

# MATLAB

# Simulink - PID

Kun-Yen Chiu

# Problem 1

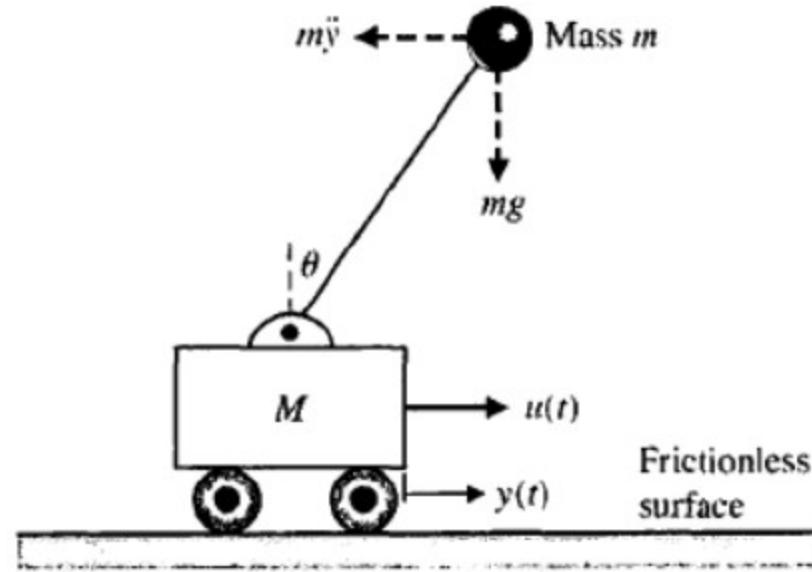


Figure 1 A cart and an inverted pendulum.

Force (horizontal):

$$(M + m)\ddot{y}(t) = u(t) - (mL\ddot{\theta}(t) \cos \theta(t) - mL\dot{\theta}(t)^2 \sin \theta(t))$$

Torque:

$$mL^2\ddot{\theta}(t) = mgL \sin \theta(t) - mL\ddot{y}(t) \cos \theta(t)$$

Assume that  $M=100\text{Kg}$ ,  $m=10\text{Kg}$ ,  $L=1\text{m}$ ,  $g=9.81\text{m/s}^2$  and  $G_c(s) = K_P + \frac{K_I}{s} + sK_D$ ,

\*This is a guideline for Problem 1.

\*The equations are not exactly the same, but same concepts

1) Use ode45 to simulate the output response in time-domain.

$$\ddot{y} = -\frac{mg}{M}\theta + \frac{1}{M}u$$

$$\ddot{\theta} = \frac{(M+m)g}{ML}\theta - \frac{1}{ML}u$$

$$x_1 = y, x_2 = \dot{y}, x_3 = \int \theta dt, x_4 = \theta, x_5 = \dot{\theta}$$

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -\frac{mg}{M}x_4 + \frac{1}{M}u$$

$$\dot{x}_3 = x_4$$

$$\dot{x}_4 = x_5$$

$$\dot{x}_5 = \frac{(M+m)g}{ML}x_4 - \frac{1}{ML}u$$

$$\text{PID control } U(s) = G_c(s)(R(s) - \theta(s))$$

$$u(t) = -K_P\theta(t) - K_I \int \theta(t)dt - K_D \frac{d\theta(t)}{dt} = -K_P x_4 - K_I x_3 - K_D x_5$$

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -\frac{mg}{M}x_4 + \frac{1}{M}(-K_P x_4 - K_I x_3 - K_D x_5)$$

$$\dot{x}_3 = x_4$$

$$\dot{x}_4 = x_5$$

$$\dot{x}_5 = \frac{(M+m)g}{ML}x_4 - \frac{1}{ML}(-K_P x_4 - K_I x_3 - K_D x_5)$$

$$x_{10} = 0, x_{20} = 0, x_{30} = 0, x_{40} = 1, x_{50} = 0$$

q(s) can be obtained by (sl-A).

$$q(s) = MLs^3 - K_D s^2 - [(M+m)g + K_P]s - K_I$$

# Problem 1 - Reference Code

```
clear all
clc
% Parameters:
m=10;
M=100;
L=1;
g=9.81;

% PID tuning
KP = ???
KI = ???
KD = ???

% Define reference signal
r = 0;
fun = @(t,x) [x(2); -m*g*x(4)/M; x(4); x(5); (M+m)*g*x(4)/(M*L)] + (-KP*x(4)-KI*x(3)-KD*x(5))*[0; 1/M; 0; 0; -1/(M*L)];
T = linspace(0, 15, 1e3);
x0 = [0; 0; 0; 1; 0];
[t, Y] = ode45(@(t,x) fun(t,x), T, x0);

figure
%%%%%%%%plot Yourself%%%%%%%%
xlabel('Time [s]')
ylabel('Angular rotation [rad]')

syms x;
digits(6)
eqn = M*L*x^3-KD*x^2-((M+m)*g+KP)*x-KI == 0;
s = solve(eqn,x);
vpa(s)
```

## Problem 2

2) Use Simulink to simulate the output response in s-domain.

Assume that  $\theta(t) \rightarrow 0^\circ \Rightarrow \sin \theta(t) \approx \theta(t), \cos \theta(t) \approx 1, \dot{\theta}(t) \approx 0,$

$$(M + m)\ddot{y}(t) + mL\ddot{\theta}(t) = u(t)$$

$$\ddot{y}(t) + L\ddot{\theta}(t) - g\theta(t) = 0$$

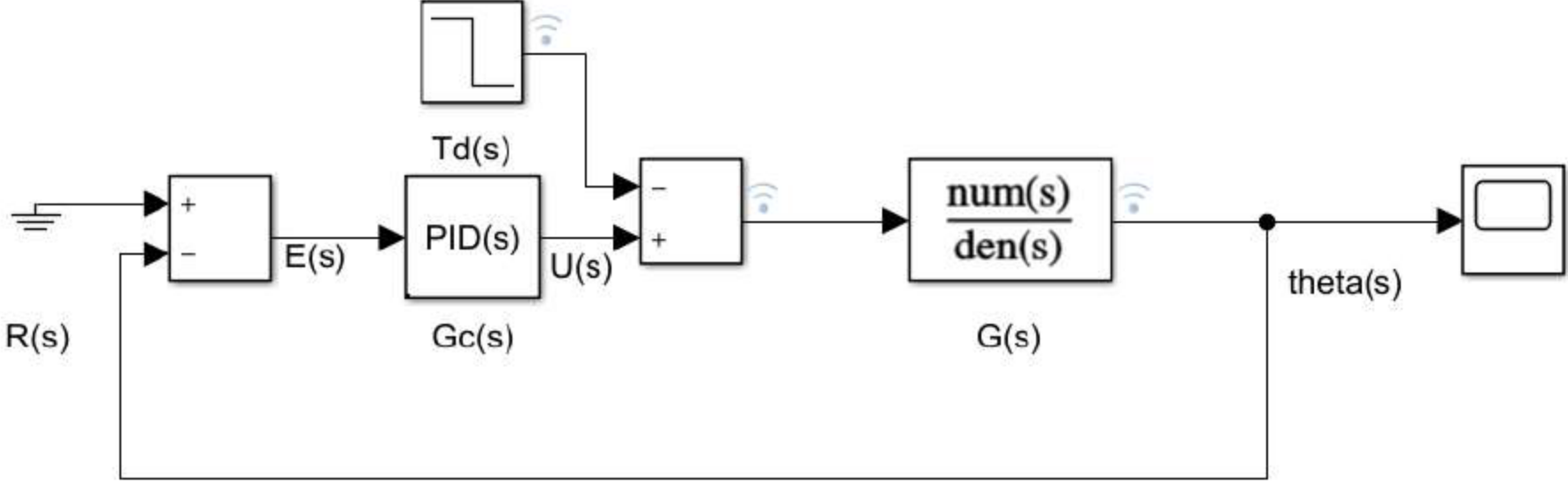
$$G(s) = \frac{-1/ML}{s^2 - \frac{(M+m)g}{ML}}$$

Applying PID control to the inverted pendulum.

Assume that  $T_d(s) = 1(T_d(t) = \delta(t))$

# Create Block Diagram

- > PID Controller
- > Add
- > Ground
- > Step
- > Transfer Fcn
- > Scope (see output result)



# Set parameters

Block Parameters: PID Controller

PID 1dof (mask) (link)

This block implements continuous- and discrete-time PID control algorithms and includes advanced features such as anti-windup, external reset, and signal tracking. You can tune the PID gains automatically using the 'Tune...' button (requires Simulink Control Design).

Controller:  Form:

Time domain:  
 Continuous-time  
 Discrete-time

Discrete-time settings  
Sample time (-1 for inherited):

Compensator formula

$$P + I \frac{1}{s} + D \frac{N}{1 + N \frac{1}{s}}$$

Main Initialization Output Saturation Data Types State Attributes

Controller parameters

Source:

Proportional (P):

Integral (I):

Derivative (D):

Use filtered derivative

Filter coefficient (N):

Automated tuning

Select tuning method:

Enable zero-crossing detection

Block Parameters: Transfer Fcn

Transfer Fcn

The numerator coefficient can be a vector or matrix expression. The denominator coefficient must be a vector. The output width equals the number of rows in the numerator coefficient. You should specify the coefficients in descending order of powers of s.

Parameters

Numerator coefficients:

Denominator coefficients:

Absolute tolerance:

State Name: (e.g., 'position')

# Set Parameters (Cont.)

Block Parameters: Step

Step

Output a step.

Parameters

Step time:  
0.1

Initial value:  
1

Final value:  
0

Sample time:  
0

Interpret vector parameters as 1-D

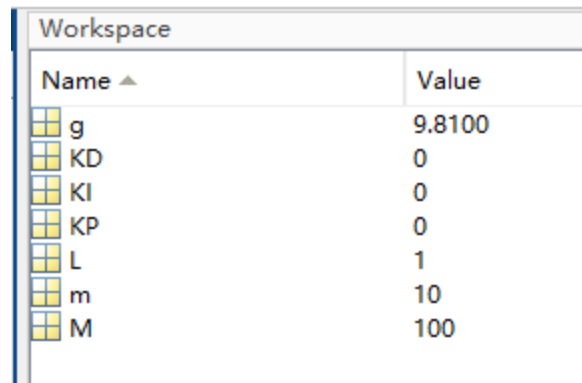
Enable zero-crossing detection

? OK Cancel Help Apply



# How to run Simulink with matlab parameter

- In parameter file, choose the parameter  $K_D$ ,  $K_I$ , and  $K_P$
- Next, Run the parameter.m file, in order to save the data in workspace



The screenshot shows the MATLAB Workspace window with a table of variables and their values. The variables are g, KD, KI, KP, L, m, and M. The values are 9.8100, 0, 0, 0, 1, 10, and 100 respectively.

Name ▲	Value
g	9.8100
KD	0
KI	0
KP	0
L	1
m	10
M	100

- After run the parameter.m, you can press run in Simulink