106 學年第一學期 普通物理 B 第二次段考試題

[Wolfson Ch. 10-14, 32]　2017/12/5, ***8:20am – 9:50am***

(i)答案卷第一張正面為封面。第一張正、反兩面<u>**不要寫任何答案**</u>。

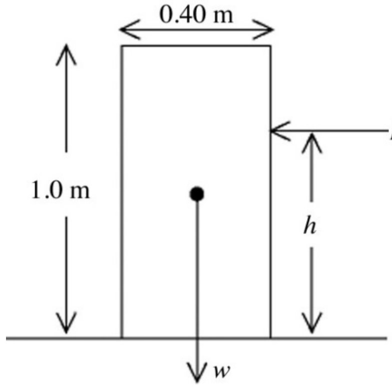
(ii)依空格號碼順序在第二張<u>**正面**</u>寫下所有填充題答案，不要寫計算過程。

(iii)依計算題之題號順序在第二張<u>**反面**</u>以後寫下演算過程與答案，<u>**每題從新的一頁寫起**</u>。

Universal Constant: Gravitational constant G = $6.67\times10^{-11} Nm^2/kg^2$, gravitational field on Earth g = 9.81 m/s^2, 1 erg = 100 nJ. 1 Å = 10^{-10}m. speed of sound = 343 m/s. Rotational Inertia of a ball: I = 2/5 MR^2. Volume of a ball: V = 4/3 π r^3. Rotational Inertia of a cylinder / disk: I = 1/2 MR^2

Part I. Filling the blank (5 points per blank)

• In the figure, a uniform rectangular crate 0.40 m wide and 1.0 m tall rests on a horizontal surface. The crate weighs 930 N, and its center of gravity is at its geometric center. A horizontal force F is applied at a distance h above the floor. If h = 0.61 m, what minimum value of F is required to make the crate start to tip over? Static friction is large enough that the crate does not start to slide. <u>【1】</u>N



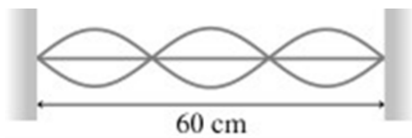
• A uniform solid sphere of mass M and radius R rotates with an angular speed ω about an axis through its center. A uniform solid cylinder of mass M, radius R, and length 2R rotates through an axis running through the central axis of the cylinder. What must be the angular speed of the cylinder so it will have the same rotational kinetic energy as the sphere?【2】

• While spinning down from 500.0 rpm to rest, a solid uniform flywheel does 5.1 kJ of work. If the radius of the disk is 1.2 m, what is its mass?【3】kg

• A mass M is attached to an ideal massless spring. When this system is set in motion, it has a period T. What is the period if the mass is doubled to $2M$? 【4】

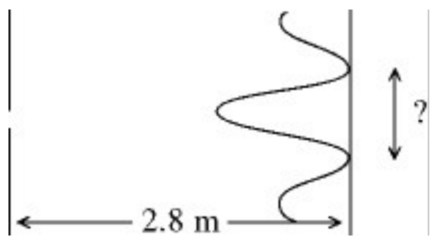
• A certain source of sound waves radiates uniformly in all directions. At a distance of 20 m from the source the intensity level is 51 db. What is the total power output of the source, in watts?
【5】W (Note: The reference intensity I_0 is 1.0×10^{-12} W/m^2.)

• A standing wave is oscillating at 690 Hz on a string, as shown in the figure. What is the wave number k? 【6】1/cm

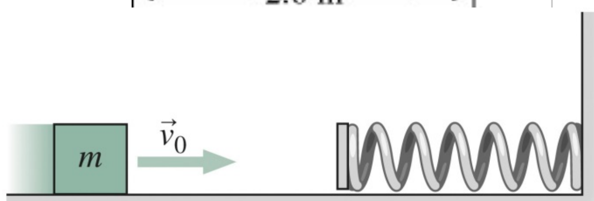


• The angle that a swinging simple pendulum makes with the vertical obeys the equation $\Theta(t)$ = (0.150 rad) cos[(2.85 rad/s)t + 1.66]. What is the length of the pendulum? 【7】m

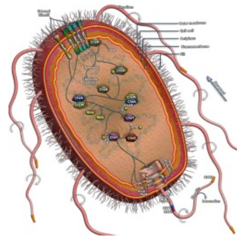
• In the figure, a slit 0.30 mm wide is illuminated by light of wavelength 426 nm. A diffraction pattern is seen on a screen 2.8 m from the slit. What is the linear distance on the screen between the first diffraction minima on either side of the central diffraction maximum ? 【8】mm



• A mass m slides with velocity v_0 over a surface and strikes elastically a spring (spring constant k), which is attached to a rigid wall. After compressing the spring, the mass is accelerated back in the opposite direction. The time of spring-mass contact is t = 【9】 in units of m,k,v.



• The optics of your mobile phone has a diameter of D = 5 mm. You make an image of a sheet with red lines (λ = 640 nm) kept at a distance of 100 cm from your phone. The minimum line separation, you are able to resolve is 【10】mm.



• The flagella of the bacterium E. Coli spins at 600 rad/s, propelling the bacterium at speeds of around 25 μm/s. E. Coli's flagella makes 【11】revolutions, while it crosses a microscope with 150 μm range of view.

• You slip a wrench over a bolt. Taking the origin at the bolt, the other end of the wrench is at x= 10 cm, y =5 cm. You apply a force F = $90\hat{\imath}$ $-20\hat{\jmath}$ N to the end of the wrench. The torque on the bolt is t = 【12】Nm $\hat{k}$.

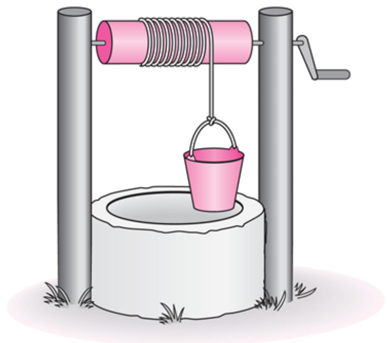
•The Lennard-Jones Potential $U(R) = 2\epsilon[A\left(\frac{\sigma}{R}\right)^{12} - B\left(\frac{\sigma}{R}\right)^{6}]$ describes the interaction between two inert gas atoms in a crystal structure (A = 12.1 and B = 14.45). R is the separation between atoms. The first term describes the repulsive interaction due to the nuclear core - the 2nd term the van-der-Waals interaction. For Argon ε = 167 x10^{-16} erg and σ = 3.40 Å. The expected Ar-Ar distance in the crystal is【13】Å.

• In a spring-mass system, a mass m= 1500g is oscillating at a frequency of 3.5 Hz and an amplitude of 0.35 m. The energy of the entire system is E = 【14】J .

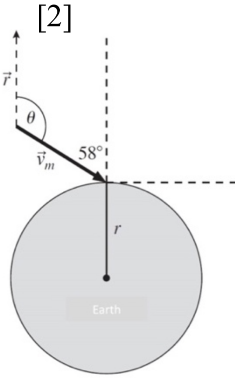
•A police car in motion (speed of v_0) is recording with radar (frequency f, speed v) the car ahead. The car ahead travels in the same direction at a speed of v_1. The frequency shift ($f - f_{recorded}$) of the radar as recorded by the police is 【15】Hz.

Part II Problems (10 points per problem)

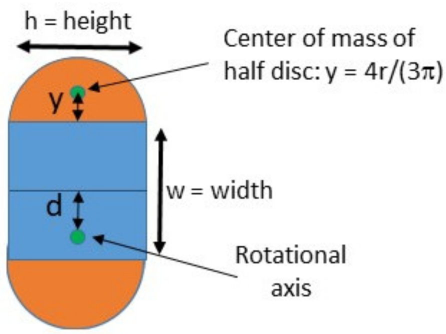
[1]



[2]



[3]



【1】A bucket of mass m drops into a well, its rope unrolling from a cylinder of mass M and radius R. What's its acceleration? (Express your answer as a function of m, M, g)

【2】The earth (m = 6.0x10^{24}kg, r = 6378 km, solid, uniform, spherical) is rotating with a period of 24h. The earth is hit by a moving, spherical meteoroid (v=15.5 km/s) within the equatorial plane under an angle of 58 (see Figure). After the collision, the rotation period of the earth is 23h59min. Find the radius of the meteoroid (density= 5500 kg/m^3)!

【3】Calculate the rotational inertia of the object indicated in the figure [3] with the rotational axis offset by *d* relative to the center of mass. *I* for a disk is given above. The mass of the rectangle is *M* and the mass of the disk is also *M*. The center of mass of a half disk is at the indicated position – shifted relative to the center of mass of a disk by y = 4/3 * r/π.

(a) Calculate *I* for two halves of a disk separated by distance *width* (4 points) with a central rotational axis.
(b) Determine *I* for a rectangular object with sides *height* and *width* (5 points) with a central rotational axis.
(c) Move the rotational axis by distance *d* and determine the final rotational inertia (1 points)

State the results in units of M, w, h, d, y !

Part I Answer Sheet，Note: 有效位數錯誤者，扣 1 分。

A【01】	300
A【02】	$2\omega/\sqrt{5}$
A【03】	5.2
A【04】	$\sqrt{2}T$
A【05】	6.3×10^{-4}
A【06】	$\pi/20$ or 0.16
A【07】	1.21
A【08】	8.0
A【09】	$\pi\sqrt{m/k}$
A【10】	0.2
A【11】	570
A【12】	-6.5
A【13】	3.71
A【14】	44
A【15】	(see below)

B【01】	$\pi\sqrt{m/k}$
B【02】	6.3×10^{-4}
B【03】	1.21
B【04】	(see below)
B【05】	8.0
B【06】	$\sqrt{2}T$
B【07】	300
B【08】	0.2
B【09】	44
B【10】	3.71
B【11】	-6.5
B【12】	570
B【13】	$2\omega/\sqrt{5}$
B【14】	$\pi/20$ or 0.16
B【15】	5.2

A[15] and B[4] four possible (identical) solutions

$$\text{(1)}\quad \Delta f = f - \frac{(1-\frac{v_1}{v})}{(1-\frac{v_0}{v})}\frac{(1+\frac{v_0}{v})}{(1+\frac{v_1}{v})} f$$

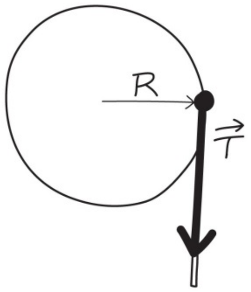
$$\text{(2)}\quad \Delta f = f - \frac{(v-v_1)}{(v-v_0)}\frac{(v+v_0)}{(v+v_1)} f$$

$$\text{(3)}\quad \Delta f = \left\{1 - \frac{(v-v_1)}{(v-v_0)}\frac{(v+v_0)}{(v+v_1)}\right\} * f$$

$$\text{(4)}\quad \Delta f = \left\{1 - \frac{\left(1-\frac{v_1}{v}\right)}{\left(1-\frac{v_0}{v}\right)}\frac{\left(1+\frac{v_0}{v}\right)}{\left(1+\frac{v_1}{v}\right)}\right\} * f$$

Part II Answer Sheet，Note: 有效位數錯誤者，扣 1 分

【A1 = B2 】



(3 points) Newton's law, bucket : $F_{\text{net}} = mg - T = ma$

(3 points) Rotational analogy of Newton's law, cylinder :

$RT = Ia / R$

Here $I = \frac{1}{2} MR^2$

(4 points) Solve the two equations to get :

$$a = \frac{mg}{m + \frac{1}{2}M}$$

【A2 = B3 】

$L_m = rp_m \sin\theta$

The initial angular momentum of the entire system is:

(3 points) $L_i = L_a + L_m = I_a\omega_a + rp_m \sin\theta = \frac{2}{5}Mr^2\omega_a + rmv_m \sin\theta$

(3 points) and the final $L_f = \left(I_a + mr^2\right)\omega_f = \left(\frac{2}{5}Mr^2 + mr^2\right)\omega_f$

(4 points) and we get $m = \frac{2Mr\left(\omega_f - \omega_a\right)}{5\left(v_m \sin\theta - r\omega_f\right)}$. With m = density * V and V = 4/3 π r^3 we get r= 140 km

【A3 = B1 】

(4 points) Rotational Inertia of a disk: ½ MR^2; each half I_h = ¼ MR^2

R = ½ height; Move rotational axis into the center of mass of half disk:

(1pt) I_{CM} = ¼ MR^2 - ½ M y^2

(2pts) Move by distance width/2 => $I_w = ¼ MR^2 + ½ M ((½ * width+y)^2 – y^2)$

1pt for $(½ * width)^2$

Minus ½ pt for forgetting ½ in $(½ * width)^2$ or in $(½ * width+y)^2$

1pt for adding y: $(½ * width+y)^2$

(1 pt) Total:$I = ½ MR^2 + M ((½ * width+y)^2 – y^2)$

This can also be written as: $½ MR^2 + M (1/4 * width^2+width*y)$

(5 points) Rotating rectangular: density den = M/(height*width)

I = int (x = -width/2 to +width/2) int (y = -height/2 to +height/2) * den * $(x^2 + y^2)$ dxdy

⇨ $I = M/12 (width^2+height^2)$

(1 points) Calculate total I

$I_{total} = ½ MR^2 + M(¼ width^2 – y^2) + M/12 (width^2+height^2)$

Shift system by offset d (total mass = 2M)

$I_{final} = ½ MR^2 + M ((½ * width+y)^2 – y^2) + M/12 (width^2+height^2)+2Md^2$

(1 pt) for $2Md^2$

Note: The final solution can be written in various ways:

(I) $I_{final} = ½ MR^2 + M ((½ * width+y)^2 – y^2) + M/12 (width^2+height^2)+2Md^2$

(II) $I_{final} = 5/24 Mh^2 + 1/3 Mw^2 + Mwy + 2Md^2$ (when realized that 2R = h)