Consider a unity feedback control system with Gc(s)=K and R(s)=0 for inverted pendulum (example 3.3) in textbook.

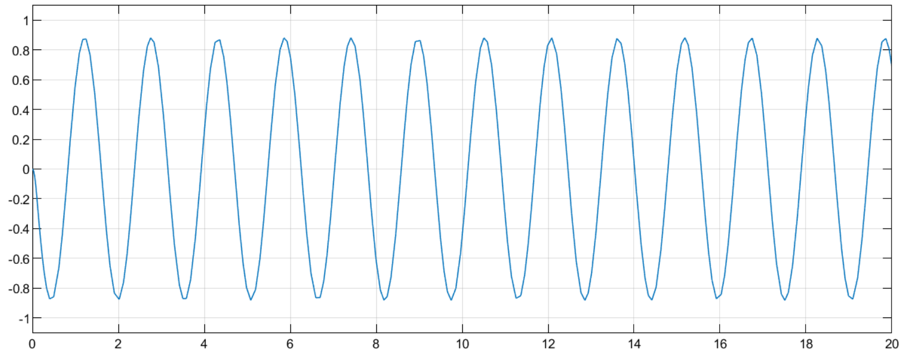
Analyze C1 = [0,0,1,0], C2 = [0,0,1,1], and C3 = [0,1,1,1] and different K.

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -\frac{mg}{M} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & \frac{g}{l} & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ \frac{1}{M} \\ 0 \\ -\frac{1}{Ml} \end{bmatrix}$$

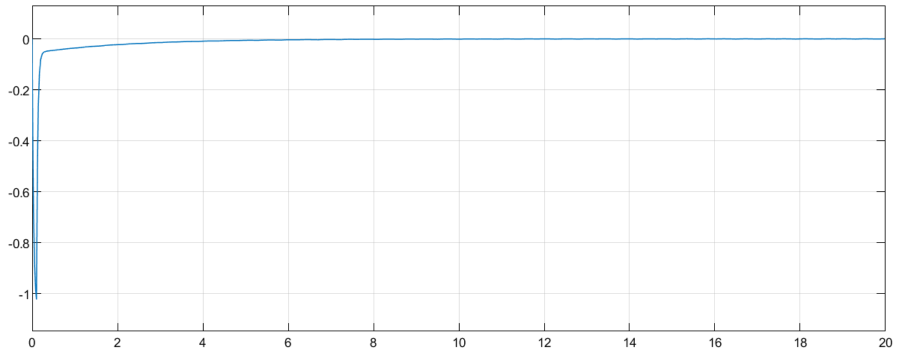
$$\dot{x}(t) = Ax(t) + Bu(t),\ y(t) = Cx(t) + Du(t)$$

1) use Simulink to simulate the output response for different K in s-domain.

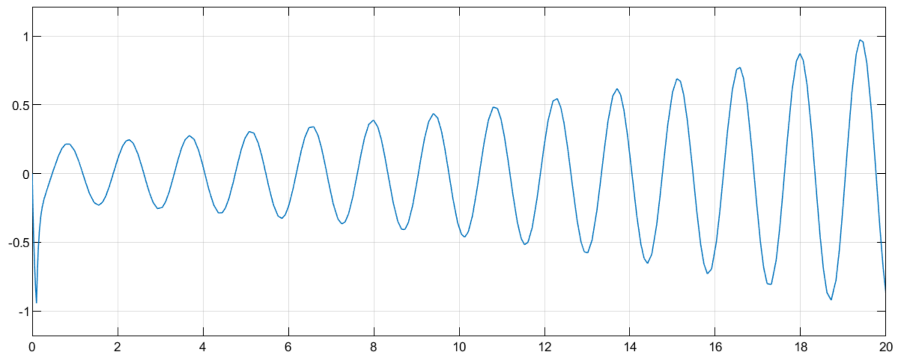
- For K = -36,
  - C = C1 = [0,0,1,0]



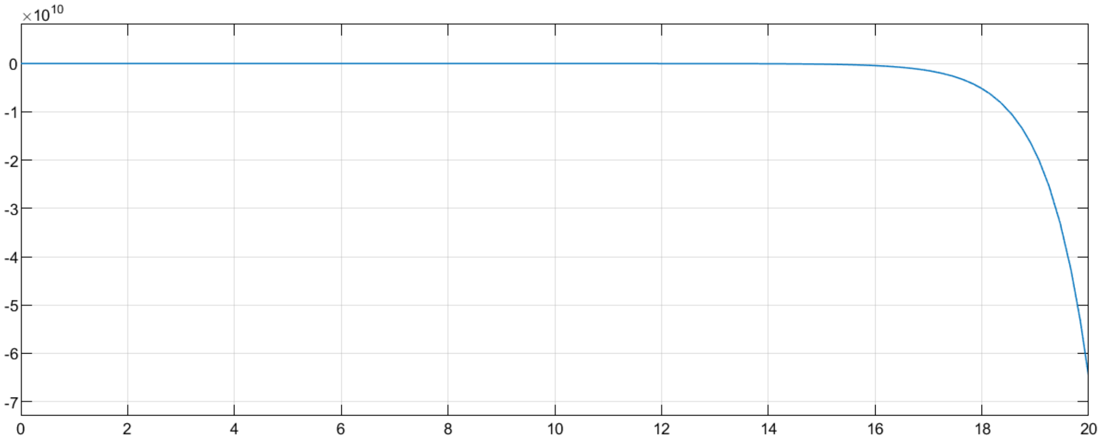
  - C = C2 = [0,0,1,1]



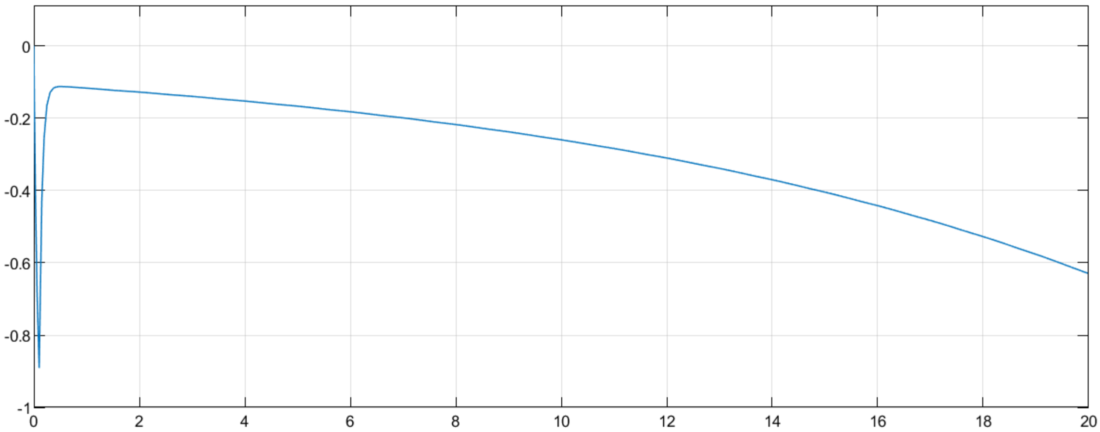
  - C = C3 = [0,1,1,1]



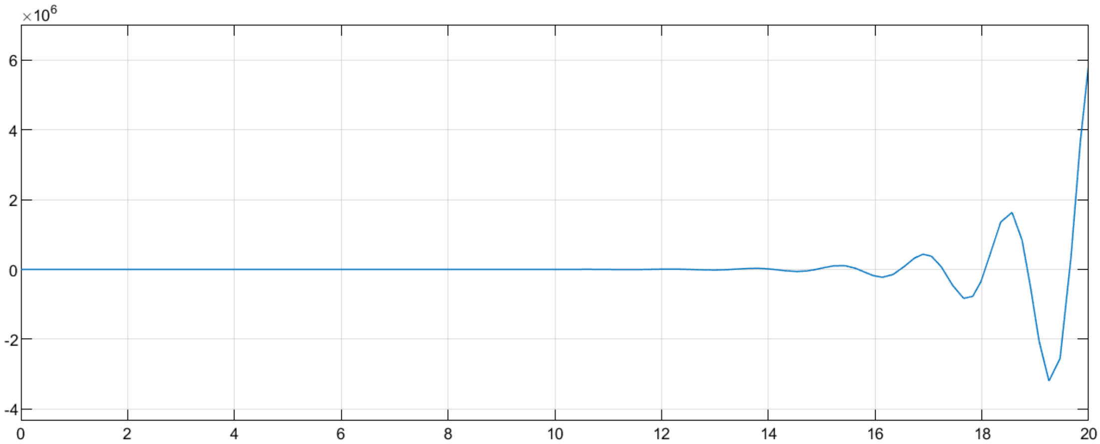
- For K = -18,
  - C = C1 = [0,0,1,0]



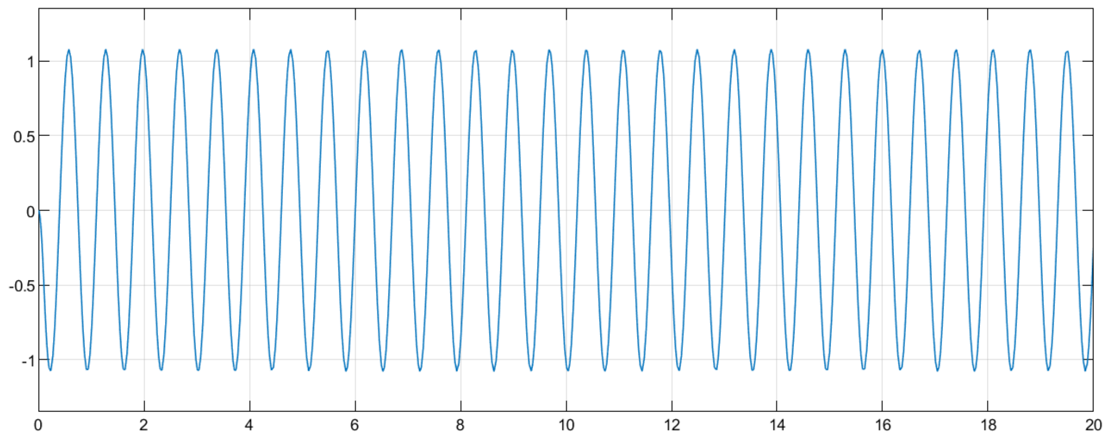
  - C = C2 = [0,0,1,1]



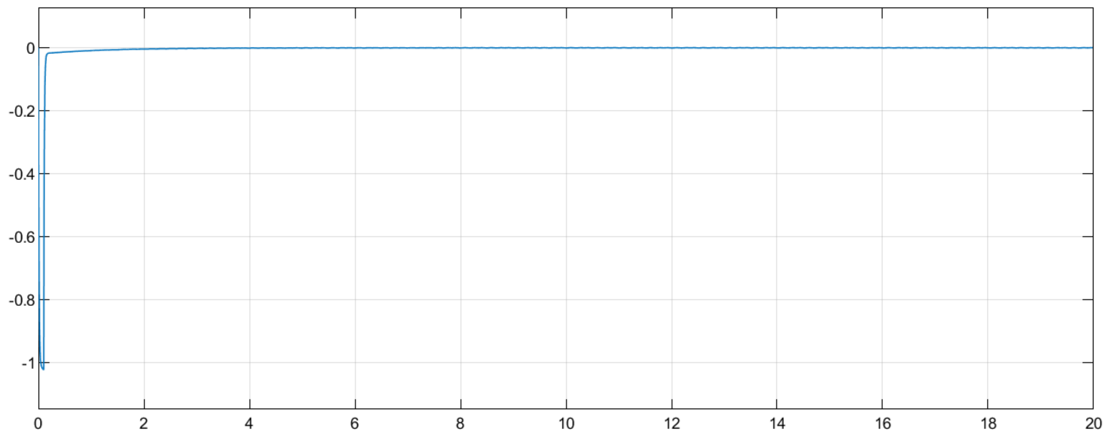
  - C = C3 = [0,1,1,1]



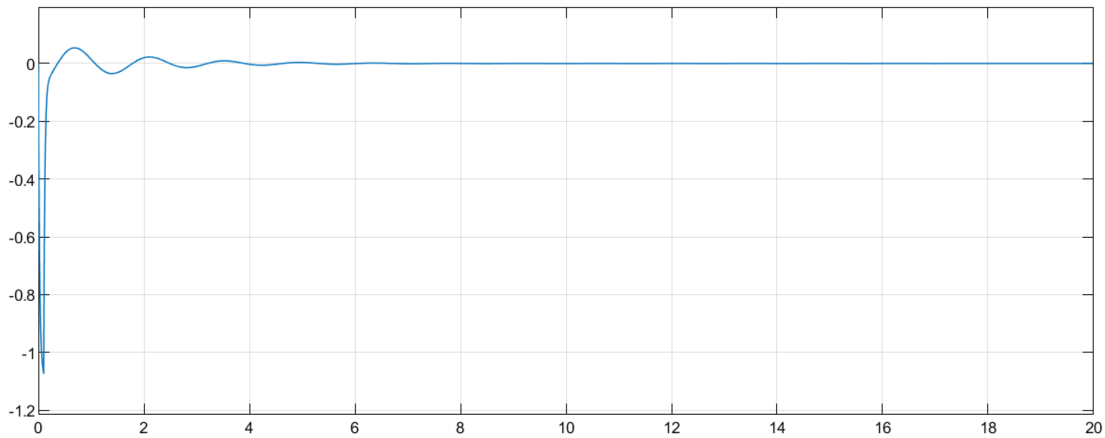
- For K = -100,
  - C = C1 = [0,0,1,0]



  - C = C2 = [0,0,1,1]



  - C = C3 = [0,1,1,1]



EE3510 HW4 105060012張育菘

Consider a unity feedback control system with Gc(s)=K and R(s)=0 for inverted pendulum (example 3.3) in textbook.

Analyze C = [0,0,1,0],[0,0,1,1] and [0,1,1,1] and different K.

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -\frac{\mathrm{mg}}{M} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & \frac{g}{l} & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ \frac{1}{M} \\ 0 \\ -\frac{1}{\mathrm{Ml}} \end{bmatrix}$$

$$\dot{x}(t) = \mathrm{Ax}(t) + \mathrm{Bu}(t), y(t) = \mathrm{Cx}(t) + \mathrm{Du}(t)$$

1) use Simulink to simulate the output response for different K in s-domain.

```
close all; clear;

% initial parameter
g = 9.8; l = 0.5; m = 0.01; M = 2;

% system matrix
A = [0 1 0 0; 0 0 -m*g/M 0; 0 0 0 1; 0 0 g/l 0;];
B = [0; 1/M; 0; -1/(M*l)];
C1 = [0 0 1 0]; C2 = [0 0 1 1]; C3 = [0 1 1 1];
D = [0];
K = -36; % K = -18, -36, -100

[num1, den1] = ss2tf(A,B,C1,D)
```

```
num1 = 1×5
         0         0   -1.0000    0.0000    0.0000
den1 = 1×5
    1.0000         0  -19.6000         0         0
```

```
[num2, den2] = ss2tf(A,B,C2,D)
```

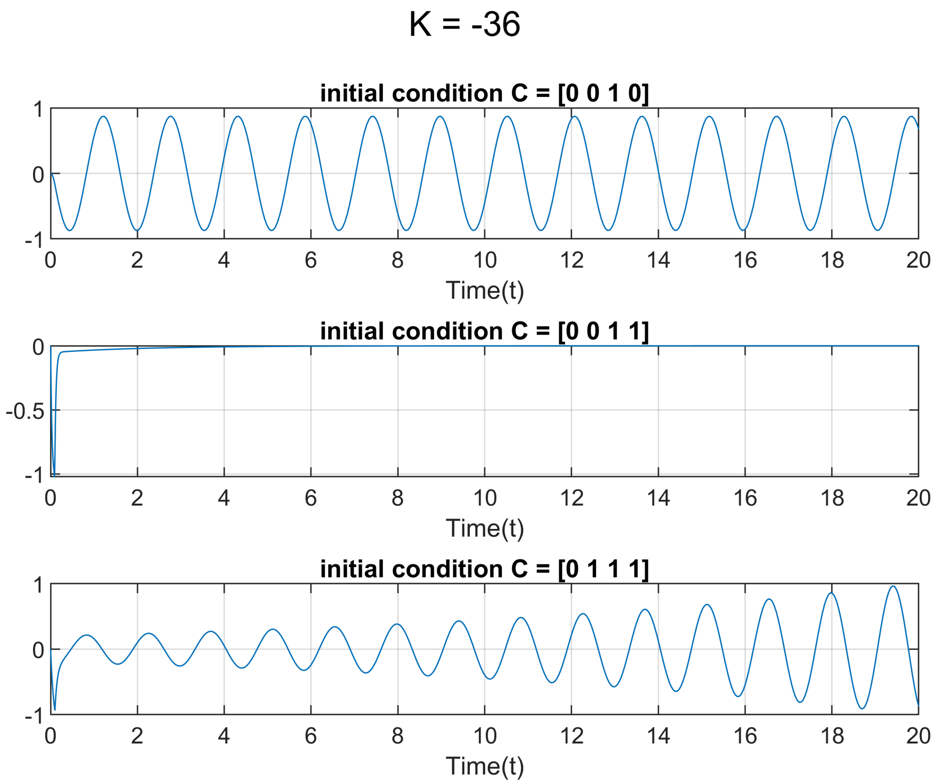
```
num2 = 1×5
         0   -1.0000   -1.0000         0         0
den2 = 1×5
    1.0000         0  -19.6000         0         0
```

```
[num3, den3] = ss2tf(A,B,C3,D)
```

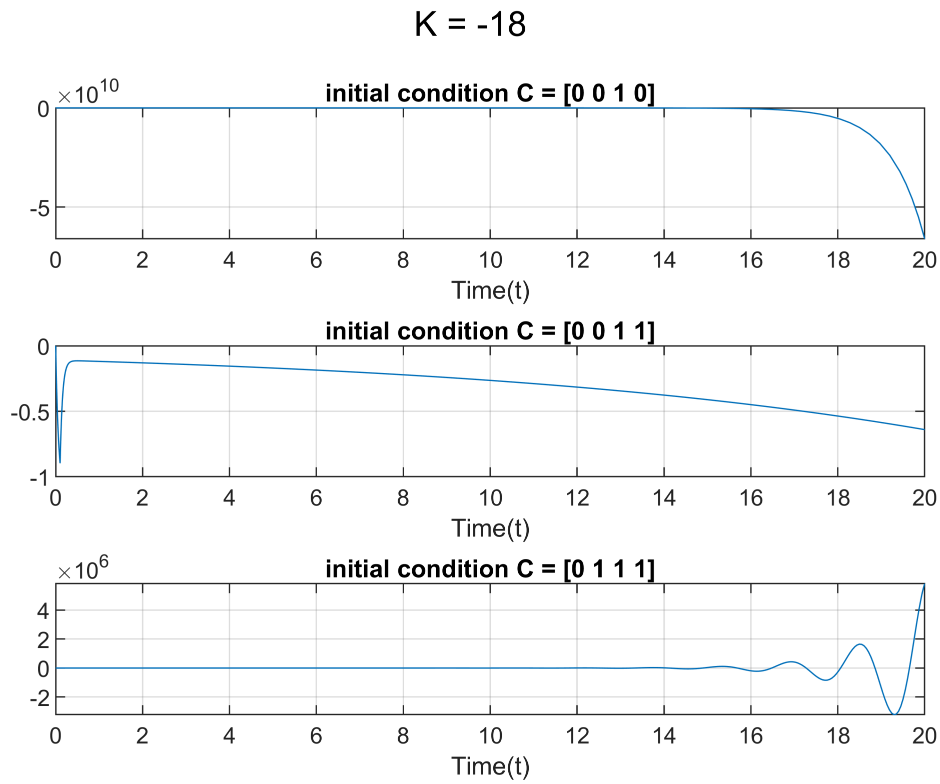
```
num3 = 1×5
         0   -0.5000   -1.0000   -9.7510         0
den3 = 1×5
    1.0000         0  -19.6000         0         0
```

2) use ode45 to simulate the output response for different K in time -domain.

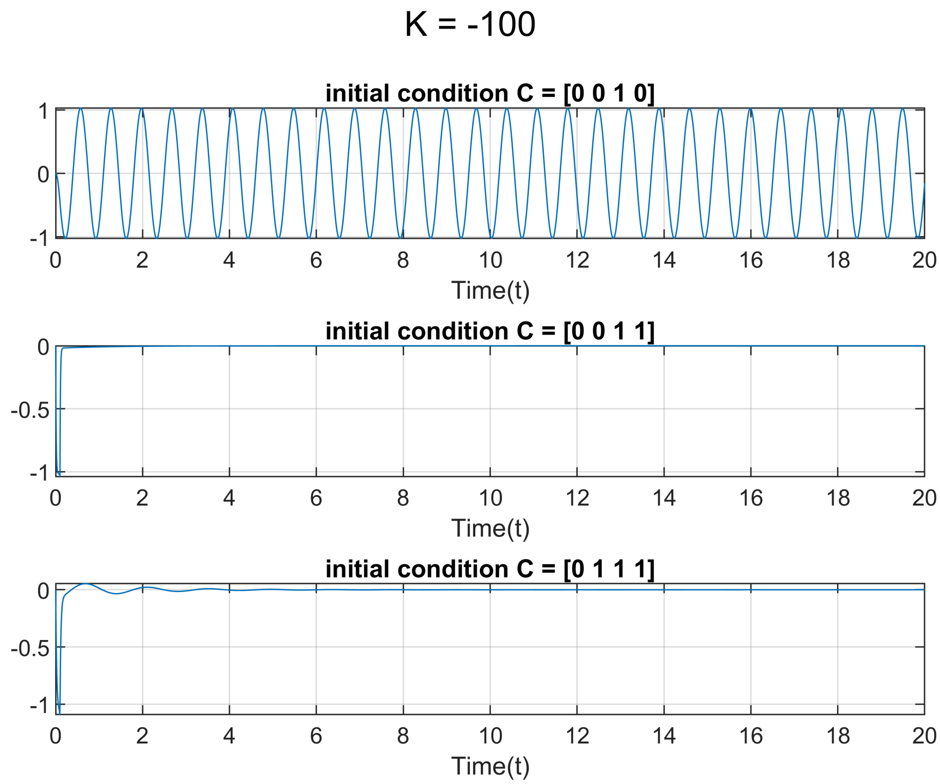
```
close all; clear;

tspan = [0 20]; % time interval from 0 ~ 20
iniCon = [0; 0; 0; 0;]; % initial condition

C1 = [0 0 1 0]; C2 = [0 0 1 1]; C3 = [0 1 1 1];
K1 = -36;   K2 = -18;   K3 = -100;

[t1, x1] = ode45(@(t, x) sysf(t, x, C1, K1), tspan, iniCon);
[t2, x2] = ode45(@(t, x) sysf(t, x, C2, K1), tspan, iniCon);
[t3, x3] = ode45(@(t, x) sysf(t, x, C3, K1), tspan, iniCon);

y1 = C1.*x1;
y2 = C2.*x2;
y3 = C3.*x3;

y1_a = y1(:, 1) + y1(:, 2) + y1(:, 3) + y1(:, 4);
y2_a = y2(:, 1) + y2(:, 2) + y2(:, 3) + y2(:, 4);
y3_a = y3(:, 1) + y3(:, 2) + y3(:, 3) + y3(:, 4);

figure; sgtitle('K = -36');
subplot(3, 1, 1); plot(t1, y1_a); grid on;
title('initial condition C = [0 0 1 0]');
xlabel('Time(t)');
subplot(3, 1, 2); plot(t2, y2_a); grid on;
title('initial condition C = [0 0 1 1]');
xlabel('Time(t)');
subplot(3, 1, 3); plot(t3, y3_a); grid on;
title('initial condition C = [0 1 1 1]');
xlabel('Time(t)');
```




```
[t1, x1] = ode45(@(t, x) sysf(t, x, C1, K2), tspan, iniCon);
[t2, x2] = ode45(@(t, x) sysf(t, x, C2, K2), tspan, iniCon);
[t3, x3] = ode45(@(t, x) sysf(t, x, C3, K2), tspan, iniCon);

y1 = C1.*x1;
y2 = C2.*x2;
y3 = C3.*x3;

y1_a = y1(:, 1) + y1(:, 2) + y1(:, 3) + y1(:, 4);
y2_a = y2(:, 1) + y2(:, 2) + y2(:, 3) + y2(:, 4);
y3_a = y3(:, 1) + y3(:, 2) + y3(:, 3) + y3(:, 4);

figure; sgtitle('K = -18');
subplot(3, 1, 1); plot(t1, y1_a); grid on;
title('initial condition C = [0 0 1 0]');
xlabel('Time(t)');
subplot(3, 1, 2); plot(t2, y2_a); grid on;
title('initial condition C = [0 0 1 1]');
xlabel('Time(t)');
subplot(3, 1, 3); plot(t3, y3_a); grid on;
title('initial condition C = [0 1 1 1]');
xlabel('Time(t)');
```




```
[t1, x1] = ode45(@(t, x) sysf(t, x, C1, K3), tspan, iniCon);
[t2, x2] = ode45(@(t, x) sysf(t, x, C2, K3), tspan, iniCon);
[t3, x3] = ode45(@(t, x) sysf(t, x, C3, K3), tspan, iniCon);

y1 = C1.*x1;
y2 = C2.*x2;
y3 = C3.*x3;

y1_a = y1(:, 1) + y1(:, 2) + y1(:, 3) + y1(:, 4);
y2_a = y2(:, 1) + y2(:, 2) + y2(:, 3) + y2(:, 4);
y3_a = y3(:, 1) + y3(:, 2) + y3(:, 3) + y3(:, 4);

figure; sgtitle('K = -100');
subplot(3, 1, 1); plot(t1, y1_a); grid on;
title('initial condition C = [0 0 1 0]');
xlabel('Time(t)');
subplot(3, 1, 2); plot(t2, y2_a); grid on;
title('initial condition C = [0 0 1 1]');
xlabel('Time(t)');
subplot(3, 1, 3); plot(t3, y3_a); grid on;
title('initial condition C = [0 1 1 1]');
xlabel('Time(t)');
```




```
function dx = sysf(t, x, C, K)
    % initial parameter
    g = 9.8; l = 0.5; m = 0.01; M = 2;

    pulse = rectangularPulse(0, 0.1, t);

    % system matrix
    A = [0 1 0 0; 0 0 -m*g/M 0; 0 0 0 1; 0 0 g/l 0;];
    B = [0; 1/M; 0; -1/(M*l)];

    C_a = C*x;

    Gc = (0-(C_a+pulse))*K;
    u = Gc*heaviside(t);
    dx = A*x + B*u;
end
```