EE3510 HW5 105060012張育菘

In this homework, you are going to design a PID controller to control the inverted pendulum model in homework 4.

1) Use ode45 to simulate the output response in time-domain.

Problem 1

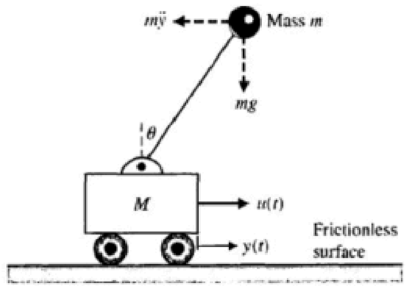


Figure 1 A cart and an inverted pendulum.

Force (horizontal):

$$(\mathrm{M}+\mathrm{m})\ddot{y}(t) = u(t) - \left(mL\ddot{\theta}(t)\cos\theta(t) - mL\dot{\theta}(t)^2\sin\theta(t)\right)$$

Torque:

$$\mathrm{mL}^2\ddot{\theta}(t) = mgL\sin\theta(t) - mL\ddot{y}(t)\cos\theta(t)$$

Assume that M=100Kg, m=10Kg, L=1m, g=9.81m/s^2 and $G_c(s) = K_P + \frac{K_I}{s} + sK_D$,

*This is a guideline for Problem 1.
*The equations are not exactly the same, but same concepts

1) Use ode45 to simulate the output response in time-domain.

$$\ddot{y} = -\frac{mg}{M}\theta + \frac{1}{M}u$$

$$\ddot{\theta} = \frac{(M+m)g}{ML}\theta - \frac{1}{ML}u$$

$$x_1 = y, x_2 = \dot{y}, x_3 = \int \theta dt, x_4 = \theta, x_5 = \dot{\theta}$$

$$\dot{x_1} = x_2$$

$$\dot{x_2} = -\frac{mg}{M}x_4 + \frac{1}{M}u$$

$$\dot{x_3} = x_4$$

$$\dot{x_4} = x_5$$

$$\dot{x_5} = \frac{(M+m)g}{ML}x_4 - \frac{1}{ML}u$$

PID control $U(s) = G_c(s)(R(s) - \theta(s))$

$$u(t) = -K_P\theta(t) - K_I\int \theta(t)dt - K_D\frac{d\theta(t)}{dt} = -K_P x_4 - K_I x_3 - K_D x_5$$

$$\dot{x_1} = x_2$$

$$\dot{x_2} = -\frac{mg}{M}x_4 + \frac{1}{M}(-K_P x_4 - K_I x_3 - K_D x_5)$$

$$\dot{x_3} = x_4$$

$$\dot{x_4} = x_5$$

$$\dot{x_5} = \frac{(M+m)g}{ML}x_4 - \frac{1}{ML}(-K_P x_4 - K_I x_3 - K_D x_5)$$

$$x_{10} = 0, x_{20} = 0, x_{30} = 0, x_{40} = 1, x_{50} = 0$$

q(s) can be obtained by (sI-A).

$$q(s) = MLs^3 - K_D s^2 - [(M+m)g + K_P]s - K_I$$

$$G(s) = \frac{-0.01}{(s+3.285)(s-3.285)}, \mathrm{PID}(s) = \frac{K_D s^2 + K_P s + K_I}{s} = \frac{K_D(s-z_1)(s-z_2)}{s}$$

$$q(s) = \mathrm{ML}s^3 - K_D s^2 - [(M+m)g + K_P]s - K_I = 0$$

#For Stable system, $-K_D > 0, -[(M+m)g + K_P] - \frac{K_I}{K_D}\mathrm{ML} > 0, -K_I > 0$

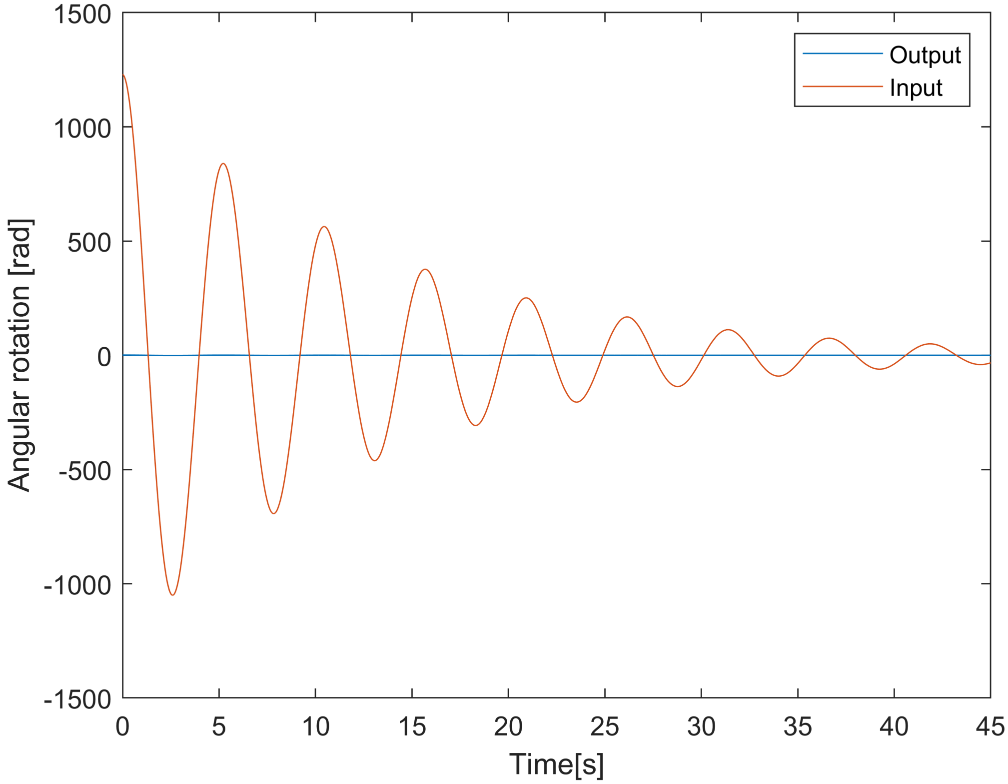
#For Marginally Stable system, $K_D < 0, K_I < 0, K_P = -[(M+m)g] - \frac{K_I}{K_D}\mathrm{ML}$

#I design a stable system with $K_D = -50, K_I = -50, K_P = -1229.1$

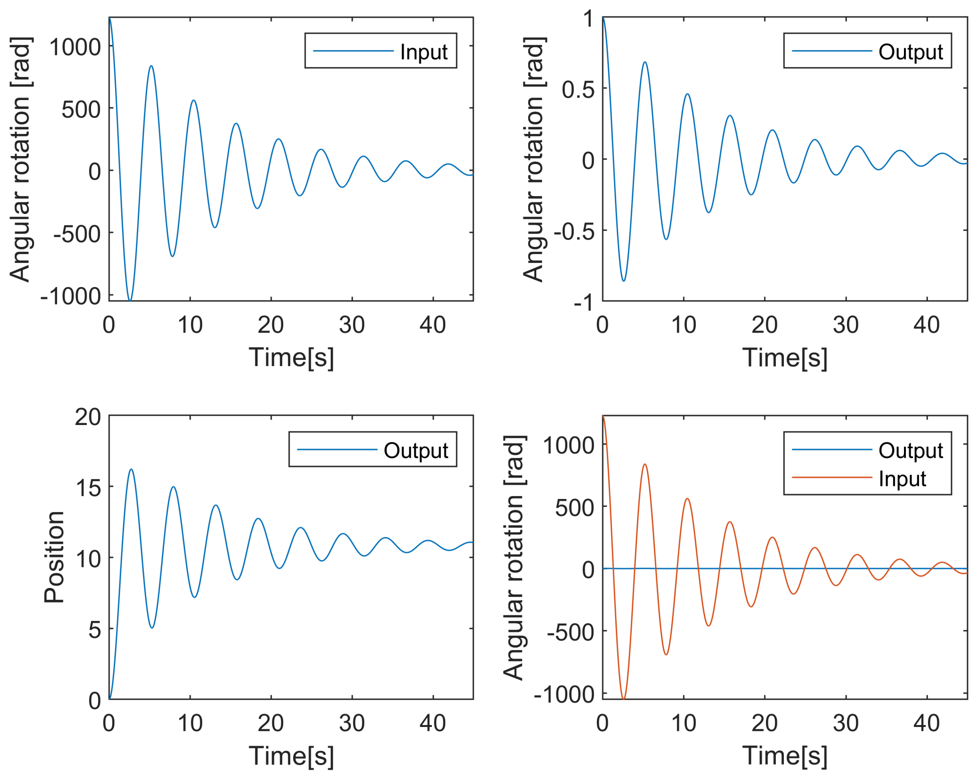
```
close all; clear;
fprintf("Q1 \n");
```

```
Q1
```

```

% Parameters
m = 10; M = 100; L = 1; g = 9.81;

% PID tuning
KD = -50; KI = -50; KP = -((M+m)*g)-((KI/KD)*M*L)-50;

% Reference signal
r = 0; c_in = [0;1/M;0;0;-1/(M*L)];
ref_sig=@(t, x)[x(2);-m*g*x(4)/M;x(4);x(5);(M+m)*g*x(4)/(M*L)]+(-KP*x(4)-KI*x(3)-KD*x(5))*c_in
T = linspace(0, 45, 3*1e3);
x0 = [0; 0; 0; 1; 0];
[t, Y] = ode45(@(t, x) ref_sig(t, x), T, x0);

u = (-KP.*Y(:,4)) + (-KI.*Y(:,3)) + (-KD.*Y(:,5));

% Plot
figure; plot(t, Y(:,4), t, u);
xlabel('Time[s]');
ylabel('Angular rotation [rad]');
legend(["Output"], ["Input"]);
```




```
figure;
subplot(2,2,1); plot(t, u);
xlabel('Time[s]'); ylabel('Angular rotation [rad]');
legend(["Input"]);
subplot(2,2,2); plot(t, Y(:,4));
xlabel('Time[s]'); ylabel('Angular rotation [rad]');
legend(["Output"]);
subplot(2,2,3); plot(t, Y(:,1));
xlabel('Time[s]'); ylabel('Position');
legend(["Output"]);
subplot(2,2,4); plot(t, Y(:,4), t, u);
xlabel('Time[s]'); ylabel('Angular rotation [rad]');
legend(["Output"], ["Input"]);
```



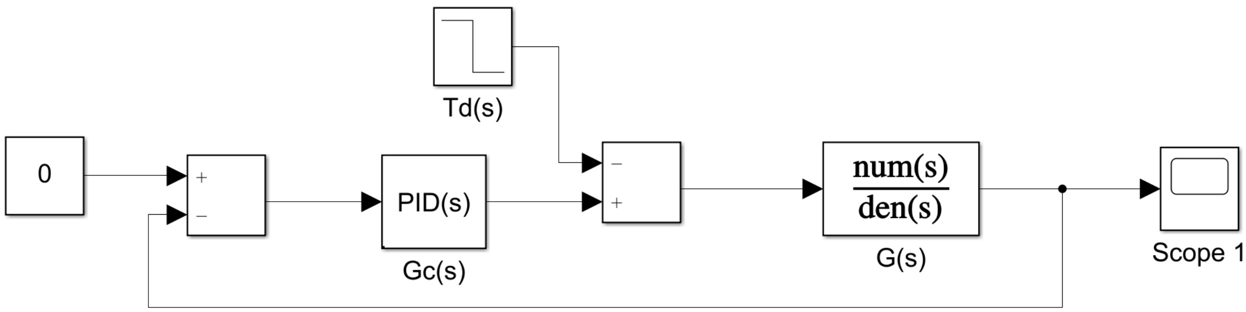

```
syms x;
eqn = M*L*x^3 - KD*x^2 - ((M+m)*g+KP)*x - KI == 0;
s = solve(eqn, x);
digits(6) % sets the precision used by vpa to 6 significant decimal digits.
vpa(s) % Variable-precision arithmetic
```

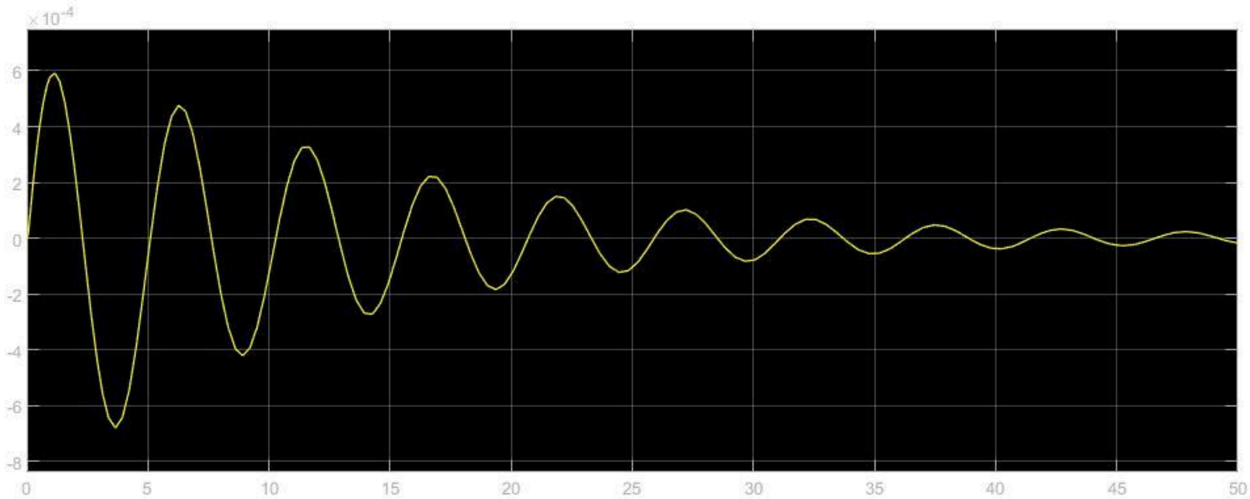
ans =

$$\begin{pmatrix} -0.345627 \\ -0.0771863 - 1.20029\,\mathrm{i} \\ -0.0771863 + 1.20029\,\mathrm{i} \end{pmatrix}$$

由模擬結果顯示，output 確實會漸漸 decay 至 0，這顯示此為一個穩定的系統。

2) Use Simulink to simulate the output response in s-domain.



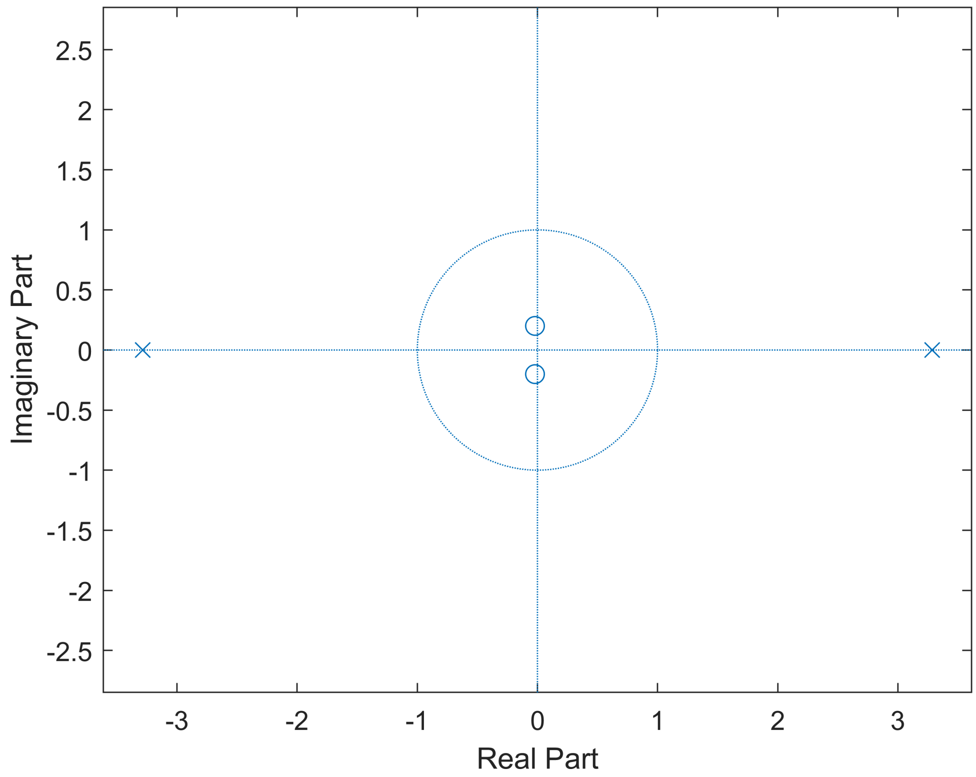


由 Simulink 模擬結果顯示，此為一個穩定的系統。

```
close all; clear;
fprintf("Q2 \n");
```

Q2

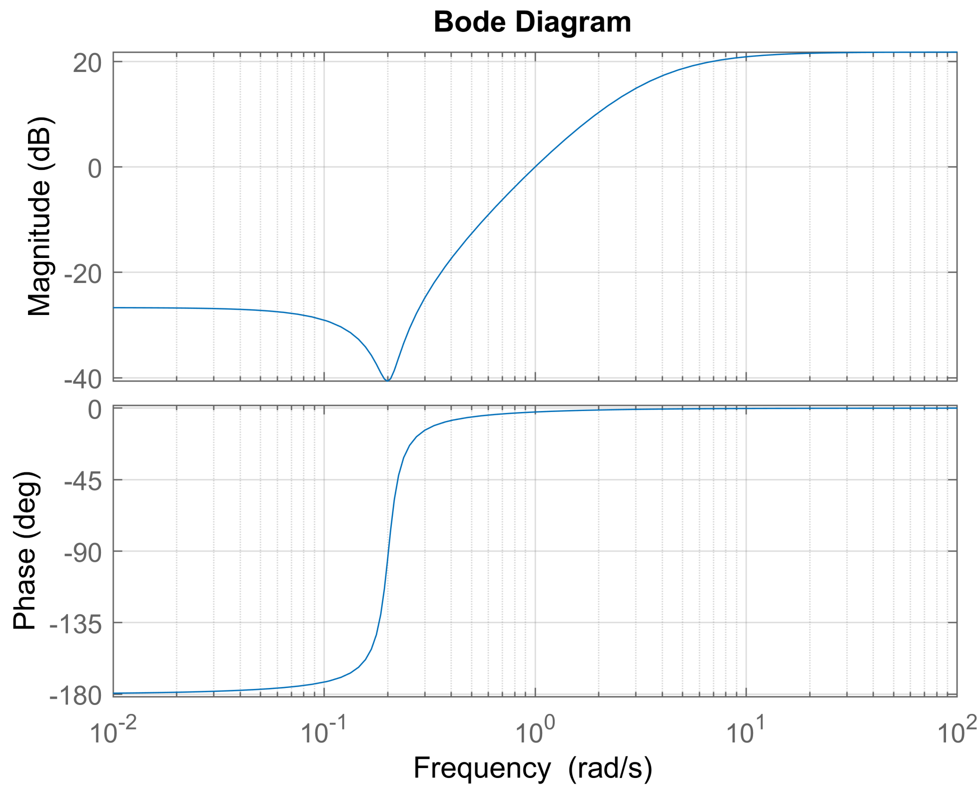
```

% Parameters
m = 10; M = 100; L = 1; g = 9.81;

% PID tuning
KD = -50; KI = -50; KP = -((M+m)*g)-((KI/KD)*M*L)-50;

b = [-1/(M*L)]; % -0.01
a = [1 0 -(M+m)*g/(M*L)]; % [1 0 -10.7910]

b_pid = [KP KI KD]; a_pid = [1];
b_all = conv(b_pid, b); a_all = conv(a_pid, a);
figure; zplane(b_all,a_all);
```



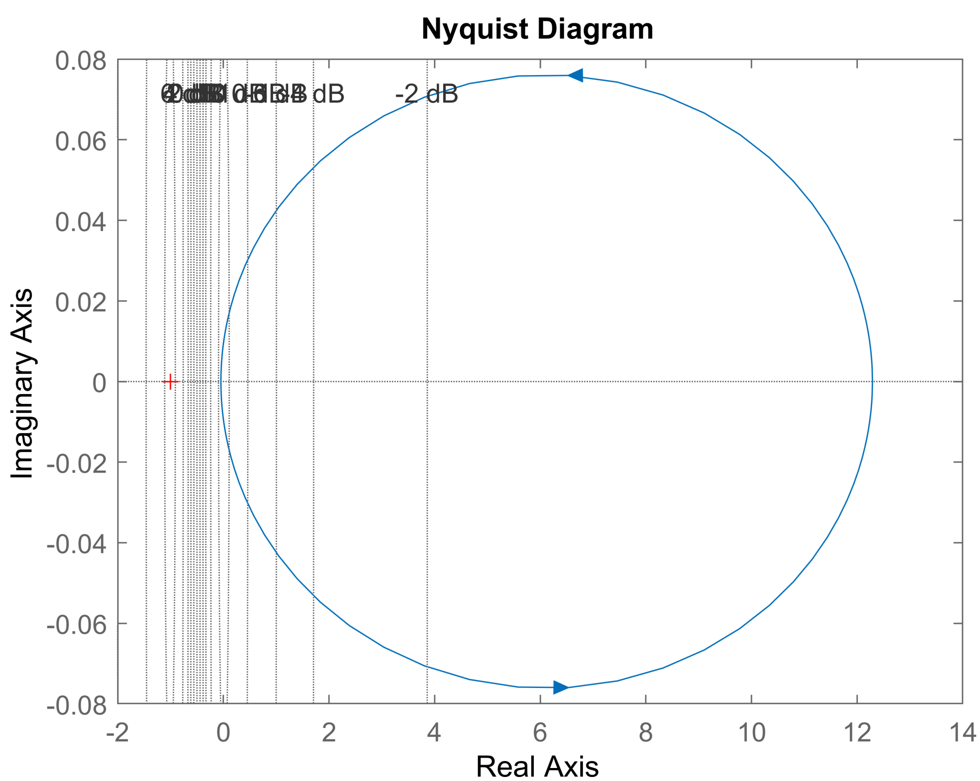

```
% open-loop system
fprintf("open-loop system \n");
```

```
open-loop system
```

```
sys = tf(b_all, a_all);
figure; bode(sys); grid on;
```



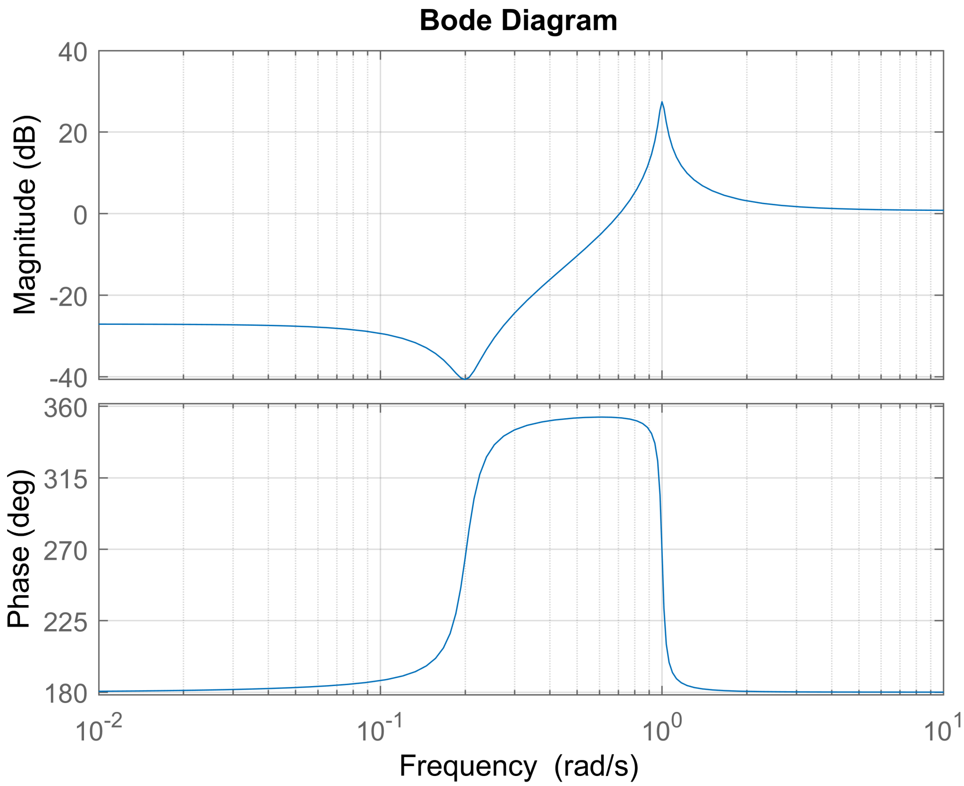

```
figure; nyquist(sys); grid on;
```



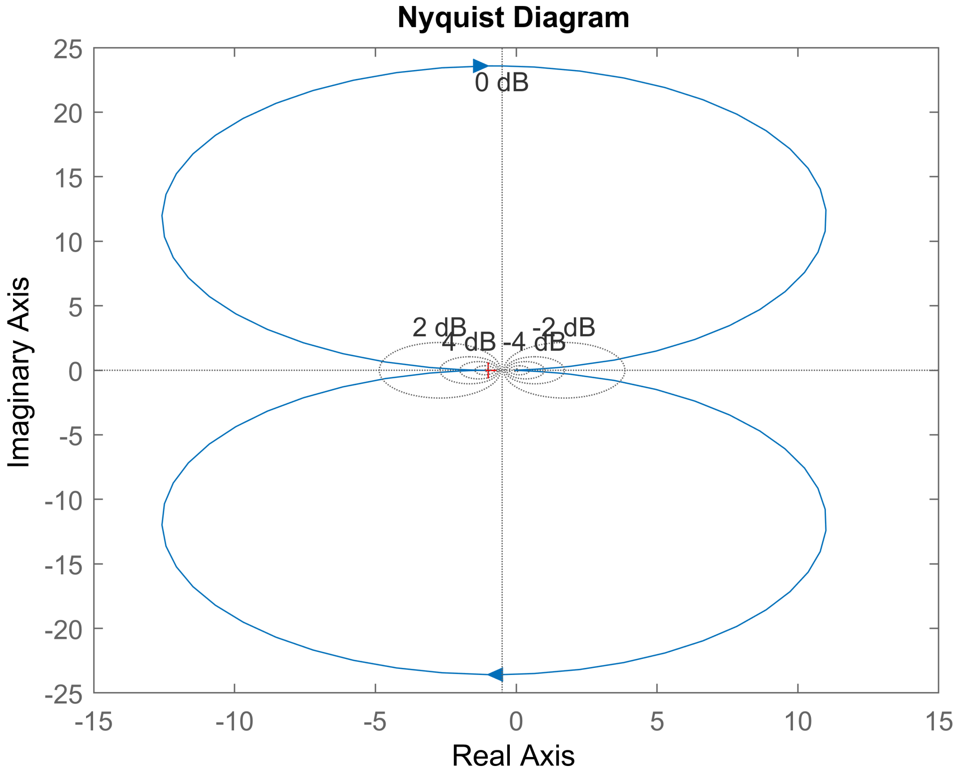
```
% closed-loop system
fprintf("closed-loop system \n");
```

```
closed-loop system
```

```
T_sys = feedback(sys,1,+1);
figure; bode(T_sys); grid on;
```




```
figure; nyquist(T_sys); grid on;
```


Nyquist Diagram
Imaginary Axis
Real Axis
0 dB
2 dB
4 dB
-4 dB
-2 dB
25
20
15
10
5
0
-5
-10
-15
-20
-25
-15
-10
-5
0
5
10
15