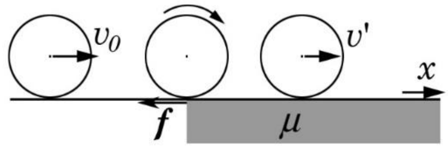
1.

The differential shell of radius $r$ and mass $dm$ inside the sphere will possess differential inertial $dI = \frac{2}{3} dm \cdot r^2$. Therefore, the total inertial $I = \int dI = \frac{2}{3}\int dm \cdot r^2 = \frac{2\rho}{3}\int dV \cdot r^2$, where ρ = density = $M$ / $V$ = $\frac{M}{\frac{4}{3}\pi R^3}$ and volume of shell $dV = 4\pi r^2 \cdot dr$.

So $I = \frac{2\rho}{3} 4\pi \int_0^R r^4 \cdot dr = \frac{2}{5}MR^2$.

2.




(a) Linear motion: $-f = -\mu N = -\mu Mg = Ma, \therefore a = -\mu g$

Rotational motion: $\tau = fR = I\alpha = \frac{2}{5}MR^2 \cdot \alpha, \therefore \alpha = \frac{5}{2R}\mu g$

If $v_0$ is reduced to $v'$ to start pure rolling, then

Linear motion: $v' = v_0 + at = v_0 - \mu g t$

Rotational motion: $\omega' = \alpha t = \frac{5}{2R}\mu g t$

And rolling starts when $v' = R\omega'$, that is $v_0 - \mu g t = \frac{5}{2R}\mu g t R. \therefore t = \frac{2v_0}{7\mu g}$ $and$ $v' = \frac{5}{7}v_0$.

(b) $v'^2 = v_0^2 + 2ax,$ where $v' = \frac{5}{7}v_0$ $and$ $a = -\mu g, \therefore x = \frac{12v_0^2}{49\mu g}$.

3.

(a) Total energy of the system $E$ is the sum of (i) potential energy of spring $U = kx^2/2$, (ii) kinetic rotational energy of pulley $K_{\text{pulley}} = I\omega^2/2$, and (iii) gravitational potential energy and kinetic energy of $m$ $U + K = \pm mgx + mv^2/2$.

Therefore, $E = \frac{1}{2}kx^2 + \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2 \pm mgx,$ where $I = \frac{1}{2}MR^2$ $and$ $\omega = \frac{v}{R}$.

So $E = \frac{1}{2}\left(m + \frac{M}{2}\right)v^2 + \frac{1}{2}kx^2 \pm mgx$. (少寫最後一項扣 2 分)

(b) By using $\frac{d}{dt}E = 0,$ we get $\frac{d^2x}{dt^2} + \frac{2k}{2m+M}[x \pm \frac{mg(2m+M)}{2k}] = 0$.

Let $x' = x \pm \frac{mg(2m+M)}{2k},$ then $\frac{d^2x'}{dt^2} + \frac{2k}{2m+M}x' = 0$

So the angular frequency $\omega = \sqrt{\frac{2k}{2m+M}}$.

4.

(a) Tension at position $x$ is $F = m'g = \mu \cdot x \cdot g = M \cdot x \cdot g/L$, where $\mu$ is the linear density of rope.

(b) $v(x) = \sqrt{\frac{F}{\mu}} = \sqrt{g \cdot x}$

(c) Total travel time $t = \int \frac{dx}{v(x)} = \int_0^L \frac{dx}{\sqrt{g \cdot x}} = 2\sqrt{\frac{L}{g}}$.