Before Ch 28, 討論的都是DC, but AC才是日常所遭遇。
∴ R, C, L 对 AC (Alternating-current 交流電)的反应为本ch討論主題。

(I) What is AC?

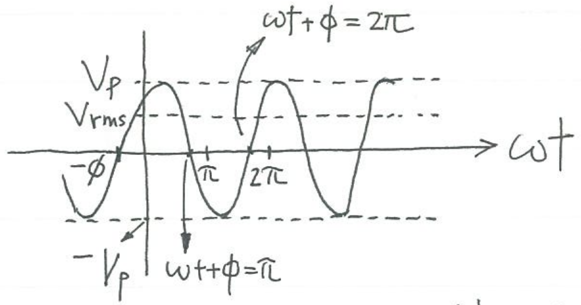
→ Sine waves of voltage or current.

Why sine wave? ∵任何波形皆可以 sine waves 合成 — Fouries th.

Like in SHM 的解为 sine wave 形式：三個描述量为

振幅 (amplitude)、頻率 (frequency, 用 f 表示) or 週期 及

phase constant ϕ : Fig. 28.1



(i) 振幅如左圖为 V_p，亦可用方均根 (root-mean-square = rms) V_{rms} 表示，$V_{rms} = V_p/\sqrt{2}$ = 電錶測量值.

Note: $\left[\int_0^T \sin^2(\omega t+\phi)\,dt\right]^{1/2} = \frac{1}{\sqrt{2}}$。 $\left(T=\frac{2\pi}{\omega}\right)$

(ii) 頻率 = f，$[f] = \sec^{-1} = Hz$ (hertz) 为常用單位.
數學上则以角頻率 ω (angular frequecy) 为便利的表示
→ $\omega = 2\pi f$ 與轉動及SHM的表示相同. $\left(f=\frac{1}{T}\right)$

(iii) phase constant ϕ : 正 slope 的 sine wave 何時與垂直軸相交

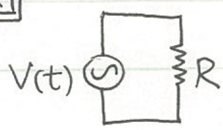
∴ AC voltage $V(t) = V_p \sin(\omega t + \phi_V)$
AC current $I(t) = I_p \sin(\omega t + \phi_I)$

羅

(2) R、C、L 对 AC 的反应

o R、C、L 個別接上 AC 電源, $V(t)=V_p \sin\omega t$, then check 電路上的電流 $I(t)$ 與 $V(t)$ 的關係.

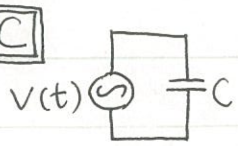
$\boxed{R}$



$$I(t)=\frac{V(t)}{R}=\frac{V_p}{R}\sin\omega t=I_p\sin\omega t$$

∴ $I(t)$ 與 $V(t)$ are in phase ($\phi=0$), i.e. 同時到達 extrems 且 $I_p=\frac{V_p}{R}$ ∴ $I_{rms}=V_{rms}/R$。

$\boxed{C}$



在 capacitor 上 $q=CV(t)$

∴ $\frac{dq(t)}{dt}=I(t)=C\frac{d}{dt}V(t)$, where $I(t)$ = capacitor current, 雖然沒有 charge across C。

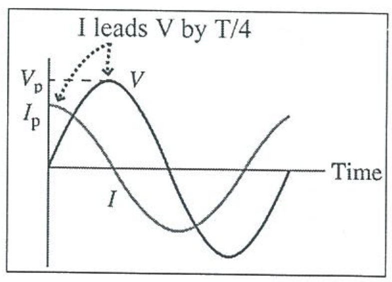
$$\therefore I(t)=\omega CV_p\cos\omega t=\omega CV_p\sin(\omega t+\tfrac{\pi}{2})=I_p\sin(\omega t+\tfrac{\pi}{2})$$

此處定義 $I_p=\omega CV_p=\frac{V_p}{1/\omega C}\equiv\frac{V_p}{X_c}$ (類 $I_p=V_p/R$)

$X_c=\frac{1}{\omega C}$ = capacitive reactance, $[X_c]=\Omega$.

∴ 流經 C 的電流 $I(t)$ 領先 (lead) 跨越 C 的電壓 $V(t)$ 達 $\frac{\pi}{2}$ $(=\frac{T}{4})$

理解(定性): 啟動電流才能累積 q 在 C plates 上, 又 $q=CV$, ∴ $I(t)$ 領先 $V(t)$。



$X_c=\frac{1}{\omega C}$ (看成是 C 的電阻) make sense?
電路上的意義?

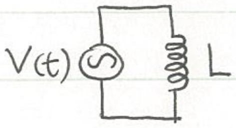
As $\omega\to 0$ ($f\to 0$), ~DC, $T\gg\tau$, ∴ C ~ 斷路 ⇒ $X_c\to\infty$

As $\omega\uparrow$ ($f\uparrow$), $T\ll\tau$, C 就像是 t=0 的狀態 i.e. C ~ 通路 (short circuit) ⇒ $X_c\to 0$。

⇒ C ~ high-pass filter (high 指的是 high frequency).

羅

$\boxed{L}$



loop rule: $V(t) - L\frac{dI}{dt} = 0$

i.e. $V_p \sin\omega t = L\frac{d}{dt}I(t)$

$\rightarrow V_p \int \sin\omega t\, dt = \int L\, dI(t) + \text{constant}$

$\therefore -\frac{V_p}{\omega}\cos\omega t = L\, I(t) + \text{constant}$, constant $=0$, ∵其代表DC量.

$\therefore I(t) = -\frac{V_p}{\omega L}\cos\omega t = \frac{V_p}{\omega L}\sin(\omega t - \frac{\pi}{2}) = I_p \sin(\omega t - \frac{\pi}{2})$

此處定義 $I_p = \frac{V_p}{\omega L} \equiv \frac{V_p}{X_L}$ (類 $I_p = V_p/R$)

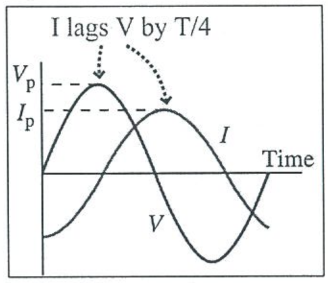
$X_L = \omega L =$ inductive reactance , $[X_L] = \Omega$.

∴流过L的電流 $I(t)$ 落後(lag) L兩端的電压 $V(t)$ 達 $\frac{\pi}{2}$ $(=\frac{T}{4})$

定性理解: ∵ $\mathcal{E}_L = -V(t)$

$= -L\frac{d}{dt}I(t)$

在 $I(t)$ 達極值前必先建立電压, ∴電压領先電流。



$X_L = \omega L$ makes sense? 電路上的意義?

As $\omega\uparrow$, i.e. $f\uparrow$, $T \ll \tau$, L ~ 斷路, $\therefore X_L \to \infty$

As $\omega \to 0$, i.e. $f \to 0$, $T \gg \tau$, L就像 $t\to\infty$ 的狀態, i.e.

L ~ 通路(short circuit), $\therefore X_L \to 0$.

$\Rightarrow$ L ~ Low-pass filter

表 28.1

電路元件	I_p vs. V_p	相位關係
R	$I_p = V_p/R$	$I(t)$ 與 $V(t)$ 同相位
C	$I_p = \frac{V_p}{X_C} = \frac{V_p}{1/\omega C}$	$I(t)$ 領先 $V(t)$ 達 $\frac{\pi}{2}$
L	$I_p = \frac{V_p}{X_L} = \frac{V_p}{\omega L}$	$I(t)$ 落後 $V(t)$ 達 $\frac{\pi}{2}$

羅

○ power in R、C、L元件

$\sim = V(t) = V_p \sin\omega t$，則經過被動元件的電流可寫成

$$I(t) = I_p \sin(\omega t + \phi)$$

∴ power in 元件 $P(t) = V(t)I(t) = I_p V_p \sin(\omega t+\phi)\cdot\sin\omega t$

$P(t)$的 mean value $\langle P(t)\rangle$ 為 $P(t)$ 一個週期的平均值

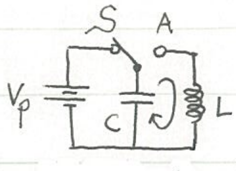
$$\therefore \langle P(t)\rangle = I_p V_p \langle \cos\phi\cdot\sin^2\omega t + \sin\phi\cdot\sin\omega t\cdot\cos\omega t\rangle$$
$$= I_p V_p\cdot\frac{1}{2}\cdot\cos\phi \qquad (\because \langle \sin\omega t\cdot\cos\omega t\rangle = 0)$$
$$= \frac{I_p}{\sqrt{2}}\cdot\frac{V_p}{\sqrt{2}}\cdot\cos\phi = I_{rms}\cdot V_{rms}\cdot\cos\phi$$

$$\therefore \cos\phi = \text{power factor}: \begin{cases} \cos\phi = 1,\ \langle P(t)\rangle = P_{max} = I_{rms}\cdot V_{rms} \\ \cos\phi = 0,\ \langle P(t)\rangle = 0. \end{cases}$$

for R: $I(t) = I_{P,R}\sin\omega t \rightarrow \phi_R = 0 \rightarrow P_{max}$: R 消耗 energy.

$$\left.\begin{array}{l} \text{for C: } I(t) = I_{P,C}\sin(\omega t + \frac{\pi}{2}),\ \langle P\rangle = 0 \\ \text{for L: } I(t) = I_{P,L}\sin(\omega t - \frac{\pi}{2}),\ \langle P\rangle = 0 \end{array}\right\}$$
C, L 不消耗 energy, but store and release energy.

∴ for system



(S→A at t=0)

$$P_C(t) = V(t)I_C(t) = \omega C V_p^2 \sin\omega t\cos\omega t = \frac{1}{2}\omega C V_p^2 \sin 2\omega t$$
$$P_L(t) = V(t)I_L(t) = \frac{-V_p^2}{\omega L}\sin\omega t\cos\omega t = -\frac{1}{2}\frac{V_p^2}{\omega L}\sin 2\omega t$$
$$= \frac{V_p^2}{2\omega L}\sin(2\omega t + \pi)$$

Energy 在 C ⟷ L transfer, $\because \frac{1}{2}\omega C V_p^2 = \frac{1}{2}\frac{V_p^2}{\omega L}$

i.e. $\omega L = \frac{1}{\omega L} \Rightarrow \omega = \frac{1}{\sqrt{LC}}$ = natural frequency.

From 能量守恆觀點: the total energy $U = U_B + U_E = \frac{1}{2}LI^2 + \frac{1}{2}CV^2$
$$= \frac{1}{2}LI^2 + \frac{q^2}{2C}$$

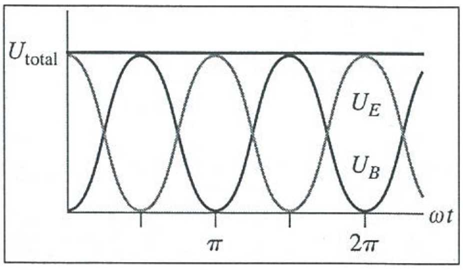
$$\frac{dU}{dt}=0=LI\frac{dI}{dt}+\frac{q}{C}\frac{dq}{dt}\text{, where } I=\frac{dq}{dt}$$

$$\therefore \frac{d^2q}{dt^2}+\frac{1}{LC}q=0 \sim \text{SHM: } \frac{d^2x}{dt^2}+\omega^2x=0$$

$$\therefore q(t)=q_p\cos\omega t,\ \omega^2=\frac{1}{LC}$$

and

$$\left.\begin{aligned} U_E&=\frac{q^2}{2C}=\frac{q_p^2}{2C}\cos^2\omega t \\ U_B&=\frac{1}{2}LI^2=\frac{L}{2}\left(\frac{dq}{dt}\right)^2=\frac{q_p^2}{2C}\sin^2\omega t \end{aligned}\right\} U_E+U_B=\frac{q_p^2}{2C}=U_{max}=\text{constant.}$$

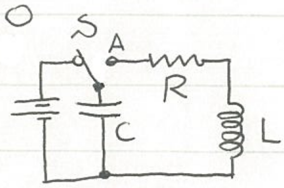


LC circuit ~ spring system (no friction)

$x \sim q$
$k \sim 1/C$
$v \sim I$
$m \sim L$

(3) RLC circuit (~ spring system with friction)

real systems have resistance and friction to dissipate energy.



at $t=0$, $S\to A$.

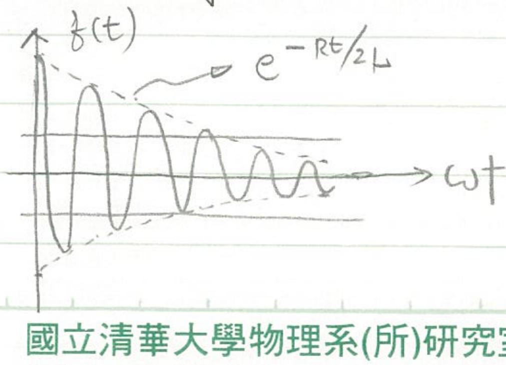
$$\therefore \frac{dU}{dt}=-I^2R=\frac{d}{dt}(U_E+U_B)=\frac{d}{dt}\left(\frac{1}{2}\frac{q^2}{C}+\frac{1}{2}LI^2\right)$$

$$\Rightarrow \frac{d^2q}{dt^2}+\frac{R}{L}\frac{dq}{dt}+\frac{q}{LC}=0$$

$\therefore q(t)=q_p e^{-Rt/2L}\cdot\cos\omega t$ = 振幅衰減的振盪 or 純衰減無振盪 (critical damping: $\frac{1}{\sqrt{LC}}=\omega=\frac{2L}{R}$)

和力學系統相同。

→使用R控制system成為振盪或衰減.



雜

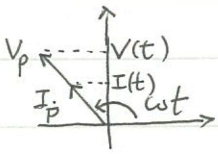
○ phasor diagram

$V(t) = V_p \sin\omega t$, $I(t) = I_p \sin(\omega t + \phi)$

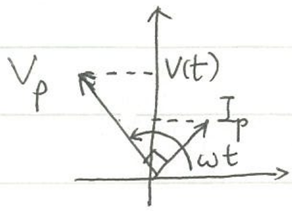
→ 以 phasor 表示:

在x-y平面上,以振幅為向量的長度: $\vec{V_p}$, $\vec{I_p}$, 與水平軸的夾角為 phase angle 如 ωt, $\omega t+\phi$, $\vec{V_p}$, $\vec{I_p}$ 以ω逆時針轉動,則 at t, V(t) or I(t) 為y軸上的投影量.

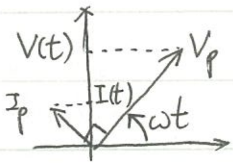
∴ For R



For L



For C



phasor的相加:

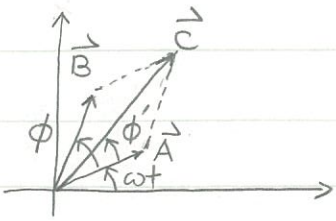
$a(t) = A\sin\omega t$, $b(t) = B\sin(\omega t+\phi)$ 則 $c(t) = a(t)+b(t)$

$= c\sin(\omega t+\phi')$

$c = ?$, $\phi' = ?$

$\Rightarrow \vec{C} = \vec{A} + \vec{B}$

→ c and ϕ'

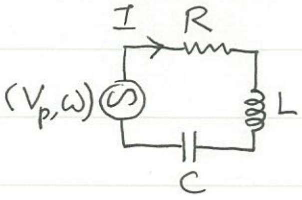


○ Driven RLC circuits and resonance

Spring-block 的力學系統,外力作功補充 friction 消耗的 energy,使 system 保持振盪狀態。

類似力學系統,RLC circuit 也可用外加的 power supply 扮演外力的角色,使 RLC circuit 保持振盪.

定性描述:



在 power supplier ⓢ 的 V_p 固定時,
變化 ω 造成的影響:

At low ω: C ~ open (∵ $X_C = \frac{1}{\omega C}$), L ~ short circuit.

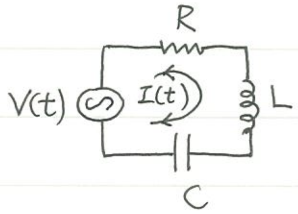
At high ω: C ~ short and L ~ open circuit (∵ $X_L = \omega L$)

In both cases, little current flows in the circuit.

∴ 有一個中間值 ω, 可使電流的振幅達 max. 此 $\omega = ?$

→ natural frequency (from 力學系統)

定量決定 $I_p(\omega) = ?$



在右圖串聯的 RLC 電路中

$$\begin{cases} V(t) = V_R(t) + V_L(t) + V_C(t) \\ I_R(t) = I_L(t) = I_C(t) = I(t) \end{cases}$$

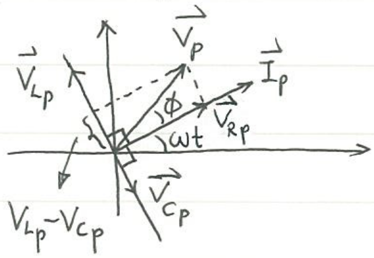
∵ 流過每個元件的電流皆相同, 即 $I(t)$, ∴ 取 $I(t)$ 的 phase angle 為零, 即 $I(t) = I_p \sin\omega t$, 則 $\underline{\underline{I_p(\omega) = ?}}$

在 phasor diagram 表達 $V(t) = V_R(t) + V_L(t) + V_C(t) = V_p \sin(\omega t + \phi)$

則 $\vec{V}_p = \vec{V}_{Rp} + \vec{V}_{Lp} + \vec{V}_{Cp}$, 其中 $\vec{V}_{Rp}$ 和 $\vec{I}_p$ 同方向

$\vec{V}_{Lp}$ 領先 $\vec{I}_p$ 90°

$\vec{V}_{Cp}$ 落後 I_p 90°



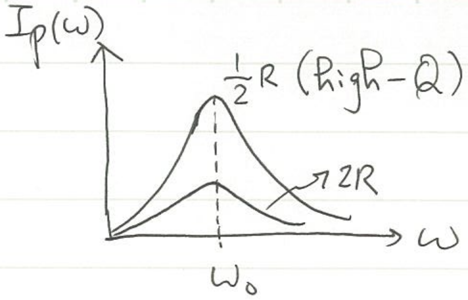
$$∴ V_p = \left[V_{Rp}^2 + (V_{Lp} - V_{Cp})^2\right]^{1/2} = \left[I_p^2 R^2 + (I_p X_L - I_p X_C)^2\right]^{1/2}$$

$$= I_p\left[R^2 + (X_L - X_C)^2\right]^{1/2} \equiv I_p Z(\omega), \text{ where}$$

$Z = [\quad]^{1/2}$ = impedance (廣義的電阻), $[Z] = \Omega$.

∴ $I_p = \frac{V_p}{Z(\omega)}$ (廣義 Ohm's law)

i.e. $I_p(\omega) = \frac{V_p}{Z(\omega)} = V_p \cdot \left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]^{-1/2}$

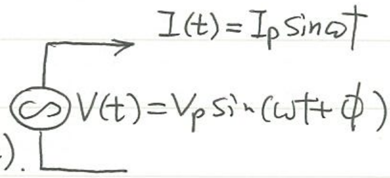


(Q-value 的定義, see problem 73)

$\Rightarrow I_p$ is max (resonance) when $X_L = X_C$

i.e. $\omega = \frac{1}{\sqrt{LC}} = \omega_0$.

又 from phasor diagram $\tan\phi = \frac{V_{Lp} - V_{Cp}}{V_{Rp}} = \frac{I(\omega L - \frac{1}{\omega C})}{IR} = \frac{\omega L - \frac{1}{\omega C}}{R}$



此處的 $\phi = \phi_V - \phi_I$ (由 phasor diagram)

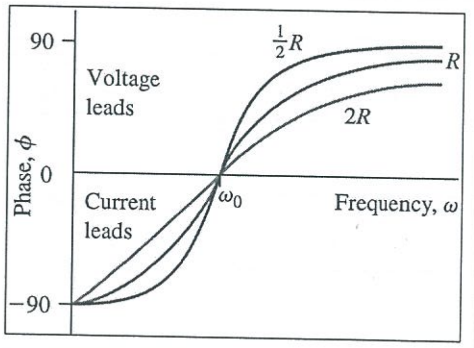
∴ when $\phi > 0$, $V(t)$ leads $I(t)$; $\phi < 0$, $V(t)$ lags $I(t)$.

At resonance, $X_L = X_C$, ∴ $\phi = 0$ →

$|IX_L| = |IX_C|$ 即 $|V_L(t)| = |V_C(t)|$, 但 $V_L(t)$ 和 $V_C(t)$ 有 180° 的相位差,

∴ 彼此抵消, 此時的 circuit 就像只有 R.

Fig. 28.18



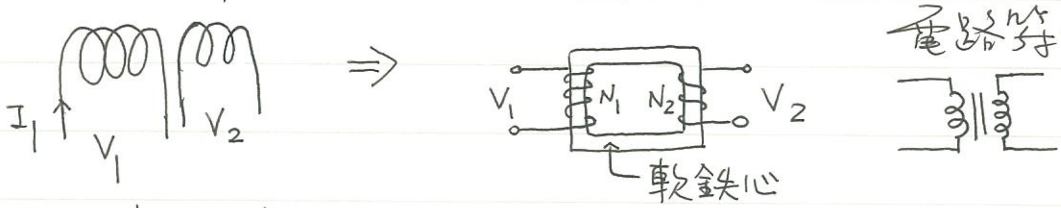
At low ω, $\omega L - \frac{1}{\omega C} < 0$, X_C dominates and $\phi < 0$, ∴ $I(t)$ 領先、

At high ω, $\omega L - \frac{1}{\omega C} > 0$, $\phi > 0$, X_L dominates, ∴ $V(t)$ 領先。

(4) Transformers and power supplies.

變壓器 (transformer)：運用廣泛的 device，大到電力輸送，小到各類的充電器。

原理：互相感應 (mutual induction)



(V_1, N_1) 為輸入端：primary coil with N_1 turns

(V_2, N_2) 為輸出端：secondary coil with N_2 turns

∴ transformer transfers electric power without direct electrical contact.

If Φ = 通過一圈線圈的磁通量，則 $V_1 = -N_1 \frac{d\Phi}{dt}$，$V_2 = -N_2 \frac{d\Phi}{dt}$

$$\therefore \frac{V_1}{V_2} = \frac{N_1}{N_2}$$

∴ { 若 $N_1 < N_2$，則輸入電壓 V_1 < 輸出電壓 V_2：step-up
若 $N_1 > N_2$，則 $V_1 > V_2$：step-down

(i) 雖 $\frac{V_1}{V_2} = \frac{N_1}{N_2}$，但 $I_1V_1 = I_2V_2$ (能量守恒)

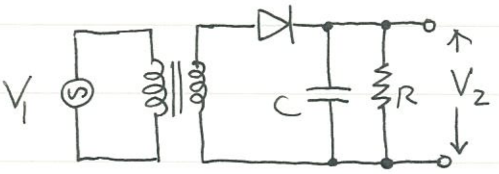
(ii) Transformers work in AC only.

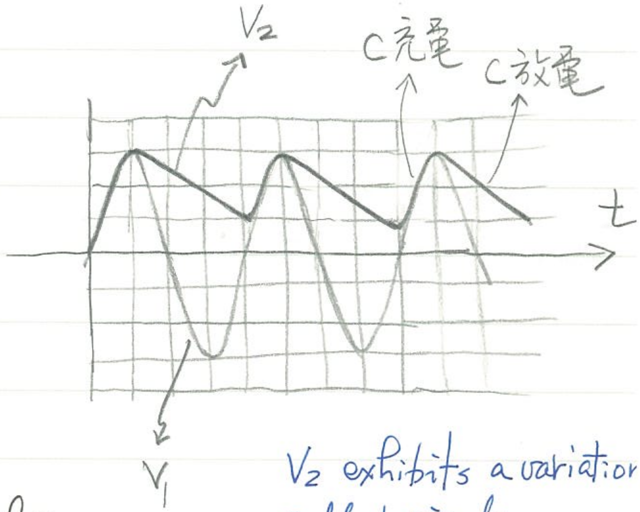
∵ 電磁感應必須在 current 隨時間變化時才會發生。

(iii) AC power 系統勝過 DC power 系統的原因：

電力傳輸以高V低I 試以減小耗損 (not 高I低V, ∵ 導線的耗損為 I^2R)，而低電压对 user 比較安全，Transformer 可以用 $\frac{V_1}{V_2} = \frac{N_1}{N_2}$ step-up and step-down 電压，達到這兩項要求。

DC power supplier





—▷|— : diode = PN junction

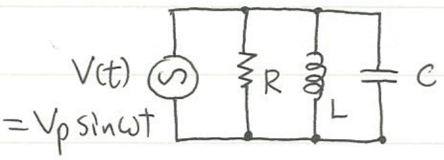
= one-way valve

砍掉(chopping off) AC 的下半部.

V2 exhibits a variation called ripple as C discharges slightly betweet cycles.

∵ C：充電→放電→充……

當 τ much longer than $\frac{1}{60}$ s (for 60-Hz AC), C 未放電至零即又開始充電。

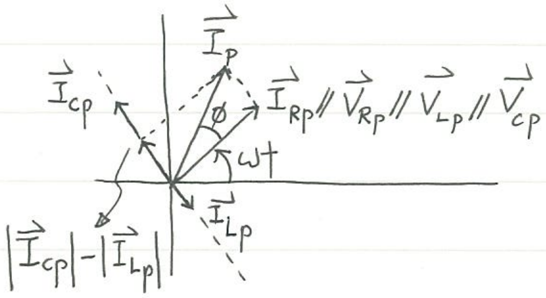


prove 左圖的 impedance

$$Z = \left[\frac{1}{R^2} + \left(\frac{1}{X_L} - \frac{1}{X_C}\right)^2\right]^{-1/2}.$$

$$V(t) = V_R(t) = V_L(t) = V_C(t) \quad \text{and} \qquad (V(t) = V_p \sin\omega t)$$

$$I(t) = I_R(t) + I_L(t) + I_C(t) = I_p$$



$$\therefore I_p^2 = I_{Rp}^2 + (I_{Lp} - I_{cp})^2$$

$$I_p = \left[\frac{V_{Rp}^2}{R^2} + \left(\frac{V_{Lp}}{X_L} - \frac{V_{cp}}{X_C}\right)^2\right]^{1/2}$$

$$= V_p\left[\frac{1}{R^2} + \left(\frac{1}{X_L} - \frac{1}{X_C}\right)^2\right]^{1/2}$$

$$\therefore V_p = I_p \cdot [\quad]^{-1/2}$$

$$= I_p \cdot Z$$

$$\therefore Z = \left[\frac{1}{R^2} + \left(\frac{1}{X_L} - \frac{1}{X_C}\right)^2\right]^{-1/2}$$

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