Wave motion is an essential feature of our physical evironment.
Waves involve a <u>disturbance</u> that propagates through space.
A wave is a traveling <u>disturbance</u> that transports energy but not matter.

(1) Wave

○ 波的分類 — 依介質

mechanical waves：obey Newton's laws，需有介質才能 propagate，
如音波、水波、繩波、地震波…。

EM waves (電磁波)：不需介質，在真空中以 C 傳播.

物質波 (matter waves)：微小粒子呈現的波動，不需介質。

— 依擾動的振動方向 vs. 傳播方向

互相垂直：transverse wave (橫波)，e.g. 繩波、EM waves.

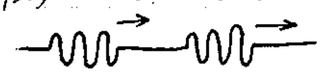
互相平行：longitudinal wave (縱波)，e.g. sound wave.

○ 波的描述

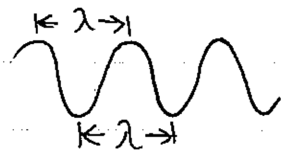
(i) Amplitude (振幅，以A代表)：擾動的最大位移量 (⇒ peak-to-peak)

(ii) Waveforms：pulse (脈衝波)：

continuous wave (連續波) 

wave train (波列)：

(iii) 波長 (用λ表示)、週期 (用T表示) 和頻率 (用f表示)



λ：wave pattern 重複的最小距離.

T：一個完整振盪的時間

f：單位時間內的 wave cycle 數 $= \frac{1}{T}$

∴ (iv) 波速 (wave speed) $v = \frac{\lambda}{T} = \lambda f$。

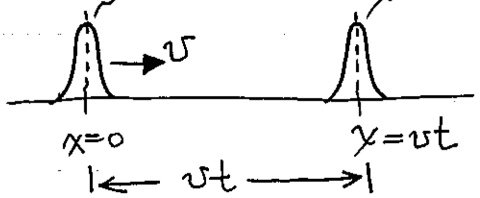
雜

(2) Traveling wave (行進波)

- Simplest case = pulse

$y(x,t=0)=f(x)$ $\quad$ $y(x,t)=f(x-vt)$

= traveling pulse toward $+x$ with speed v

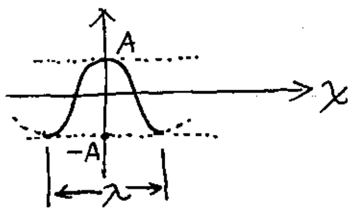


$\Rightarrow$ 向$-x$方向,以v行進的 pulse 為 $y(x,t)=f(x+vt)$.

$\Rightarrow$ 只要知道 waveform $f(x)$, 將 x 代換成 $x\pm vt$ 就是 traveling wave in 1D, i.e. traveling waves 的形式為 $y(x,t)=f(x\pm vt)$.

- Simple harmonic wave

Waveform 為 $y(x,t=0)=f(x)=A\cos kx$ 的行進波。



What is k?

$\because \cos\theta$ 在 $\theta=2\pi$ 時會 repeat 一次,

$\therefore kx$ 在 $x=\lambda$ 時 (repeat 一次) $k\lambda=2\pi$

$\therefore k=\frac{2\pi}{\lambda}$ = wave number and $[k]=m^{-1}$

$\Rightarrow +x$ 方向: $y(x,t)=A\cos k(x-vt)=A\cos(kx-kvt)$

$=A\cos(kx-\frac{2\pi}{\lambda}\cdot\frac{\lambda}{T}t)=A\cos(kx-\frac{2\pi}{T}t)$

$=A\cos(kx-\omega t)$

Where $\omega=\frac{2\pi}{T}$ = angular frequency.

$\therefore v=\frac{\lambda}{T}=\frac{2\pi/k}{2\pi/\omega}=\frac{\omega}{k}$

ω = oscillation cycles per unit time

k = oscillation cycles per unit distance

$\therefore k$ 又稱為 spatial frequency.

雜

$\therefore y(x,t) = A\cos(kx \pm \omega t)$ $\begin{cases} +: -x 方向行進 \\ -: +x 方向行進 \end{cases}$

○ 若 $y(x,t) = f(x \pm vt)$ (行進波)

then $\dfrac{\partial^2 y}{\partial x^2} = \dfrac{1}{v^2}\dfrac{\partial^2 y}{\partial t^2}$ — wave eq. in 1D (problem 71)

here, y is the wave disturbance, ~~如~~ height of water wave, pressure of sound,

v = wave speed.

(3) String wave

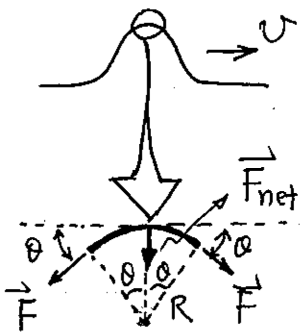
○ wave speed

考慮一向右行進的 pulse and

μ is the mass per unit length of the string.

When the string is stretched to a tension

F, v = ?



In the pulse frame, string 向左以 v 行進, 在

pule 頂端處的一小段 string 受力如右圖, like UCM with (R, v)

$\therefore F_{net} = 2F\sin\theta$ 向 curvature 中心.

for small disturbance i.e. $\theta \ll 1$ 則 $\sin\theta \cong \theta$

$$\therefore F_{net} \cong 2F\theta = \frac{mv^2}{R} = \underbrace{R \cdot 2\theta \cdot \mu}_{\text{is mass}} \cdot \frac{v^2}{R} = 2\theta\mu v^2$$

$\therefore v = \sqrt{\dfrac{F}{\mu}}$ — for small-amplitude pulses, traveling waves, and wave trains.

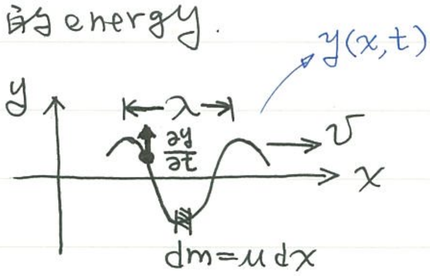
○ Wave power

for mechanical wave, 介質的 $K+U=$ 傳輸的 energy.

Consider string 上的一小段 $dm=\mu\cdot dx$

then its 動能

$$dK=\frac{1}{2}(dm)\left(\frac{\partial y}{\partial t}\right)^2=\frac{1}{2}(\mu\cdot dx)\left(\frac{\partial y}{\partial t}\right)^2$$



∴ 一段 λ 內的介質動能

$$K_\lambda=\int dK=\int_0^\lambda \frac{1}{2}(\mu\cdot dx)\left(\frac{\partial y}{\partial t}\right)^2$$

∴ wave 在單位時間內 transfer 的平均動能 $\bar{K}$ 為

$$\frac{d\bar{K}}{dt}=\frac{K_\lambda}{T}=\frac{1}{T}\int_0^\lambda \frac{1}{2}\mu\left(\frac{\partial y}{\partial t}\right)^2\cdot dx$$

For mass-spring system, SHM or simple pendulum : $\bar{K}=\bar{U}$

$$\therefore \frac{E}{T}=\frac{d}{dt}(\bar{K}+\bar{U})=2\frac{K_\lambda}{T}=\bar{P}=\text{wave transfer 的平均功率.}$$

For simple harmonic wave : $y(x,t)=A\cos(kx-\omega t)$

$$\therefore K_\lambda=\frac{\mu}{2}\int_0^\lambda\left(\frac{\partial y}{\partial t}\right)^2dx=\frac{1}{2}\mu\omega^2A^2\underbrace{\int_0^\lambda \sin^2(kx-\omega t)\,dx}_{=\pi/k}=\frac{\pi\mu}{2k}\omega^2A^2$$

$$=\frac{\mu}{4}\lambda\omega^2A^2$$

$$\Rightarrow \frac{K_\lambda}{T}=\frac{\mu}{4}\frac{\lambda}{T}\omega^2A^2=\frac{1}{4}\mu v\omega^2A^2$$

$$\therefore \bar{P}=2\frac{d\bar{K}}{dt}=\frac{1}{2}\mu v\omega^2A^2.$$

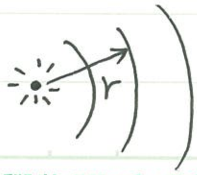
○ Wave intensity (用 I 表示)

Total power is useful in 1D wave. 3D? e.g. sound in air.

⇒ Intensity : 穿越單位面積 (⊥ wave 方向) 的 power, ∴ $[I]=\frac{W}{m^2}$

For 一個 point source, 發出球形波, 其 wavefronts (波前 — 即相同 wave phase 所連成的面) 就是同心球面。波源發出的 power 分布在球面上, ∴

$$I=\frac{P}{A}=\frac{P}{4\pi r^2}\ (\text{球形波})\propto r^{-2}.$$



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(4) Sound wave (longitudinal wave)

The disturbance of sound wave comprises a small change in air pressure and density accompainied by a back-and-forth motion of air molecules. (Figure 14.14, p231)

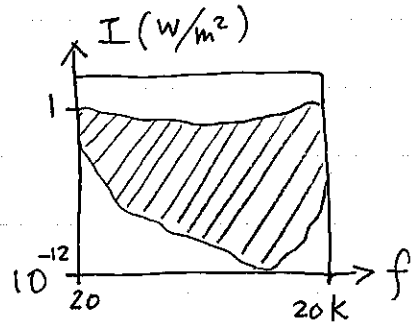
在空氣中的音速 $v=\sqrt{\frac{\gamma p}{\rho}}$
$\begin{cases} p: 空氣的背景壓力 \\ \rho: 空氣密度 \\ \gamma: \text{air constant}\ (和空氣分子种類有關) \end{cases}$

$\gamma=\frac{C_p}{C_v}\ (>1)$, $\gamma=5/3$ for 單原子氣体, $\gamma=7/5$ for 双原子氣体.

在液体及固体傳播得較快 because they are less compressible.
(可用spring的k來想, L及S的k比G大, 受扰動而前後運動的分子或原子其restoring force $F_s=-kx$, k愈大, 回到平衡點的時間愈短。)

Note: longitudinal wave 在流体 (ρ, B) 的傳播speed $v=\sqrt{\frac{B}{\rho}}$

人耳对sound的反应区域(I and f)如图14.15.



(i) 頻率: 20 Hz ~ 20 kHz
由图, 低頻及高頻的sound比較聽不到
⇒ 立体音响增強低頻及高頻音量.

(ii) $I: 10^{-12} \sim 1\ W/m^2$ 達12個數量級
人耳分辨sound的大小(=音量 loundness) 不正比於sound的I.
如 I↑10倍, 音量↑2倍.
用 decibel (dB=分貝) 分級音量:

sound intensity level $\beta \equiv 10 \log \frac{I}{I_0}$ (Note: log = 以10為底, ln = 以e為底)

where $I_0=10^{-12}\ \frac{W}{m^2}$ 為參考值.

and $[\beta]=$ dB

⇒ when $I=I_0$, $\beta=0$ dB

$I=1\ \frac{W}{m^2}$, $\beta=120$ dB (人耳的無痛苦極限值)

(5) Interference (干涉): wave 的疊加

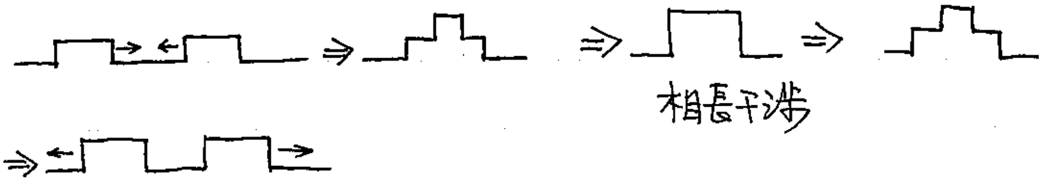
o wave 的疊加原理 (superposition principle)

when 多個 waves 在空間中傳播時, 空間的波函數 = 所有 waves 的線性相加. ⇒ $y_T = y_1 + y_2 + \cdots + y_N$

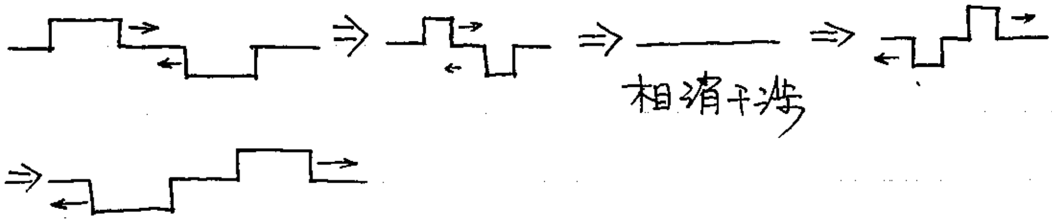
因此疊加而產生干涉 (interference)

兩种干涉現象: constructive (相長的) 及 destructive (相消的) 干涉.

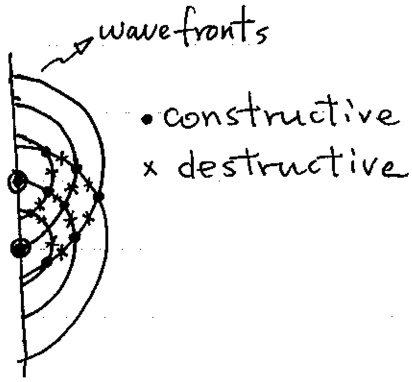
constructive:



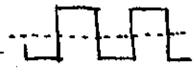
destructive:



2D inteference



o Fourier analysis

y_T 可以是複雜如音樂或簡單如方波 

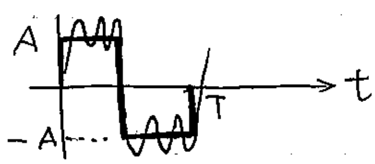
Fourier analysis: 任何週期性波函數皆可用多個 simple harmonic waves (不同的 A 及 f) 合成.

$$y_T(t) = \sum[a_n \sin(n\omega t) + b_n \cos(n\omega t)],\ \text{where } \omega = \frac{2\pi}{T} = 2\pi f.$$

a_n 和 b_n 的決定即稱為 Fourier analysis.

如方波的前 3 個 simple harmonic waves 為 $y(t) = \frac{4A}{\pi}\left(\sin\omega t + \frac{1}{3}\sin 3\omega t + \frac{1}{5}\sin 5\omega t\right)$

其大小比為 $1 : \frac{1}{3} : \frac{1}{5}$



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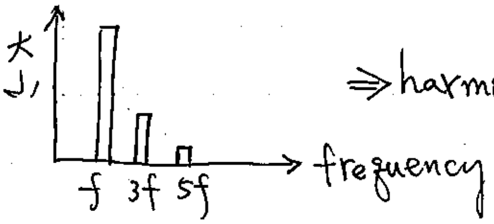


⇒ harmonic spectrum

。If v 和 λ 無關的話,各個 simple harmonic waves 皆用相同的 speed 行進,∴波形不變.

在 nondispersive medium.

但在某些介質中, v 和 λ 有關,也就是各個 simple harmonic waves 以不同的 speed 行進 ⇒ 波形改變,此為 dispersion 現象,

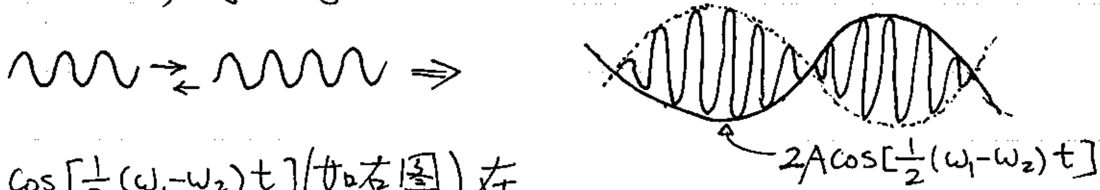
在 dispersive medium

如在深水區的表面水波 speed $v=\sqrt{\frac{\lambda g}{2\pi}}$ g=重力常數.
⇒ 波長大的水波先到達岸邊。

。Beats
兩個頻率相當接近的 waves 所產生的干涉

$$y(t)=y_1(t)+y_2(t)=A\cos\omega_1 t+A\cos\omega_2 t$$
$$=\underbrace{2A\cos\left[\tfrac{1}{2}(\omega_1-\omega_2)t\right]}_{\sim\text{ amplitude}}\cdot\cos\left[\tfrac{1}{2}(\omega_1+\omega_2)t\right]$$

∴ $y(t)$ 形成具有接近原先頻率 (ω_1 or ω_2),但振幅双倍且受到調變的 wave,時大時小,



由於 $2A\cos[\frac{1}{2}(\omega_1-\omega_2)t]$ (如右圖) 在一個週期內有 2 個重複區,∴ $y(t)$ 的頻率為 $\frac{1}{2}|\omega_1-\omega_2|\times 2=|\omega_1-\omega_2|$

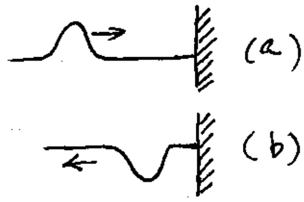
for sound waves ⇒ beats

雜

(6) Reflection(反射)、透射(transmission)與折射(refraction)

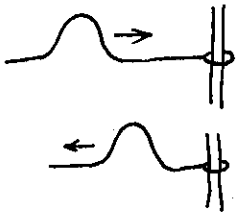
o wave的反射為一普遍常見現象、

(i) 固定端的反射



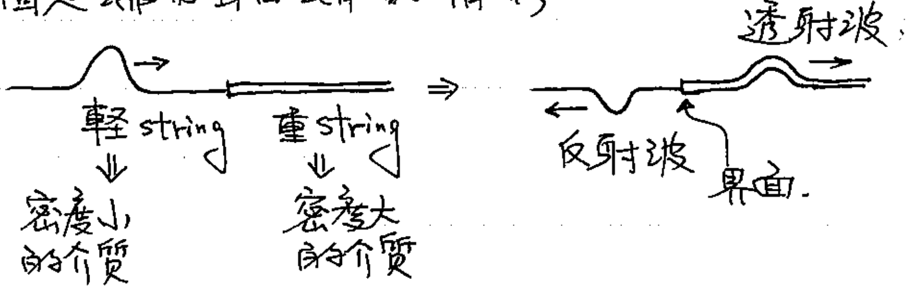
wave無法穿過固定端的wall,
∴一定要反射, or energy何處去?
反射波(b)的波形顛倒,因wave在到達固定端時,固定端施加向下的反作用力造成。

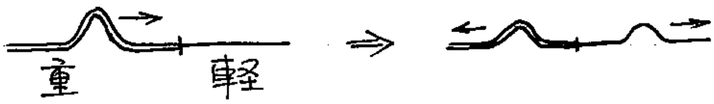
(ii) 但如是自由端點則無此反作用,∴原波形反射。自由端的max height為pulse的2倍高。



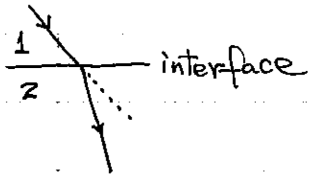
(用假想波與真實波疊加,保持boundary condition,固定端與自由端為interfaces)

(iii) 介於固定端與自由端的情形





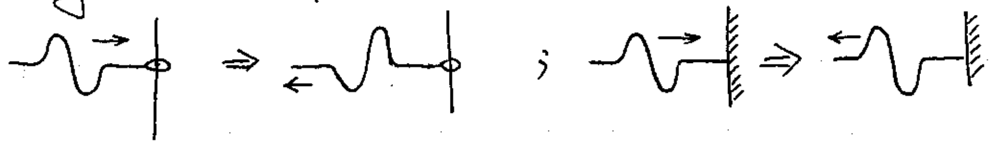
o refraction

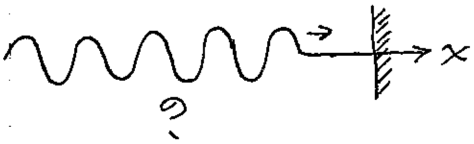


wave在medium 1及medium 2的speed不同,造成穿越界面時,方向偏移 ⇒ refraction.

雜

(7) Standing waves (駐波)





⇒ 兩個A及λ相同的waves 在空間superposition

$y_1(x,t)=A\cos(kx-\omega t)$: → +x

$y_2(x,t)=-A\cos(kx+\omega t)$: 反射波, −x

$$
\begin{aligned}
y(x,t) &= y_1(x,t)+y_2(x,t) \\
&= A[\cos(kx-\omega t)-\cos(kx+\omega t)] \\
&= 2A\sin\omega t\sin kx \\
&= A(t)\sin kx \quad , \text{where } A(t)=2A\sin\omega t
\end{aligned}
$$

$y(x,t)$ 為一個振幅隨t變動的非行進波 (harmonic waves)

[Note: $y(x,t)$ 非屬行進形式 : $f(kx\pm\omega t)$]

$y(x,t)$ = standing wave (駐波)

特徵:

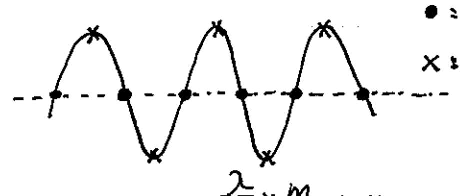
Node (節點): for any time 振幅為0的點

∴ node的位置為 $\sin kx=0 \Leftrightarrow kx=\frac{2\pi}{\lambda}x=n\pi$

∴ $x=\frac{n}{2}\lambda,\ n\in\mathbb{Z}$ 為node的位置.

$\Delta x=\frac{\lambda}{2}$ 的整數倍、

Antinode (反節點或波腹) 為振幅max.處



•: nodes

x: antinodes

•—• : $\frac{\lambda}{2}\times m=$ ×—× , $\underline{\underline{m\in N}}$

×—• : $\frac{\lambda}{4}(2m-1)$

在node位置以外的點 x_0,

其 $y(x_0,t)=2A\sin kx_0\sin\omega t$

$=A'\sin\omega t$ — SHM with amplitude $=2A\sin kx_0$

羅

○ 被 confined 的駐波

两端點是 nodes, ∴ $\boxed{L=\frac{\lambda}{2}m}$, $m\in N$

by $\lambda f = v \Rightarrow \boxed{f_m = \frac{v}{2L}\cdot m}$

For $m=1$, $f_1 = \frac{v}{2L}$ = fundamental mode

Given L, $f=\frac{v}{2L}\times m$ limits the allowed standing waves to a discrete set of wavelengths (or frequencies). 這些稱為 modes or harmonics.

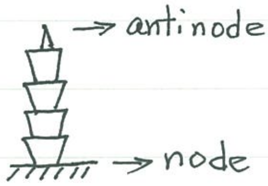
The higher modes (not $m=1$) are <u>overtones</u>.

一端固定，一端自由，則 $\boxed{L=\frac{\lambda}{4}(2m-1)}$, $m\in N$

$$\boxed{f_m = \frac{v}{4L}(2m-1)}$$

When wave 的 λ matches $L=\frac{\lambda}{2}m$ or $L=\frac{\lambda}{4}(2m-1)$ 稱為 resonance (共振)

for skyscraper:



for musical instruments: $f_m = \frac{1}{2L}\sqrt{\frac{F}{\mu}}\times m$ for 弦樂器

共振箱:  $\Rightarrow$ •—x : $f_1 = \frac{v}{4L}$, 如笛。

 $\Rightarrow$ x—x : $f_1 = \frac{v}{2L}$, 如簫。

open end = 压力的 node (i.e. p_0) = air 分子位移的 antinode.

closed end = air 分子位移的 node.

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(8) Doppler effect (for sound)

Assume air is at rest and

v = sound speed
v_s = source speed
v_o = observer speed

$\left.\begin{array}{l} \\ \\ \end{array}\right\}$ all relative to air.

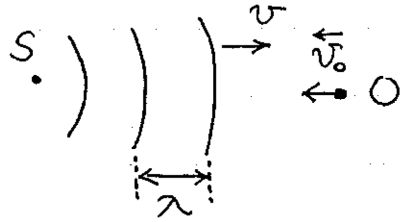
and 在 $v_s = 0 = v_o$ 時, 观察者测量到的 $f \cdot \lambda = v$ and $f = \frac{1}{T}$.

If $v_s \neq 0$ or $v_o \neq 0$, 则 $f \to f'$, then $f' = ?$

(a) $v_s = 0$ and $v_o \neq 0$ (观察者移動)

O 量到的 sound speed $v' = v + v_o$

, 但波长 λ 不变,



$\because T' = \frac{\lambda}{v'} = \frac{\lambda}{v+v_o} = \frac{1}{f'}$

$\therefore f' = \frac{v+v_o}{v/f} = \frac{v+v_o}{v} f$

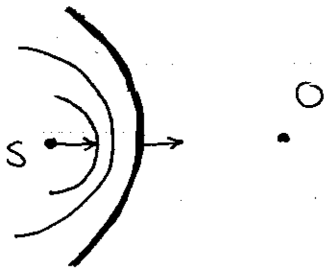
$\Rightarrow f' = \frac{v \pm v_o}{v} f$ — + : approaching; − : moving away

(b) $v_s \neq 0$ and $v_o = 0$ (观察者静止)

通过 O 的波长减小到

$\lambda' = \lambda - v_s T = vT - v_s T$

$= (v - v_s) \cdot \frac{1}{f} = \frac{v}{f'}$ → sound 在 air 中的 speed 对 O 而言仍是 v.



$\because f' = \frac{v}{v - v_s} f$

$\Rightarrow f' = \frac{v}{v \mp v_s} f$ — − : approaching; + : moving away

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Combine (a) and (b)

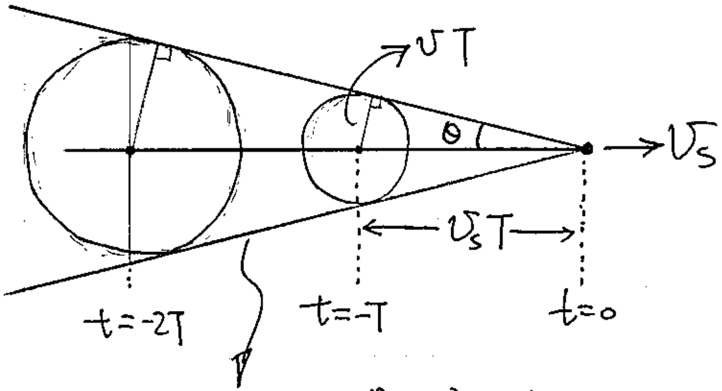
$f' = \frac{v}{v \mp v_s} \cdot \frac{v \pm v_o}{v} f = \frac{v \pm v_o}{v \mp v_s} f$ — upper: approaching ($f' \uparrow$); lower: moving away. ($f' \downarrow$)

雜

○ Shock wave

from (b) $\lambda' = (v - v_s)T \to 0$ when $v_s = v$.

v_s even is greater than v. ⇒ shock wave



波前重疊線⇒振幅巨大.
= shock wave.

$\sin\theta = \frac{vT}{v_s T} = \frac{v}{v_s}$

where $\frac{v_s}{v}$ = Mach number (馬赫數 = 音速的倍數)

θ: Mach angle.