $Q$  :

$$\begin{cases} Q = Q_1 + Q_2 \dots \textcircled{1} \textcircled{1} \\ -Q_1 + Q_2 + Q_3 = 0 \dots \textcircled{2} \textcircled{1} \end{cases}$$

about  $V$  :

$$\begin{cases} V = V_1 + V_2 \dots \textcircled{3} \textcircled{1} \\ = 2V_1 + V_3 \dots \textcircled{4} \\ = 2V_2 - V_3 \dots \textcircled{5} \\ V_1 + V_3 - V_2 = 0 \dots \textcircled{6} \textcircled{1} \end{cases}$$

by ② & ⑥  $\Rightarrow$  reduce to 2 variables !

$$\begin{cases} -Q_1 + Q_2 + Q_3 = 0 \\ \frac{Q_1}{C_1} - \frac{Q_2}{C_2} + \frac{Q_3}{C_3} = 0 \end{cases} \Rightarrow \begin{cases} -Q_1 + Q_2 + Q_3 = 0 \\ 5Q_1 - 3Q_2 + Q_3 = 0 \end{cases} \Rightarrow \underline{Q_1 = \frac{2}{3} Q_2 \dots \textcircled{7} \textcircled{1}}$$

plug ⑦ into ① & ③  $\Rightarrow$  reduce to 1 variables !

$$\begin{cases} Q = \frac{2}{3} Q_2 + Q_2 = \frac{5}{3} Q_2 \\ V = \frac{Q_1}{C_1} + \frac{Q_2}{C_2} = \frac{2}{9} Q_2 + \frac{1}{5} Q_2 = \frac{19}{45} Q_2 \end{cases} \Rightarrow C_{eq} = \frac{\frac{5}{3} Q_2}{\frac{19}{45} Q_2} = \underline{\underline{\frac{15}{19} F}} \textcircled{2}$$