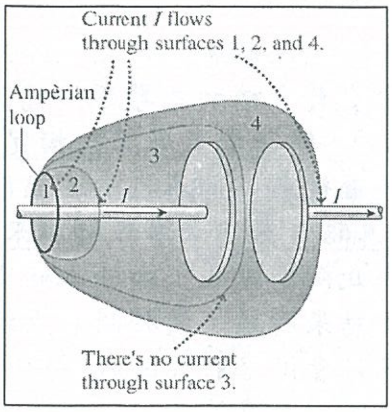
(1) Displacement current — Ampere law 的修正.

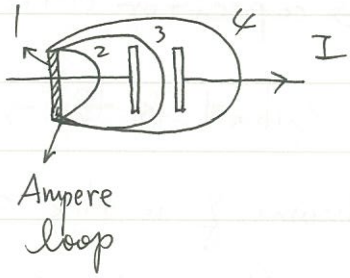
Ampere law: $\oint \vec{B}\cdot d\vec{r}=\mu_0 I$ 僅適用於 steady 電流 I.

其中的線積分為沿封閉迴路(closed loop), 而I是流過此迴路所界定(bounded)的任意曲面的電流。

Maxwell found: 封閉迴路所界定的曲面不是惟一的, i.e. 可選擇不同的曲面, 造成Ampere law 失效。如下例:



側視图



相同的Ampere loop 可界定不同的曲面1~4, 但經過 C plate 的曲面3並無電流通過.

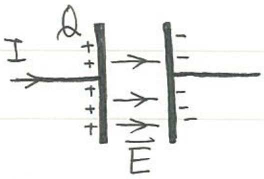
→ Ampere law does work for surface 3.

Maxwell 的修正:

displacement 電流 I_D 可在 C plates 中形成

∴ $\oint \vec{B}\cdot d\vec{r}=\mu_0(I+I_D)$

I_D = ? (Example 29.1)



設C的面積為A, 則

Q與E的關係為 $E=\frac{\sigma}{\epsilon_0}=\frac{Q}{\epsilon_0 A}$

∴ $Q=\epsilon_0 E\cdot A=\epsilon_0\Phi_E$

∵ Φ_E = 通過任意曲面的電通量, 此曲面完整包含C的一個plate, 如上圖的surface 3.

∴ $\frac{dQ}{dt}=\epsilon_0\frac{d\Phi_E}{dt}$ ⇒ I的方向與 $\frac{d\Phi_E}{dt}$ 同方向 (比較 $\mathcal{E}=-\frac{d\Phi_B}{dt}$)

$\frac{dQ}{dt}$ = wire 中由 e^- 運動形成的傳導(conduction)電流 I.

而 $\epsilon_0\frac{d\Phi}{dt}=I_D$

∵ Current 有兩種：由 charge carriers 運動形成的傳導 current I
及時變電通量 ($d\Phi_E/dt$) 形成的位移電流 I_D.

→ Ampere-Maxwell eq. $\oint \vec{B}\cdot d\vec{r} = \mu_0 \left(I + \epsilon_0 \frac{d\Phi_E}{dt} \right)$

(2) Maxwell eqs.

o Eqs:

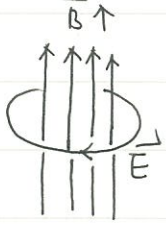
$\vec{E}$ Gauss law $\oint \vec{E}\cdot d\vec{A} = q/\epsilon_0$: q produces $\vec{E}$

$\vec{B}$ Gauss law $\oint \vec{B}\cdot d\vec{A} = 0$: No monopole

Faraday law $\oint \vec{E}\cdot d\vec{r} = -\frac{d}{dt}\Phi_B$: changing Φ_B produces $\vec{E}$

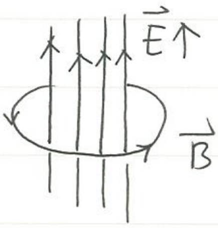
Ampere-Maxwell law $\oint \vec{B}\cdot d\vec{r} = \mu_0 I + \mu_0\epsilon_0 \frac{d\Phi_E}{dt}$: I and changing Φ_E produces $\vec{B}$

Faraday law:



$\vec{B}\perp\vec{E}$

Ampere-Maxwell law:
(A-M law)



$\vec{E}\perp\vec{B}$

{以上四方程式 + $\vec{F} = q(\vec{E} + \vec{v}\times\vec{B})$ + 電荷守恆} 可描述所有的電磁現象。

o 真空中的 Maxwell 方程式

$\oint \vec{E}\cdot d\vec{A} = 0$ ($\vec{E}$ Gauss) ; $\oint \vec{B}\cdot d\vec{A} = 0$ ($\vec{B}$ Gauss)

$\oint \vec{E}\cdot d\vec{r} = -\frac{d\Phi_B}{dt}$ (Faraday) ; $\oint \vec{B}\cdot d\vec{r} = \mu_0\epsilon_0 \frac{d\Phi_E}{dt}$ (A-M)

— the symmetry is complete

(3) EM waves

。 $$\left.\begin{aligned}\oint \vec{E}\cdot d\vec{r} &= -\frac{d}{dt}\Phi_B \\ \oint \vec{B}\cdot d\vec{r} &= \epsilon_0\mu_0\frac{d}{dt}\Phi_E\end{aligned}\right\} \vec{B}\to\vec{E}\to\vec{B}\to\vec{E}\to\cdots$$

EM waves: $\vec{E}$ and $\vec{B}$ induce each other.

The simplest EM wave: 真空中的平面波 (plane wave)

平面波:

行進方向

波前 (wave front) ⊥ 行進方向

and 波的性質在同一個波前保持固定不變。

例子: 離點波源很遠處之球面波。

∴ 真空中的EM平面波: $\vec{E}$ 與 $\vec{B}$ 互相垂直, 且都垂直於傳播方向,
所以EM wave是橫波 (transverse wave) — $\vec{E}$ or $\vec{B}$ 的擾動方向 ⊥ 行進方向。

在某一時間t的EM wave: (見Fig. 29.3, 前頁)
we prove 此形式符合 Maxwell eqs 且 EM wave 的存在.

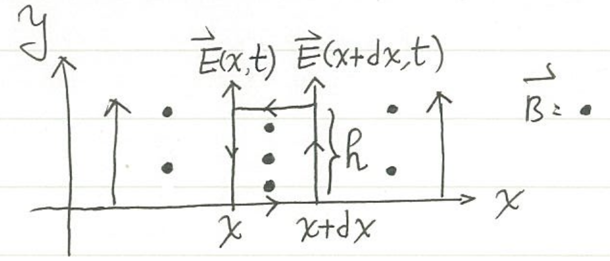
In ch 14: 向+x方向行進的 sine wave 為 $y(x,t)=A\sin(kx-\omega t)$
, where A=振幅, k=wave number$=2\pi/\lambda$,
ω=angular frequency$=2\pi f=2\pi/T$.

∴ 如圖的EM wave, we can write
$$\left.\begin{aligned}\vec{E} &= E(x,t)\,\hat{j} = E_p\sin(kx-\omega t)\,\hat{j} \\ \vec{B} &= B(x,t)\,\hat{k} = B_p\sin(kx-\omega t)\,\hat{k}\end{aligned}\right\} \vec{E} \text{ and } \vec{B} \text{ are in phase}$$

Gauss laws: 在真空中沒有 $\vec{E}$ or $\vec{B}$ 的source, ∴
$\vec{E}$ lines, $\vec{B}$ lines 都為直線.

Faraday law:

On x-y plane，積分的 loop 如右圖之逆時針長方形，



(E or B ∝ line density)

$$\therefore \oint \vec{E}\cdot d\vec{r} = [E(x+dx,t)-E(x,t)]\cdot h$$

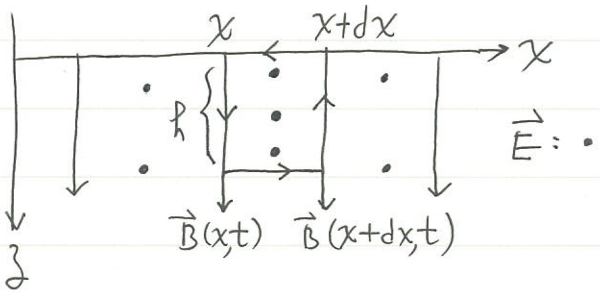
$$= -\frac{d}{dt}\Phi_B = -\frac{d}{dt}[h\cdot dx\cdot B(x,t)] = -h\cdot dx\cdot \left.\frac{dB}{dt}\right|_{\text{const. } x} = -h\cdot dx\cdot \frac{\partial B(x,t)}{\partial t}$$

$$E(x+dx,t)-E(x,t) \cong E(x,t) + dx\cdot \left.\frac{dE}{dx}\right|_{\text{const. } t} - E(x,t)$$

$$= dx\cdot \frac{\partial E(x,t)}{\partial x}$$

$$\therefore \boxed{\frac{\partial E(x,t)}{\partial x} = -\frac{\partial B(x,t)}{\partial t}} \quad \text{—(a)}$$

Similarly, using Ampere-Maxwell law on x-y plane 如右圖之積分路徑，



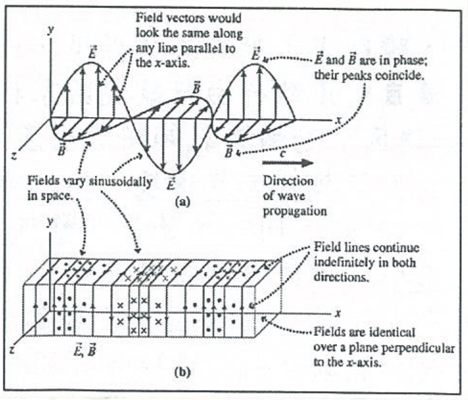
$$\therefore \oint \vec{B}\cdot d\vec{r} = -[B(x+dx,t)-B(x,t)]h$$

$$= \epsilon_0\mu_0 \frac{d}{dt}\Phi_E$$

$$= \epsilon_0\mu_0\cdot h\cdot dx\cdot \frac{\partial E(x,t)}{\partial t}$$

$$B(x+dx,t)-B(x,t) \cong dx\cdot \frac{\partial B(x,t)}{\partial x}$$

$$\therefore \boxed{\frac{\partial B(x,t)}{\partial x} = -\epsilon_0\mu_0 \frac{\partial E(x,t)}{\partial t}} \quad \text{—(b)}$$



羅

- Conditions and properties of EM waves

(i) velocity.

$$\frac{\partial}{\partial x}(a): \frac{\partial^2 E}{\partial x^2} = -\frac{\partial}{\partial x}\left(\frac{\partial B}{\partial t}\right) = -\frac{\partial^2 B}{\partial x \partial t}$$

$$-\frac{\partial}{\partial t}(b): -\frac{\partial}{\partial t}\left(\frac{\partial B}{\partial x}\right) = \epsilon_0 \mu_0 \frac{\partial^2 E}{\partial t^2}$$

$$\therefore \frac{\partial^2 E}{\partial x^2} = \epsilon_0 \mu_0 \frac{\partial^2 E}{\partial t^2}$$ 。同理可得 $$\frac{\partial^2 B}{\partial x^2} = \epsilon_0 \mu_0 \frac{\partial^2 B}{\partial t^2}$$

In Ch 14 行進速度為 v 之行進波的波方程式為

$$\frac{\partial^2 y(x,t)}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y(x,t)}{\partial t^2}$$

$\therefore$ EM wave 的 velocity $v = \frac{1}{\sqrt{\epsilon_0 \mu_0}} = 3.0 \times 10^8 \, m/s =$ 光速 c

(ii) E_p vs. B_p ; $E(x,t)$ vs. $B(x,t)$

from (a)

$$\left.\begin{aligned} \frac{\partial E}{\partial x} &= k E_p \cos(kx-\omega t) \\ \frac{\partial B}{\partial t} &= -\omega B_p \cos(kx-\omega t) \end{aligned}\right\} \Rightarrow k E_p = \omega B_p$$

or $E_p = \frac{\omega}{k} B_p = \frac{2\pi/T}{2\pi/\lambda} B_p = \frac{\lambda}{T} B_p = c B_p \Rightarrow \boxed{E_p = c B_p}$

$\therefore \boxed{E(x,t) = c B(x,t)}$

(iii) (ω, k) vs. (f, λ)

from (ii) we have $\frac{\omega}{k} = c$ and $\left.\begin{aligned} \omega &= 2\pi f \\ k &= 2\pi/\lambda \end{aligned}\right\} \Rightarrow \boxed{\lambda f = c}$

(iv) phase : $\vec{E}$ and $\vec{B}$ are in phase at time, 但 $\vec{E} \perp \vec{B}$ in space.

方向 : EM wave 的傳播方向為 $\vec{E} \times \vec{B}$

$$\vec{c} = \vec{E} \times \vec{B}$$

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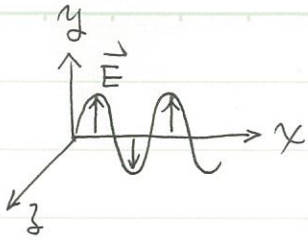
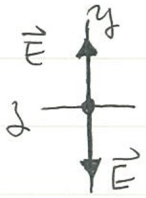
○ polarization (偏極化)



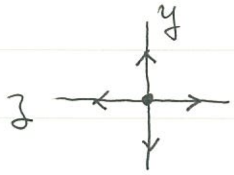
$\vec{E}$所構成之平面如右圖之x-y plane 稱為振盪平面(oscillation plane), 如此的 EM wave 為 plane-polarized wave.

polarization 定義為$\vec{E}$的振盪方向, ∴上圖的 polarization 為y軸, 用

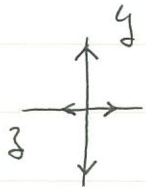


表示. 一旦 polarization 確定, $\vec{E}$的方向也確定。

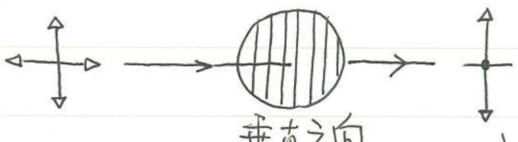
雷達波、大部分的laser都有固定的polarization, they are polarized. 但一般的光源, 如陽光、燈泡發出的光是 unpolarized or polarized randomly → 分解$\vec{E}$在振盪平面上的兩垂直軸, 則分量相等大小. (∵是randomly)

∴ 

: unpolarized

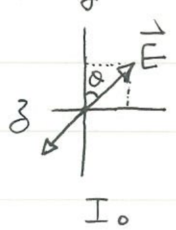
partially polarized: 

unpolarized light → polarizer(偏極片) → polarized light

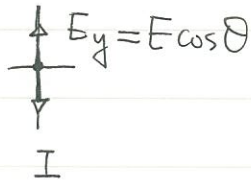


平行偏極片穿透軸的$\vec{E}$通过.
⊥偏極片穿透軸的$\vec{E}$被吸收

When $\vec{E}$和穿透軸夾θ角時.



→



∵ 強度 $I \propto$ 振幅^2

$$\therefore \frac{I}{I_0} = \frac{E^2\cos^2\theta}{E^2} = \cos^2\theta \qquad \therefore I = I_0\cos^2\theta \text{ — Malus law}$$

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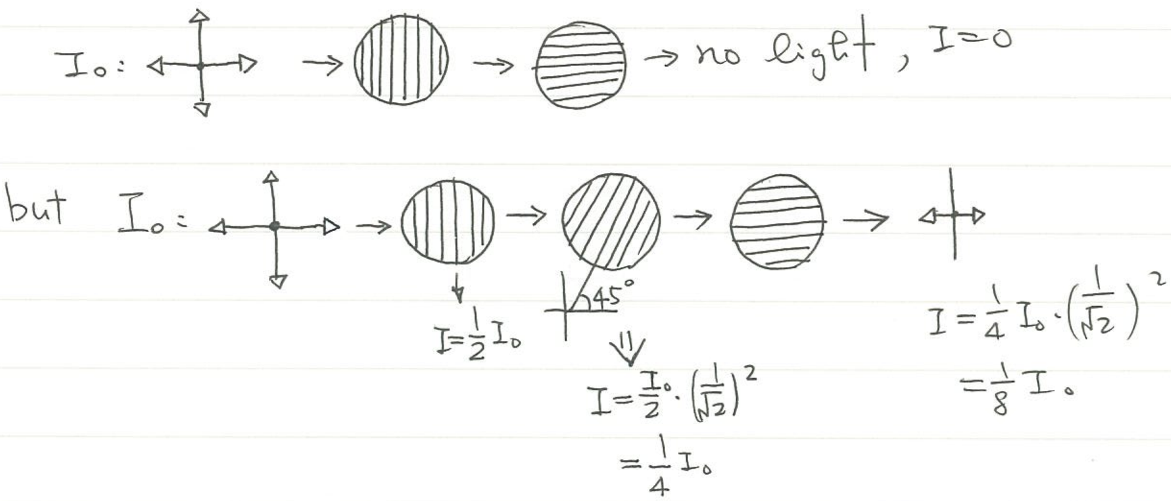
反射(reflection)或散射(scattering)都可造成光的偏極化.

觀念例題29.1



(4) EM波光譜和EM wave的製造

○ All EM wave: $f\cdot\lambda = c$ (光速)

f和λ並無極限，但不同的區段有不同的名稱.

以波長(nm)區分:

700 400

Radio ← 微波 ← IR ← $\boxed{\text{可見光}}$ → UV → X-ray → γ-ray

UV (ultraviolet)紫外線: 400 ~ 10

X-ray: 10 ~ 0.01 (固體原子間距 0.1 nm, 1 nm = 10 Å)

IR(infrared)紅外線: 700 ~ 10^3 — 溫暖物体所發射

microwave (微波): 1 mm ~ 15 cm

Radio wave: 15 cm ~ 200 m — 長距離通訊，如太空。

(i) 不同波長的EM waves和物質有不同的交互作用，一般而言，越短波長和越小尺寸的物質交互作用。

(ii) 地球大氣可讓可見光及大部分的Radio及microwave穿透，但可阻擋其他除γ-ray以後的EM wave. → 地球生命的保障.

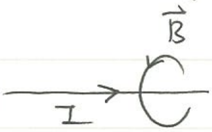
(iii) $E = hf$

羅

。EM waves的製造和接收

A changing $\vec{E}(t)$ or $\vec{B}(t)$ produces EM waves.

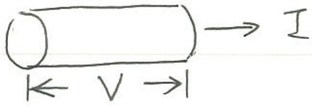
$$\left.\begin{array}{l}\text{靜電荷 } (\vec{v_d}=0) \\ \vec{v_d}=\text{constant} \rightarrow \text{steady } I\end{array}\right\} \text{ no varying } \vec{E} \text{ or } \vec{B}$$

check: (i) 

$B=\frac{\mu_0 I}{2\pi r}$ where $I=n\cdot A\cdot q\cdot v_d$

$$\therefore \frac{dB}{dt}=\frac{\mu_0}{2\pi r}\frac{dI}{dt}\propto\frac{dv_d}{dt}$$

(ii) 一段長l，電阻R，有I流通的導線

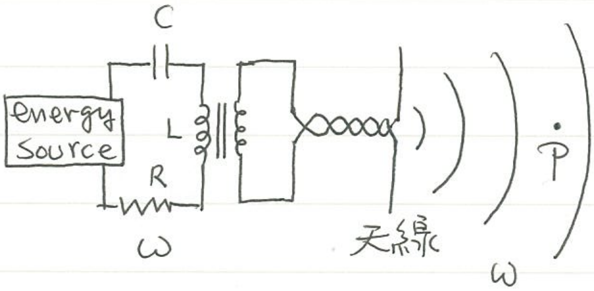


$$E=\frac{V}{l}=\frac{R}{l}\cdot I$$

$$\therefore \frac{dE}{dt}=\frac{R}{l}\frac{dI}{dt}\propto\frac{dv_d}{dt}$$

∴ changing $\vec{E}(t)$ or $\vec{B}(t)$ $\leftrightarrow$ $\frac{dE}{dt}\neq 0$ or $\frac{dB}{dt}\neq 0$ $\leftrightarrow$ $\frac{d}{dt}v_d\neq 0$

∴作加速度運動的charges產生EM waves!



charge and I 在RLC電路中振盪 → 天線中的charge振盪，天線～振盪中的electric dipole. EM wave垂直天線以LC circuit的振盪頻率輻射而出.

在場點P放置∥EM wave $\vec{E}$的導線可使導線中的e^-振盪而達到接收EM wave的目的.

雜

(5) EM wave 的能量和動量.

○ Energy of EM wave

曬太陽会熱 → EM wave = 光傳遞 energy.

地球上的石化燃料 = 累積數百萬年的太陽能.

→ EM wave 的功率:

E and B 的 energy density $u_E=\frac{1}{2}\epsilon_0 E^2$, $u_B=\frac{1}{2\mu_0}B^2$

∴ EM wave 中的 energy density $u=u_E+u_B$

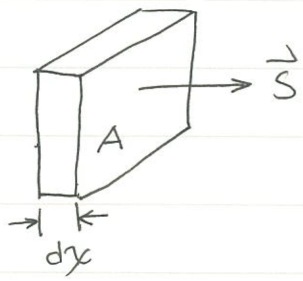
Note: $\begin{cases} E(x,t)=E_p\sin(kx-\omega t) \\ B(x,t)=B_p\sin(kx-\omega t) \end{cases}$ and $E(x,t)=cB(x,t)$, $E_p=cB_p$

$$\therefore u=\epsilon_0E^2=\frac{1}{\mu_0}B^2=\sqrt{\frac{\epsilon_0}{\mu_0}}\,EB$$

u 的平均值 $\bar{u}=\epsilon_0\overline{E^2}=\frac{1}{2}\epsilon_0E_p^2=\epsilon_0E_{rms}^2$

能量的流動率 S:

Consider 單位時間內, 通過垂直 EM wave 方向上, 單位面積的 energy =? (右圖)



體積 $dV=A\cdot dx$ 內的 energy $dU=u\cdot dV$

$\therefore dU=u\cdot A\cdot dx$

$$\rightarrow S=\frac{1}{A}\frac{dU}{dt}=\frac{1}{A}\frac{d}{dt}(u\cdot A\cdot dx)$$

$$=u\cdot\frac{dx}{dt}=u\cdot c=\sqrt{\frac{\epsilon_0}{\mu_0}}\cdot E\cdot B\cdot\frac{1}{\sqrt{\mu_0\epsilon_0}}=\frac{1}{\mu_0}EB$$

S = 能量流動的方向 // 行進方向

$\therefore \vec{S}=\frac{1}{\mu_0}\vec{E}\times\vec{B}$ = Poynting vector

S 的其他形式: (using $E=cB$)

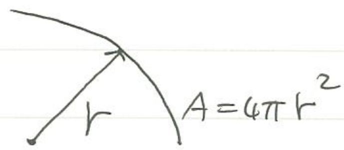
$$S=\frac{1}{\mu_0}EB=\frac{1}{c\mu_0}E^2=\frac{c}{\mu_0}B^2=cu$$

S的平均值 $\bar{S}$ = EM wave 的 intensity I
= 行進方向上單位面積的平均功率

$$\therefore \bar{S} = \frac{1}{\mu_0}\overline{EB} = \frac{1}{c\mu_0}\frac{E_p^2}{2} = \frac{c}{\mu_0}\frac{B_p^2}{2} = \frac{E_p \cdot B_p}{2\mu_0} = c\bar{u}$$

for point source → 球面波

$$\therefore I = \bar{S} = \frac{\text{power}}{\text{area}} = \frac{P}{4\pi r^2}$$



○ 動量 of EM wave

狹義相对論：$E^2 = p^2c^2 + (mc^2)^2$ → 能量 E ~ 動量 p

Maxwell proves: $\Delta p = e \cdot \frac{\Delta U}{c}$ where $e = 1$ (or 2) for 完全吸收 (or 反射) EM wave 的物体.

Δp = EM wave 的動量變化 = −(物体的動量變化)

ΔU = 物体在 Δt 内所 gain 的 energy

Newton II: $F = \frac{\Delta p}{\Delta t} = \frac{1}{c} \cdot e \cdot \frac{\Delta U}{\Delta t} = \frac{e}{c} \cdot \text{power} = \frac{e}{c} \cdot \bar{S} \cdot A$

$= \frac{e}{c} I \cdot A$

$\therefore \frac{F}{A} = P_{rad}$ = radiation pressure

$= \frac{\bar{S}}{c} \cdot e = \bar{u} \cdot e$

$\boxed{\text{Example 29.4}}$ 手机的平均発射功率为 0.6W。if 基地台接收到 EM wave 訊号的 min. E_p 值为 1.2 mV/m，則手机的 max 通訊距離 = ?

$$\frac{P}{4\pi r^2} = \bar{S} = \frac{E_p^2}{2c\mu_0}, \quad \therefore r = \sqrt{\frac{P}{4\pi\bar{S}}} = \sqrt{\frac{P}{4\pi} \cdot \frac{2c\mu_0}{E_p^2}} = 5 \text{ km}.$$

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