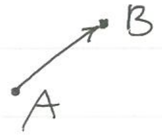
(1) ΔU

potential energy difference $\Delta U_{AB} = U_B - U_A$
$= -W_{F_c} = W_{F_{ext}}$, where F_c = 保守力,
F_{ext} = 外力.



e.g. 重力 (g=const.): $F_c \propto r^0$, 萬有引力: $F_c \propto r^{-2}$, spring force: $F_c \propto r^1$ ...

庫侖力 $\propto r^{-2}$ is also a F_c, ∵ Consider a charge q in $\vec{E}$ (created by Q)，則 q 所受庫侖力 $\vec{F}_c = q\vec{E}$, ∴ when q $A \to B$，其間的

$\Delta U_{AB} = U_B - U_A = -W_{F_c}$

$$= -\int_A^B \vec{F}_c \cdot d\vec{r} = -q\int_A^B \vec{E}\cdot d\vec{r}$$

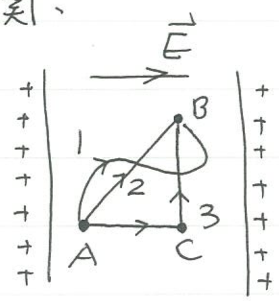
∴ $\Delta U \propto q$.
定義單位正電荷的 ΔU 為 electric potential difference ΔV (電位差)

∴ $\Delta V_{AB} \equiv \dfrac{U_{AB}}{q} = V_B - V_A = -\int_A^B \vec{E}\cdot d\vec{r}$ (與 q 無關, 與 $\vec{E}$ 有關)

∵ $\vec{E}$ 是保守場, ∴ ΔV_{AB} 與 $A \to B$ 的路徑無關.
例: for uniform $\vec{E}$ (右圖)

path 1, 2, 3 的 $\Delta V_{AB} = -\int_A^B \vec{E}\cdot d\vec{r}$

$= -\vec{E}\cdot\int_A^B d\vec{r} = -\vec{E}\cdot\Delta\vec{r}_{AB} = -E\cdot\overline{AC}$



與 path 無關。
特別是路徑 CB where $\overrightarrow{CB}\perp\vec{E}$, ∴ $\int_C^B \vec{E}\cdot d\vec{r} = 0$, i.e. $\vec{E}$ 在 $\overline{BC}$ 不对 q 作功, ∴ q 的電位保持不變, i.e., $\overline{BC}$ 上的點皆有相同的電位 ⇒ $\overline{BC}$ 為等位面 (equipotential surface): 相同電位的點所構成的平面。
⇒ 等位面 $\perp \vec{E}$。

$[V] = J/C = V$ (volt) , voltage = potential difference.

∵ When a charge q 由 $A \to B$, 則 q 所獲得的 energy $= q\cdot\Delta V_{AB} = U_{AB}$.

(~ 重力位 gh, $\Delta U_g = mg\Delta h$)

electron volt

帶 $|e|$ 的 charge 經历 $\Delta V = 1$ volt 所得到的 energy $\equiv 1$ eV (電子伏特)

∴ $1\,eV = |e|\cdot 1\,V = 1.6\times10^{-19}\,C\cdot 1\,\frac{J}{C} = 1.6\times10^{-19}\,J \Rightarrow$ eV is energy unit.

化學鍵的 bond energy ~ several eV.

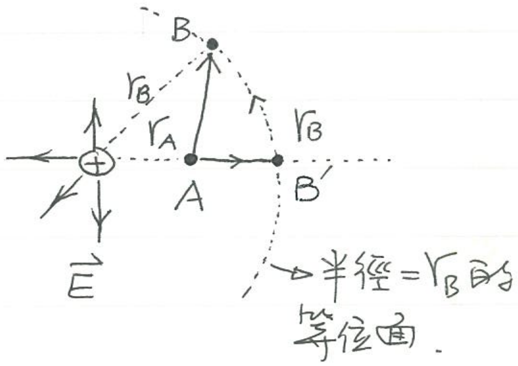
(2) ΔV 的計算

○ point charge

For 點電荷 $\vec{E} = \frac{kq}{r^2}\hat{r}$, ∴

$\Delta V_{AB} = V_B - V_A = V_{B'} - V_A$

where B and B′ 位於同是 r_B 的等位球面上.



$$\therefore \Delta V_{AB} = \int_A^{B'} \vec{E}\cdot d\vec{r} = -\int_{r_A}^{r_B} \frac{kq}{r^2}\hat{r}\cdot d\vec{r} = -kq\int_{r_A}^{r_B}\frac{dr}{r^2}$$

$$= kq\left(\frac{1}{r_B} - \frac{1}{r_A}\right) = V_B - V_A$$

$$\therefore V_B = \frac{kq}{r_B} + \text{constant}$$

⇒ choose V 的參考點: check $r_B \to \infty$, $V_B = 0 + \text{constant}$

∴ choose $r = \infty$ 為 V 的參考點, then constant $= 0$.

$$\Rightarrow V_{\infty r} = V(r) - V(\infty) = V(r) = \frac{kq}{r}.$$

[$r \to \infty$, 零交互作用, 作用力 $= 0$ like $\vec{F}_G$, $\vec{F}_{spring}$].

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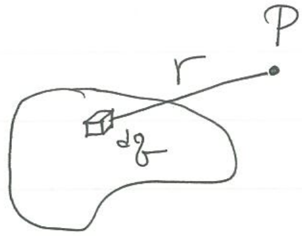
For a system of point charges q_i at r_i (w.r.t. field point P), then the total potential at P is $V(P)=\sum V_i=\sum \frac{kq_i}{r_i}$.

◦ 電荷連續体的 V

Similar to $\vec{E}$, $dV=k\frac{dq}{r}$



∴ $V=\int dV= k\int\frac{dq}{r}$ (純量相加)

Examples

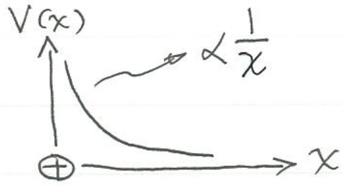
◦ V 與 $\vec{E}$ 的關係:

保守力 $\vec{F_c}$ 與 U: $\vec{F_c}=-\vec{\nabla}U$, where $\vec{\nabla}=\hat{i}\frac{\partial}{\partial x}+\hat{j}\frac{\partial}{\partial y}+\hat{k}\frac{\partial}{\partial z}$.

Here $\vec{F_c}=q\vec{E}=-\vec{\nabla}U=-\vec{\nabla}(qV)$

∴ $\vec{E}=-\vec{\nabla}V$ where $\vec{\nabla}V$: gradient (梯度) of V ⇒ V → $\vec{E}$

For 1D: $E=-\frac{dV}{dx}$



1D 的梯度 = 斜率 $\frac{dV}{dx}$

⇒ slope 越大的地方, |E| 越強.

(3) 帶電導体

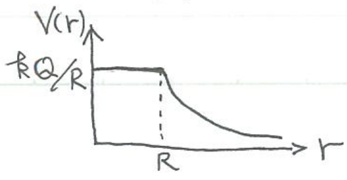
charged conductor 的特徵: E=0 inside and $\vec{E}$ lines ⊥ surface,

⇒ 在達靜電平衡時, charged conductor 的 surface 為等位面.

內部的 E=0 ⇒ ∴ V= constant inside conductor.

For a charged sphere (Q, R)

$V(R)=\frac{kQ}{R}=k\cdot(4\pi R^2\sigma)/R=\frac{\sigma R}{\epsilon_0}$, where σ = surface charge density.



(Example 22.3)

將不同半徑R_1, R_2的金屬球以導線連接，如右圖、

⇒ 兩個球面及導線皆為等位面，

$\therefore \frac{kQ_1}{R_1} = \frac{kQ_2}{R_2}$ 且 $Q = \sigma \cdot \pi R^2$

$\Rightarrow \sigma_1 R_1 = \sigma_2 R_2$ 即 $\frac{\sigma_1}{\sigma_2} = \frac{R_2}{R_1}$ ($\because E \propto \sigma$)

$\therefore \sigma \propto \frac{1}{R}$，半徑愈小的球面，$\sigma$愈高 ⇒ E愈高

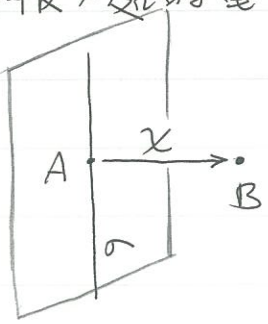
∴ 愈尖的地方（半徑愈小），產生的E愈高.

⇒ 避雷針原理

越小顆粒的dust越容易產放電現象.

$\boxed{22.2}$ 表面電荷密度 σ 的無限大平板，離平板 x 處的電位 = ?

$E = \frac{\sigma}{2\epsilon_0}$ = constant, 向外.



$\therefore \Delta V_{AB} = V_B - V_A = -\int_A^B \vec{E}\cdot d\vec{r}$

$= -\vec{E}\cdot\int_A^B d\vec{r} = -\vec{E}\cdot\vec{AB}$, where $\vec{E} \parallel \vec{AB}$

$= -E\cdot\overline{AB} = -E\cdot x = -\frac{\sigma}{2\epsilon_0}x$

$\therefore$ for positive charge　　　　for negative charge

$\boxed{22.3}$ 金屬帶電球 (Q,R), 則 (a) 球面的電位 $V(R) = ?$

(b) $W = ?$ for bringing p^+ from ∞ to R. (c) $\Delta V_{R\to 2R} = ?$

(a) 金屬球的 $\vec{E}$ ~ point charge 的 $\vec{E} = \frac{kQ}{r^2}\hat{r} \rightarrow V(r) = \frac{kQ}{r}$.

$\therefore$ 金屬球的球面電位 $V(R) = kQ/R$.

(b) Note: $V(R) = V_{\infty R}$ in (a)

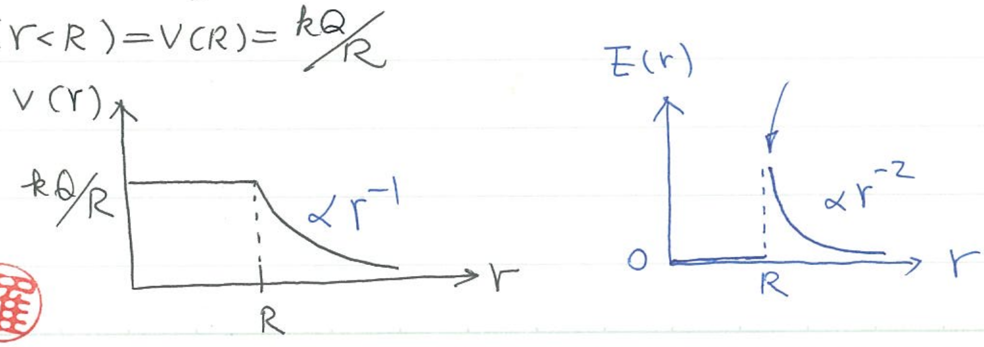
$\therefore W = |e|\cdot V_{\infty R} = kQ/R$ (eV).

(c) $\Delta V_{R\to 2R} = V(2R) - V(R) = kQ\left(\frac{1}{2R} - \frac{1}{R}\right) = -\frac{kQ}{2R}$.

Notice: $E(r<R) = 0$, $\therefore V(r<R)$ = constant = ?

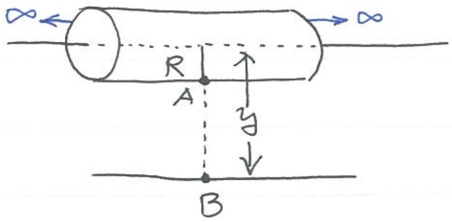
$V(r<R) = V(R) = kQ/R$

$\Rightarrow$



22.4 半径R的無限长電力線，帶有 linear charge density λ，則距線中心 y 公尺處地線的電压=？

如右图之



$$\Delta V_{AB} = -\int_A^B \vec{E}\cdot d\vec{r}$$

where $E = \dfrac{\lambda}{2\pi\epsilon_0 r}$，

方向：以線為中心輻射向外，∴ 平行 $\overrightarrow{AB}$ 的 $d\vec{r}$

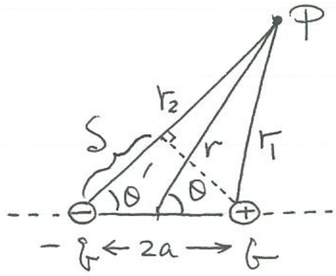
$$\therefore \Delta V_{AB} = -\int_A^B \frac{\lambda}{2\pi\epsilon_0 r}\cdot dr$$ where at A, $r=R$, and B, $r=y$.

$$= -\frac{\lambda}{2\pi\epsilon_0}\int_R^y \frac{dr}{r} = \frac{\lambda}{2\pi\epsilon_0}\ln\frac{R}{y} \quad (<0,\ \because R<y)$$

22.5 Potential of dipole

如右圖的 dipole，則 $V(p)=$？

在 $r \gg a$ 時 $V(p)=$？



$$V(p) = \sum \frac{kq_i}{r_i} = \frac{kq}{r_1} - \frac{kq}{r_2}$$

$$= kq(r_2 - r_1)/r_1 r_2 .$$

Far field: $r \gg a \Rightarrow r_1 \sim r \sim r_2$ and $\theta \sim \theta'$

$$\therefore r_2 - r_1 = S \cong 2a\cos\theta$$

$$\therefore V(p) = V(r,\theta) = kq\cdot 2a\cos\theta / r^2$$

$$= \frac{kp}{r^2}\cos\theta ,\ \text{where } p = 2aq$$

$$\propto r^{-2} \quad (\text{Note: } E \propto r^{-3} \text{ for far-field})$$

在中垂線上 $(\theta = \frac{\pi}{2})$，$V(p) = 0$。

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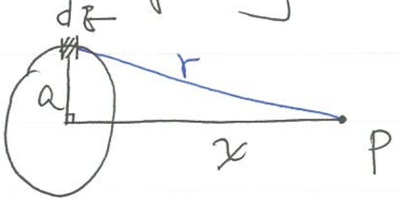
$\boxed{22.6}$ A uniformly charged ring (Q, a) 的中心軸上，距 ring 中心 x 處的 $V(x) = ?$



$$dV = \frac{k\,dq}{r} = \frac{k\,dq}{\sqrt{a^2+x^2}}$$

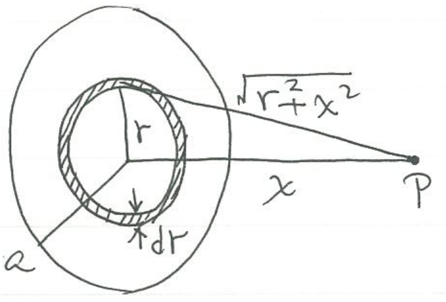
$$\therefore V = \int dV = \frac{k}{\sqrt{a^2+x^2}} \int dq = \frac{kQ}{\sqrt{a^2+x^2}}$$

Check: (i) Near field at $x=0 \Rightarrow V(x=0) = \frac{kQ}{a}$ = constant. (純量相加)

(ii) Far field when $x \gg a \Rightarrow V(x) \cong \frac{kQ}{x}$ ~ point charge.

(iii) $E(x) = -\frac{dV}{dx} = \frac{kQx}{(a^2+x^2)^{3/2}}$ = results of Example 20.6.

$\boxed{22.7}$ A uniformly charged disk (Q, a)，中心軸 x 處的 $V(x) = ?$



dq = 半徑 r，寬度 dr 的 ring 所帶 charge

$= \sigma \cdot dA$ (dA = ring 的面積)

$= \sigma \cdot 2\pi r \cdot dr$, where $\sigma = \frac{Q}{\pi a^2}$ = surface charge density

$$\therefore dV = \frac{k\,dq}{\sqrt{r^2+x^2}} = 2\pi k\sigma \cdot \frac{r\,dr}{\sqrt{r^2+x^2}}$$

$$V(x) = \int_{r=0}^{r=a} dV = 2\pi k\sigma \int_0^a \frac{r\,dr}{\sqrt{r^2+x^2}} = \frac{2kQ}{a^2}\left(\sqrt{a^2+x^2} - |x|\right)$$

$$= 2\pi k\sigma\left(\sqrt{a^2+x^2} - x\right) \text{ for positive } x.$$

Check: (i) Near field at disk center, $V(x=0) = 2kQ/a = \frac{\sigma}{2\epsilon_0} \cdot a$

(ii) Far field when $x \gg a$ (for positive x)

$$V(x) = \frac{2kQ}{a^2} x \left[\left(1+\frac{a^2}{x^2}\right)^{1/2} - 1\right] \simeq \frac{2kQ}{a^2} x \left[\left(1+\frac{1}{2}\frac{a^2}{x^2}\right) - 1\right]$$

$= kQ/x$ ~ point charge of Q.

(iii) $E(x) = -\frac{dV}{dx}$ (for positive x)

= Example $\boxed{22.8}$

$$= -\frac{2kQ}{a^2}\left[\frac{1}{2}(a^2+x^2)^{-1/2} \cdot 2x - 1\right] = \frac{2kQ}{a^2}\left(1 - \frac{x}{\sqrt{a^2+x^2}}\right)$$

$= 2\pi k\sigma(\quad)$

= results of problem 20.71

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