Ans:

1.

(a)

$$p = \frac{dF}{dA} = \rho gh, \text{where } h = H - y \text{ and } dA = W \cdot dy. \therefore F = \int dF = \int_0^H \rho h(H-y) \cdot W \cdot dy = \frac{1}{2}\rho gWH^2.$$

(b)

$$d\tau = dF \cdot y = \rho gy(H-y) \cdot dA. \therefore \tau = \int d\tau = \int_0^H \rho gWy(H-y) \cdot dy = \frac{1}{6}\rho gWH^3.$$

2.

$n = 1$ and $\gamma = 5/3$.

1→2 is isothermal at T_2, so $\Delta U = 0 = Q_{12} - W_{12}$. $\Delta S = S_2 - S_1 = Q_{12}/T_2 > 0$, therefore, $Q_{12} = W_{12} > 0$.

Gas does positive work W_{12} by adsorbing heat Q_{12}.

2→3: because $\Delta S = 0$, means $Q_{23} = 0$. So 2→3 is adiabatic and $\Delta T < 0$.

Gas does positive work W_{23}.

(a) $Q = Q_{12} + Q_{23} + Q_{31}$

Q_{12}: $\Delta S = S_2 - S_1 = \int_1^2 \frac{dQ}{T} = \frac{1}{T_2}\int_1^2 dQ = \frac{Q_{12}}{T_2}$, so $Q_{12} = T_2(S_2 - S_1)$

$Q_{23} = 0$

Q_{31}: $dS = dQ/T$, so $dQ = T \times dS$ and $T = M(S - S_2) + T_1$, where M = slope = $(T_2 - T_1)/(S_1 - S_2)$.

So $dQ = [M(S - S_2) + T_1] \times dS$

$$Q_{31} = \int_3^1 dQ = \int_3^1 [M(S - S_2) + T_1] \cdot dS = \frac{1}{2}(T_1 + T_2)(S_1 - S_2)$$

$Q = Q_{12} + Q_{23} + Q_{31} = \frac{1}{2}(T_2 - T_1)(S_2 - S_1)$ = area enclosed by the cycle.

(b) isothermal: 1→2, $W_{12} = Q_{12} = T_2(S_2 - S_1)$ (> 0)

Adiabatic: 2→3, $W_{23} = \left|\frac{p_3V_3 - p_2V_2}{\gamma - 1}\right| = \frac{nR}{\gamma - 1}|T_3 - T_2| = \frac{3R}{2}(T_2 - T_1)$ (> 0)

(c) from (b) $W_{12} = nRT_2\ln(V_2/V_1) = T_2(S_2 - S_1)$, where $n = 1$. So $V_2 = V_1 \cdot \exp(\frac{S_2 - S_1}{R})$.

Sine 2→3 is adiabatic, so $T_2 \cdot V_2^{\gamma-1} = T_3 \cdot V_3^{\gamma-1}$, where $T_3 = T_1$.

Therefore, $V_3 = V_2 \cdot (\frac{T_2}{T_1})^{\frac{1}{\gamma-1}} = V_1 \cdot (\frac{T_2}{T_1})^{3/2} \cdot \exp(\frac{S_2 - S_1}{R})$

(d) $\Delta U = Q - W = 0$ for a full cycle.