MATLAB
Simulink - PID

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Problem 1

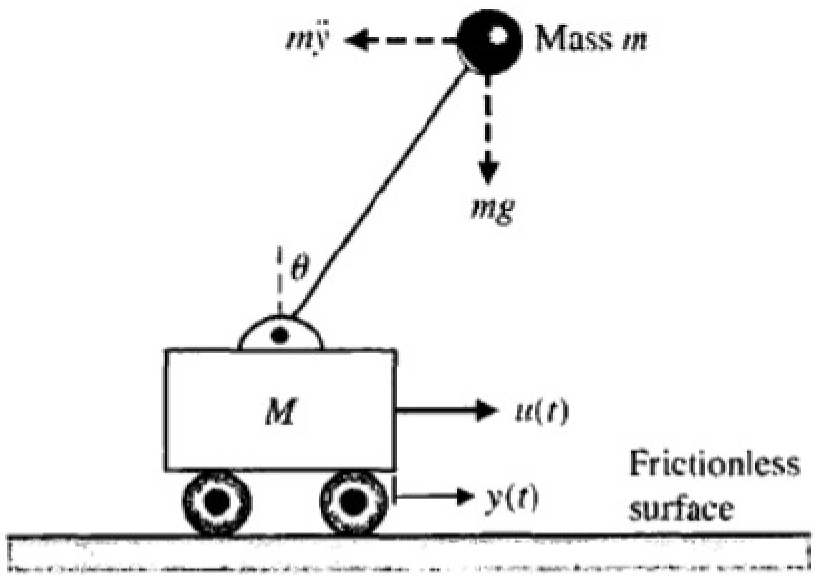


Figure 1 A cart and an inverted pendulum.

Force (horizontal):

$$(\mathrm{M}+\mathrm{m})\ddot{y}(t)=u(t)-\left(mL\ddot{\theta}(t)\cos\theta(t)-mL\dot{\theta}(t)^2\sin\theta(t)\right)$$

Torque:

$$\mathrm{m}\mathrm{L}^2\ddot{\theta}(t)=mgL\sin\theta(t)-mL\ddot{y}(t)\cos\theta(t)$$

Assume that M=100Kg, m=10Kg, L=1m, g=9.81m/s^2 and $G_c(s)=K_P+\frac{K_I}{s}+sK_D$,

*This is a guideline for Problem 1.
*The equations are not exactly the same, but same concepts

1) Use ode45 to simulate the output response in time-domain.

$$\ddot{y} = -\frac{mg}{M}\theta + \frac{1}{M}u$$

$$\ddot{\theta} = \frac{(M+m)g}{ML}\theta - \frac{1}{ML}u$$

$$x_1 = y, x_2 = \dot{y}, x_3 = \int \theta dt, x_4 = \theta, x_5 = \dot{\theta}$$

$$\dot{x_1} = x_2$$

$$\dot{x_2} = -\frac{mg}{M}x_4 + \frac{1}{M}u$$

$$\dot{x_3} = x_4$$

$$\dot{x_4} = x_5$$

$$\dot{x_5} = \frac{(M+m)g}{ML}x_4 - \frac{1}{ML}u$$

PID control $U(s) = G_c(s)(R(s) - \theta(s))$

$$u(t) = -K_P\theta(t) - K_I\int \theta(t)dt - K_D\frac{d\theta(t)}{dt} = -K_P x_4 - K_I x_3 - K_D x_5$$

$$\dot{x_1} = x_2$$

$$\dot{x_2} = -\frac{mg}{M}x_4 + \frac{1}{M}(-K_P x_4 - K_I x_3 - K_D x_5)$$

$$\dot{x_3} = x_4$$

$$\dot{x_4} = x_5$$

$$\dot{x_5} = \frac{(M+m)g}{ML}x_4 - \frac{1}{ML}(-K_P x_4 - K_I x_3 - K_D x_5)$$

$$x_{10} = 0, x_{20} = 0, x_{30} = 0, x_{40} = 1, x_{50} = 0$$

q(s) can be obtained by (sI-A).

$$q(s) = MLs^3 - K_D s^2 - [(M+m)g + K_P]s - K_I$$

Problem 1 - Reference Code

```
clear all
clc
% Parameters:
m=10;
M=100;
L=1;
g=9.81;

% PID tuning
KP = ???
KI = ???
KD = ???

% Define reference signal
r = 0;
fun = @(t,x) [x(2); -m*g*x(4)/M; x(4); x(5); (M+m)*g*x(4)/(M*L)] + (-KP*x(4)-KI*x(3)-KD*x(5))*[0; 1/M; 0; 0; -1/(M*L)];
T = linspace(0, 15, 1e3);
x0 = [0; 0; 0; 1; 0];
[t, Y] = ode45(@(t,x) fun(t,x), T, x0);

figure
%%%%%%plot Yourself%%%%%%%%%%
xlabel('Time [s]')
ylabel('Angular rotation [rad]')

syms x;
digits(6)
eqn = M*L*x^3-KD*x^2-((M+m)*g+KP)*x-KI == 0;
s = solve(eqn,x);
vpa(s)
```

Problem 2

2) Use Simulink to simulate the output response in s-domain.

Assume that $\theta(t) \to 0^o \Longrightarrow \sin\theta(t) \approx \theta(t), \cos\theta(t) \approx 1, \dot{\theta}(t) \approx 0$,

$$(\mathrm{M}+\mathrm{m})\ddot{y}(t) + mL\ddot{\theta}(t) = u(t)$$

$$\ddot{y}(t) + \mathrm{L}\ddot{\theta}(t) - g\theta(t) = 0$$

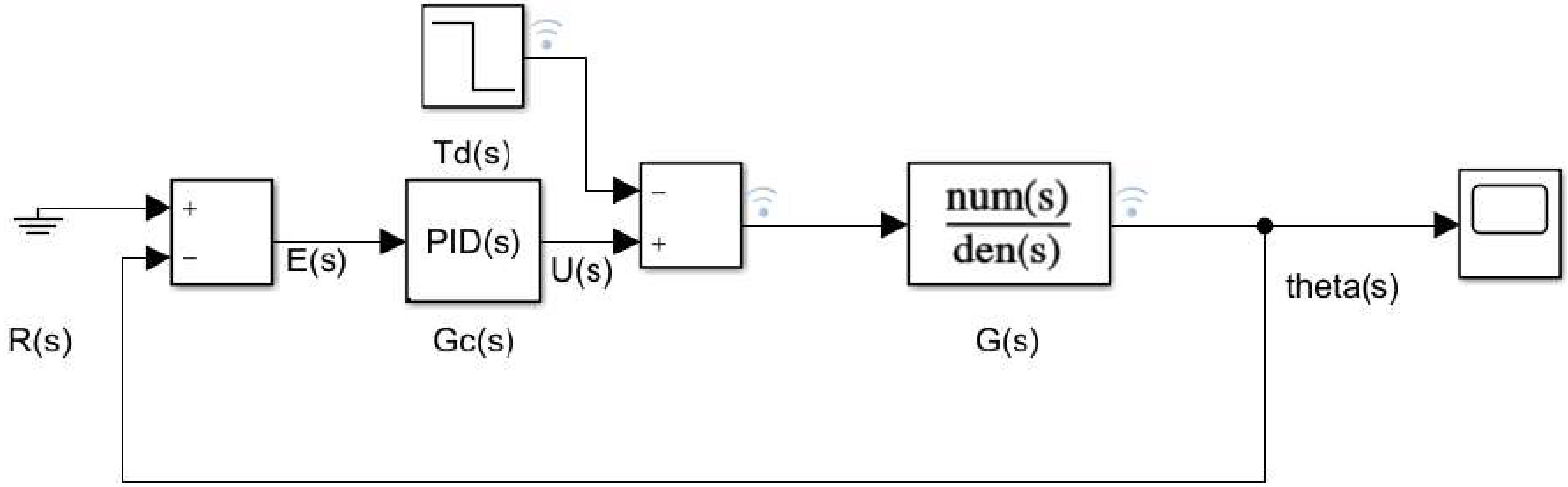
$$\mathrm{G(s)} = \frac{-1/_{ML}}{s^2 - \frac{(M+m)g}{ML}}$$

Applying PID control to the inverted pendulum.

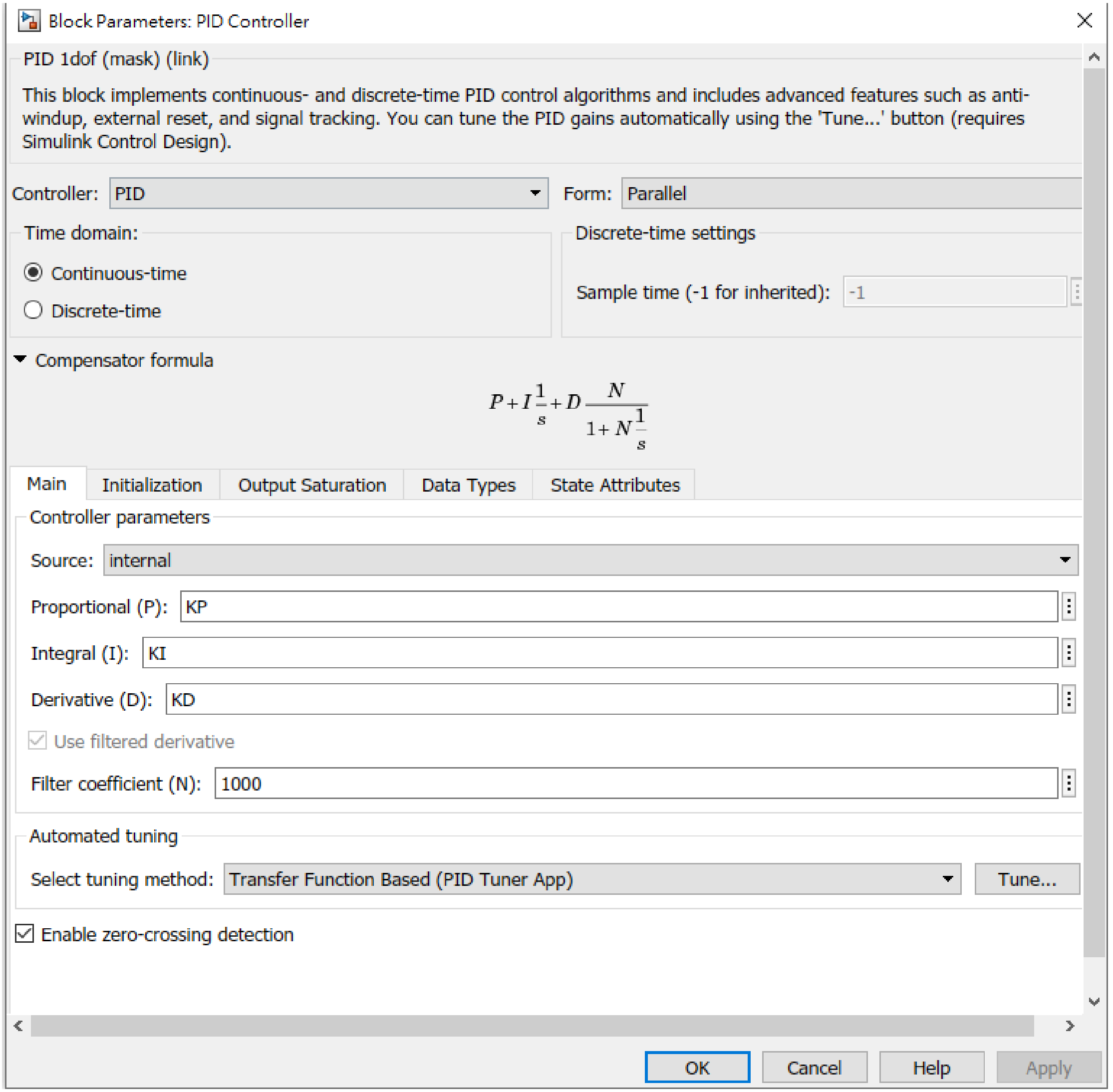
Assume that $T_d(s) = 1(T_d(t) = \delta(t))$

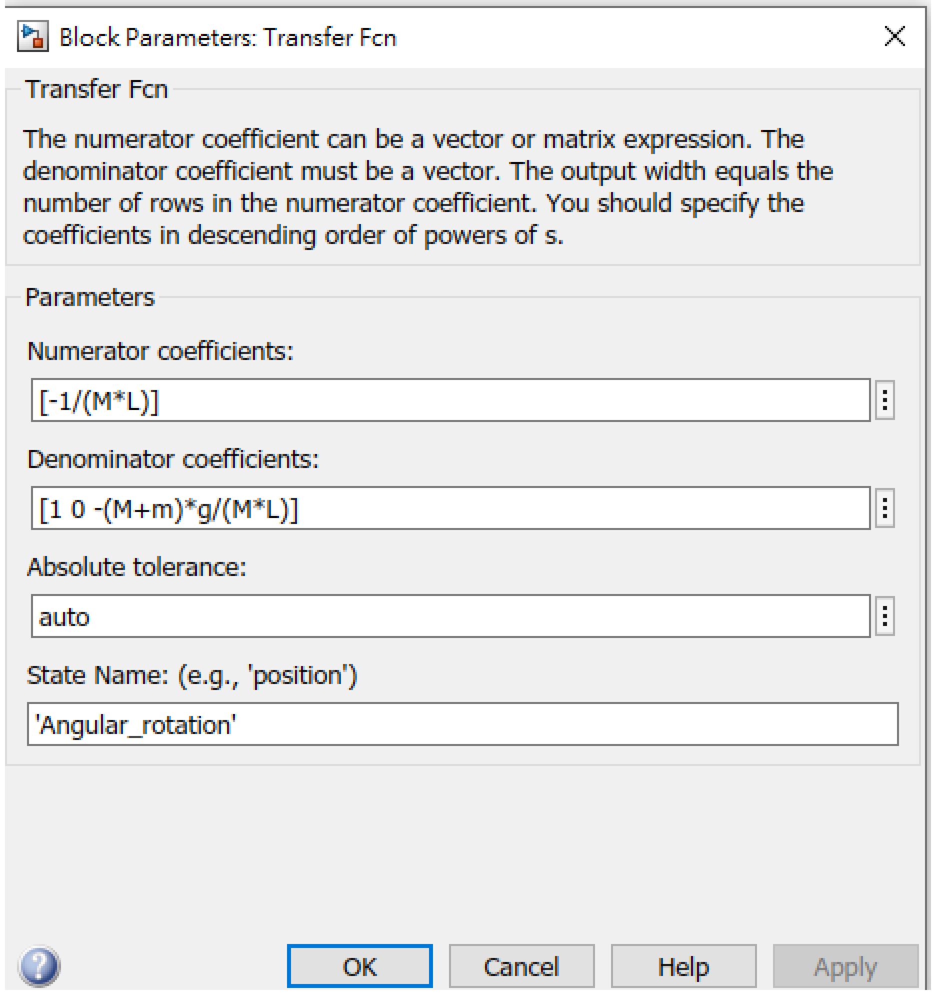
Create Block Diagram

You will need the following function blocks:
-> PID Controller
-> Add
-> Ground
-> Step
-> Transfer Fcn
-> Scope (see output result)

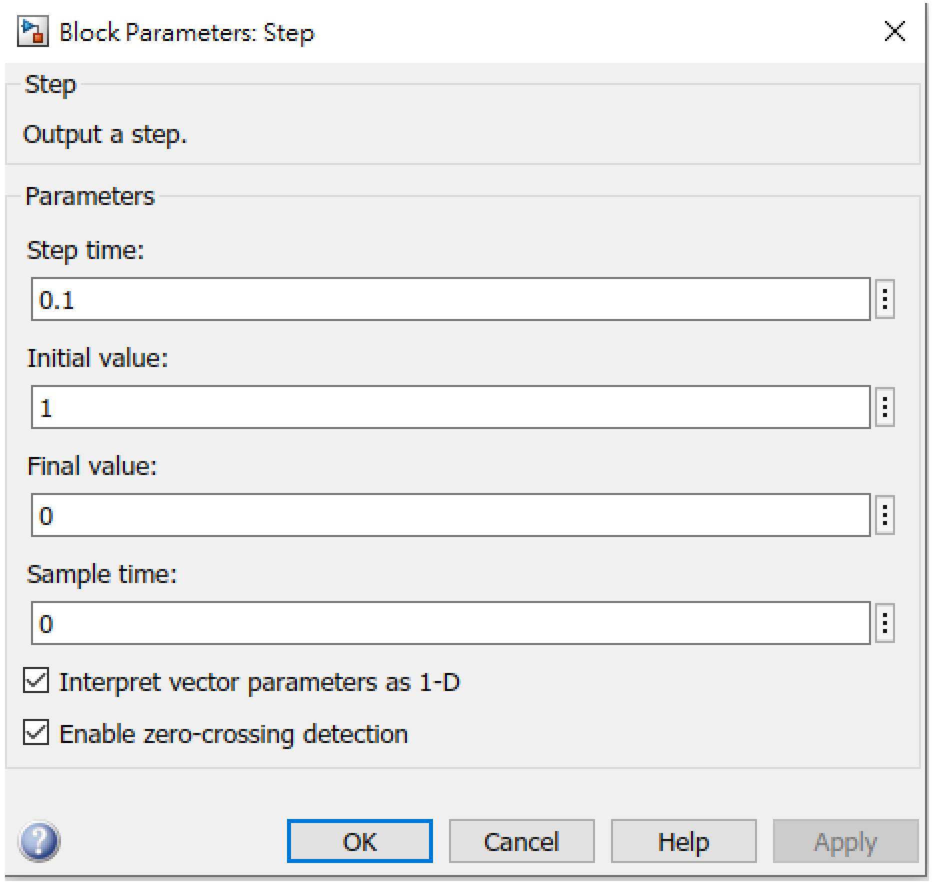


Set parameters





Set Parameters (Cont.)



How to run Simulink with matlab parameter

- In parameter file, choose the parameter $K_D, K_I, and\ K_P$
- Next, Run the parameter.m file, in order to save the data in workspace

Workspace

Name	Value
g	9.8100
KD	0
KI	0
KP	0
L	1
m	10
M	100

- After run the parameter.m, you can press run in Simulink