104 學年第二學期 普通物理 B 期末考試題

Ch 29, 33-35 (2012 版) or Ch 10, 14-16 (2014 版)；2016/06/14, ***8:20 am – 09:50 am***

(i)答案卷第一張正面為封面。第一張正、反兩面<u>不要寫任何答案</u>。

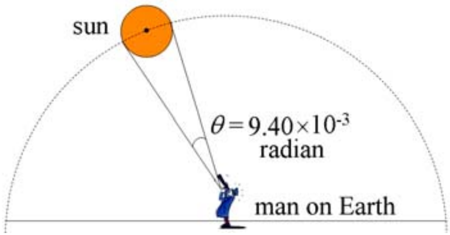
(ii)依空格號碼順序在第二張<u>正面</u>寫下所有填充題答案，不要寫計算過程。

(iii)依計算題之題號順序在第二張<u>反面</u>以後寫下演算過程與答案，<u>每題從新的一頁寫起</u>。

Useful constants: Planck constant $h = 6.63\times10^{-34}$ m^2 kg/s, speed of light $c = 3.00\times10^8$ m/s, Stefan-Boltzmann const $\sigma = 5.67\times10^{-8}$ W m^{-2} K^{-4}, electron mass = 9.11×10^{-31} kg, $\varepsilon_0 = 8.85\times10^{-12}$ A²·s⁴·kg⁻¹·m⁻³

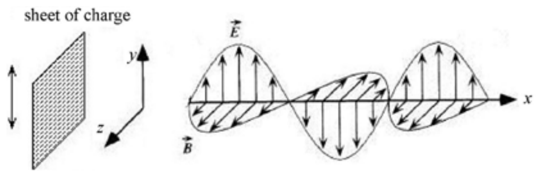
Part I. Filling the blank (5 points per blank)

- The peak in the radiation from the sunlight occurs at wavelength λ_{peak} = 500 nm. The radiation from the brightest star, Sirius, has λ_{peak} = 290 nm. Knowing the surface temperature of the sun is 5800 K, determine the surface temperature of Sirius. 【1】K.
- Which statement(s) is (are) correct? 【2】 (A) In Compton effect, the scattered photon has wavelength longer than that of the incident photon. (B) In photoelectric effect, the stopping potential does not depend on the type of cathode material. (C) At the same temperature, the de Broglie wavelength of an electron is shorter than that of a proton. (D) In an electromagnetic wave, the energy stored in its electric field is larger than that in its magnetic field. (E) The special relativity sets an upper limit on the speed of a particle (< *c*). However, there is no upper limit on the kinetic energy that a particle can have.
- To estimate the temperature of the sun (treated as ideal black body), you have measured the sunlight intensity on Earth to be 1.40 kW/m^2. Although you do not know the sun's radius nor the sun-earth distance, you have measured the sun's angle of field of view (視角) to be 9.40×10^{-3} radian. Find the sun's surface temperature. 【3】K.

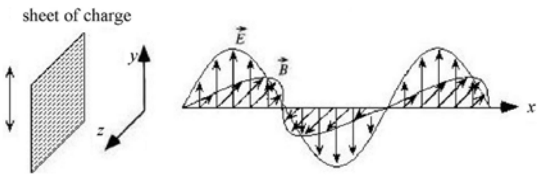


- In the hydrogen spectrum, what is the ratio of wavelength in Lyman-α line (n = 2 to n =1) to the wavelength of Balmer-α line (n = 3 to n = 2)? $\lambda_{Lyman-\alpha}/\lambda_{Balmer-\alpha}$ = 【4】.
- In Bohr model of hydrogen atom, the angular momentum of the ground state is quantized as $L = \hbar$. We can then derive the radius of the orbit to be $r = a_0$, where $a_0 = \frac{\hbar^2}{mke^2}$ = 5.29 × 10^{-11} m, is the Bohr radius. Determine the speed of the electron in the ground state. 【5】m/s.
- On a typical summer day, the intensity of sunlight at the Earth's surface is approximately I = 1000 W/m^2. What is the peak amplitude of the electric field associated with this electromagnetic wave? 【6】 V/m.
- The electric field of a plane electromagnetic wave is described as: $\vec{E}(x,t) = E_0\cos[(10m^{-1})x + (3\times10^9 s^{-1})t]\hat{k}$, where *k* is the unit vector along the *z*-axis. (a) (2 pts) In which direction does the wave propagate? Indicate the direction with a unit vector (*i*, *j*, or *k* for *x*, *y*, and *z*-direction) and an appropriate sign (+ or -).【7a】 (b) (3 pts) Write an expression for the magnetic field of the wave in terms of the quantities given and the speed of light *c*. Be sure to indicate the direction. 【7b】
- The average lifetime of muons at rest is t'=2.20 μs. You measure the decays of a beam of muons in flight in a lab, and get an average lifetime t=6.60 μs. (a) What is the speed of the muons in the lab? 【8】*c*. (b) The rest mass of the muon is 100 MeV/c^2. What is the ***kinetic*** energy of the muons in the lab? 【9】MeV
- A friend of yours who is the same age as you travels to the star Alpha Centauri, which is 4 light-years away and returns immediately. He claims that the entire trip took just 6 years. How fast did he travel? 【10】*c*.

- In your inertial reference frame, event A happens before event B, and you measure the time difference to be Δt_{AB} and the two events separated by a distance Δx_{AB}. Suppose another observer is moving along the same direction as Δx_{AB} with velocity v, how large is v if event A and B happen at the same time ($\Delta t'_{AB}=0$) in the observer's reference frame? (hint: use Lorentz transformation) 【11】
- An infinite sheet of positive charge in the y-z plane is shaken up and down in the y direction. Which of the following is a possible representation of the electromagnetic wave generated to the ***right*** of the sheet? 【12】.



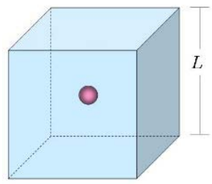
(a)



(b)

Part II Problems

1. (10 pts) 1 mW (10^{-3} watt) of light from a red laser pointer (wavelength = 650 nm) is shined on a 0.1 kg black plate and is totally absorbed. (a) Find the number of photons emitted per second from the laser pointer. (b) If the plate is on a frictionless surface, find the acceleration of the plate.
2. (15 pts) An artificial atom (人造原子) is made by putting an electron in a cubic box. Its wave function $\psi(x,y,z)$ satisfying the boundary condition,



$$\psi = 0 \ \ at \ \begin{cases} x=0, \ \ y=0, \ \ z=0 \\ x=L, \ \ y=L, \ \ z=L \end{cases}$$, has the form of a 3-dimensional standing wave:

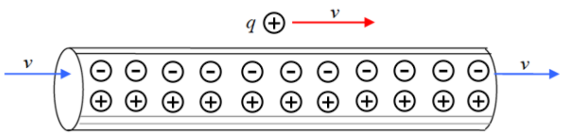
$$\psi(x,y,z) = A\sin\left(\frac{n_x \pi x}{L}\right)\cdot\sin\left(\frac{n_y \pi y}{L}\right)\cdot\sin\left(\frac{n_z \pi z}{L}\right)$$, where n_x, n_y, n_z are positive integers = 1, 2, 3….

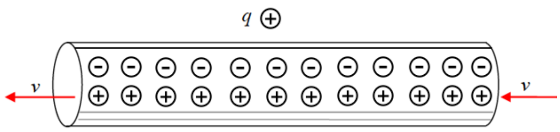
(a) Use Schrödinger equation $-\frac{\hbar^2}{2m}\left(\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}+\frac{\partial^2\psi}{\partial z^2}\right) = E\cdot\psi(x,y,z)$ to find the allowed energies of the atom.

(b) Find the normalization constant A.

(c) If L = 1.00 nm, what is the wavelength of the light emitted if it makes a transition from its first excited state to the ground state? (The value of electron mass is on the first page.)

3. (15 pts) Magnetic force as a consequence of Special Relativity: In class we briefly mentioned how the Lorentz force $\mathbf{F_B} = q\mathbf{v} \times \mathbf{B}$ arises from going from the "lab" frame to the rest frame of the moving charge. Now we'll derive it step by step. (a) The right figure shows what it looks like in the lab frame. Assume a positive external charge q is moving at the same speed of the drift velocity $\mathbf{v}$ of the negative charge careers in the wire. Let λ denote the charge density (C/m) of the positive and negative charges of the wire (they are the same in the lab frame because the wire is charge neutral in the lab.) We know that the charge q feels a magnetic force $\mathbf{F_B}=q\mu_0\lambda v^2/2\pi r$. Now the figure below shows the view from the rest frame of the external charge q. Since q is at rest, it no longer feels a magnetic force. But the charge density of the positive charges in the wire is different from the lab frame due to Lorentz contraction. Let's call it λ_+'. (a) Is λ_+' larger or smaller than λ? (2 pts) Express λ_+' in terms of λ, v, and the speed of light c. (3 pts) (b) Let λ_-' be the density of the negative charge in the wire in q's rest frame. Is λ_-' larger or smaller than λ? (2 pts). Express λ_-' in terms of λ, v, and the speed of light c. (3 pts) (c) This results in a net electrostatic force in the rest frame of q. Expand λ_+' and λ_-' using $(1+x)^n \sim 1+nx$ for a small number x, and write down the electric force $\mathbf{F_E}$ that q feels in its rest frame (2 pts). What is the condition that $\mathbf{F_E} = \mathbf{F_B}$ in the lab frame? (3 pts) Hint: The electric field at distance r away from a line charge of density λ is $E = \lambda/(2\pi\varepsilon_0 r)$.





Part I Answer Sheet，<u>**Note: 有效位數錯誤者，扣 1 分**</u>。

A【01】	10000		B【01】	a
A【02】	AE 每個選項獨立計分		B【02】	$c^2(\Delta t_{AB}/\Delta x_{AB})$
A【03】	5780		B【03】	AE 每個選項獨立計分
A【04】	5/27		B【04】	5780
A【05】	2.19×10^6		B【05】	0.95
A【06】	870		B【06】	200
A【07a】	$-i$		B【07】	10000
A【07b】	$\vec{\mathbf{B}}(x,t)=\frac{E_0}{c}\cos\left[\left(10\text{ m}^{-1}\right)x+\left(3\times10^9\text{ s}^{-1}\right)t\right]\hat{\mathbf{j}}$		B【08】	5/27
A【08】	0.95		B【09】	2.19×10^6
A【09】	200		B【10】	870
A【10】	0.8		B【11a】	$-i$
A【11】	$c^2(\Delta t_{AB}/\Delta x_{AB})$		B【11b】	$\vec{\mathbf{B}}(x,t)=\frac{E_0}{c}\cos\left[\left(10\text{ m}^{-1}\right)x+\left(3\times10^9\text{ s}^{-1}\right)t\right]\hat{\mathbf{j}}$
A【12】	a		B【12】	0.8

Part II Answer Sheet，<u>**Note: 有效位數錯誤者，扣 1 分。**</u>

【A1 = B2】

(a) power = energy/time = (dn/dt)hf, where (dn/dt) = number of photons per second

10^{-3}= (dn/dt) × 6.63 × 10^{-34} ×(3.00×10^8)/(650×10^{-9}) find (dn/dt) = 3.27 × 10^{15} photons/sec

(b) F= (dn/dt)*(h/λ)=ma, find a = 3.34 × 10^{-11} m/s^2

【A2 = B3】

(a) put the solution into the S.E., one finds:

$E = \frac{\pi^2\hbar^2}{2mL^2}\left({n_x}^2+{n_y}^2+{n_z}^2\right) = \frac{h^2}{8mL^2}\left({n_x}^2+{n_y}^2+{n_z}^2\right)$. n's are positive integers.

(b) $\iiint_{-L}^{L}\psi^2$ dxdydz=1, so $A^2\times\frac{L}{2}\times\frac{L}{2}\times\frac{L}{2}$=1, find $A = (\frac{2}{L})^{3/2}$

(c) ground state: (n_x, n_y, n_z) = (1,1,1), first excited state = (1,1,2), (1,2,1) or (2,1,1).

Energy difference = $\Delta E = \frac{3h^2}{8mL^2} = \frac{hc}{\lambda}$. Find λ = 1100 nm

【A3 = B1】

(a)$\lambda_+' > \lambda$, $\lambda_+' = \lambda * 1/\text{sqrt}(1-v^2/c^2)$

(b)$\lambda_-' < \lambda$, $\lambda_-' = \lambda * \text{sqrt}(1-v^2/c^2)$ ("Lorentz un-contracted")

(c)$E = \lambda /(2\pi * \varepsilon_0 * r) * v^2/c^2$

$F_E = F_B$ implies that $\varepsilon_0*\mu_0 = 1/c^2$