

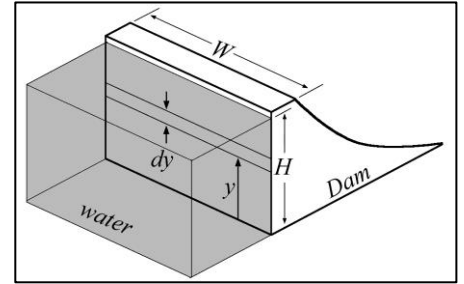
Open book. 寫下詳細的計算過程。

學號：_____，姓名：_____

1.

A dam has a width of W as shown in right figure. The surface of the freshwater behind the dam is filled to a height H of the dam.

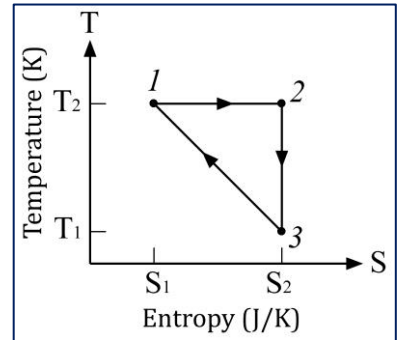
- (a) Find the net horizontal force exerted by the water on the dam.
- (b) Find the torque exerted by the water about an axis along the bottom of the dam. (20%)



2.

One mole of a diatomic ideal gas is taken reversibly around the cycle shown in the T - S diagram (right figure). **The molecules do not rotate or oscillate.** Assume that the volume in state 1 is V_1 .

- (a) Find the heat Q transferred into the gas system for paths $1 \rightarrow 2$ and $2 \rightarrow 3$, and the full cycle.
- (b) Find the works done by the gas for the isothermal and adiabatic processes.
- (c) Find the volumes in state 2 and state 3.
- (d) Find the change of internal energy ΔU for the full cycle? (40%)



期末考場地分配、考試時間第二次段考相同。

Ans:

1.

(a)

$$p = \frac{dF}{dA} = \rho gh, \text{ where } h = H - y \text{ and } dA = W \cdot dy. \therefore F = \int dF = \int_0^H \rho h(H - y) \cdot W \cdot dy = \frac{1}{2} \rho g W H^2.$$

(b)

$$d\tau = dF \cdot y = \rho g y(H - y) \cdot dA. \therefore \tau = \int d\tau = \int_0^H \rho g W y(H - y) \cdot dy = \frac{1}{6} \rho g W H^3.$$

2.

 $n = 1$ and $\gamma = 5/3$.1→2 is isothermal at T_2 , so $\Delta U = 0 = Q_{12} - W_{12}$. $\Delta S = S_2 - S_1 = Q_{12}/T_2 > 0$, therefore, $Q_{12} = W_{12} > 0$.Gas does positive work W_{12} by adsorbing heat Q_{12} .2→3: because $\Delta S = 0$, means $Q_{23} = 0$. So 2→3 is adiabatic and $\Delta T < 0$.Gas does positive work W_{23} .(a) $Q = Q_{12} + Q_{23} + Q_{31}$

$$Q_{12}: \Delta S = S_2 - S_1 = \int_1^2 \frac{dQ}{T} = \frac{1}{T_2} \int_1^2 dQ = \frac{Q_{12}}{T_2}, \text{ so } Q_{12} = T_2(S_2 - S_1)$$

$$Q_{23} = 0$$

$$Q_{31}: dS = dQ/T, \text{ so } dQ = T \times dS \text{ and } T = M(S - S_2) + T_1, \text{ where } M = \text{slope} = (T_2 - T_1)/(S_1 - S_2).$$

$$\text{So } dQ = [M(S - S_2) + T_1] \times dS$$

$$Q_{31} = \int_3^1 dQ = \int_3^1 [M(S - S_2) + T_1] \cdot dS = \frac{1}{2}(T_1 + T_2)(S_1 - S_2)$$

$$Q = Q_{12} + Q_{23} + Q_{31} = \frac{1}{2}(T_2 - T_1)(S_2 - S_1) = \text{area enclosed by the cycle.}$$

(b) isothermal: 1→2, $W_{12} = Q_{12} = T_2(S_2 - S_1) (> 0)$

$$\text{Adiabatic: } 2 \rightarrow 3, W_{23} = \left| \frac{p_3 V_3 - p_2 V_2}{\gamma - 1} \right| = \frac{nR}{\gamma - 1} |T_3 - T_2| = \frac{3R}{2} (T_2 - T_1) (> 0)$$

(c) from (b) $W_{12} = nRT_2 \ln(V_2/V_1) = T_2(S_2 - S_1)$, where $n = 1$. So $V_2 = V_1 \cdot \exp\left(\frac{S_2 - S_1}{R}\right)$.Sine 2→3 is adiabatic, so $T_2 \cdot V_2^{\gamma-1} = T_3 \cdot V_3^{\gamma-1}$, where $T_3 = T_1$.

$$\text{Therefore, } V_3 = V_2 \cdot \left(\frac{T_2}{T_1}\right)^{\frac{1}{\gamma-1}} = V_1 \cdot \left(\frac{T_2}{T_1}\right)^{3/2} \cdot \exp\left(\frac{S_2 - S_1}{R}\right)$$

(d) $\Delta U = Q - W = 0$ for a full cycle.