EE3510 HW3 105060012張育菘

CP3.1 Determine a state variable representation for the following transfer functions (without feedback) using the SS function:

(a) $G(s) = \dfrac{1}{s+10}$

```
close all; clear;

g1_num = [1]; g1_den = [1 10];
g1_tf = tf(g1_num, g1_den);
g1_ss = ss(g1_tf)
```

```
g1_ss =

  A = 
        x1
   x1  -10

  B = 
       u1
   x1   1

  C = 
       x1
   y1   1

  D = 
       u1
   y1   0

Continuous-time state-space model.
```

(b) $G(s) = \dfrac{s^2+5s+3}{s^2+8s+5}$

```
g2_num = [1 5 3]; g2_den = [1 8 5];
g2_tf = tf(g2_num, g2_den);
g2_ss = ss(g2_tf)
```

```
g2_ss =

  A = 
         x1    x2
   x1    -8  -2.5
   x2     2     0

  B = 
       u1
   x1   2
   x2   0
```

```
  C =
         x1    x2
   y1  -1.5  -0.5

  D =
       u1
   y1   1

Continuous-time state-space model.
```

(c) $G(s) = \dfrac{s+1}{s^3 + 3s^2 + 3s + 1}$

```
g3_num = [1 1]; g3_den = [1 3 3 1];
g3_tf = tf(g3_num, g3_den);
g3_ss = ss(g3_tf)
```

```
g3_ss =

  A =
         x1    x2    x3
   x1    -3  -1.5    -1
   x2     2     0     0
   x3     0   0.5     0

  B =
       u1
   x1   1
   x2   0
   x3   0

  C =
        x1   x2   x3
   y1    0  0.5    1

  D =
       u1
   y1   0

Continuous-time state-space model.
```

CP3.2 Determine a transfer function representation for the following state variable models using the tf function:

(a) $A = \begin{bmatrix} 0 & 1 \\ 2 & 8 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 \end{bmatrix}$

```
close all; clear;

a1 = [0 1; 2 8]; b1 = [0; 1];
c1 = [1 0]; d1 = [0];
sys1_ss = ss(a1, b1, c1, d1);
sys1_tf = tf(sys1_ss)
```

```
sys1_tf =

        1
  -------------
  s^2 - 8 s - 2

Continuous-time transfer function.
```

(b) $A = \begin{bmatrix} 1 & 1 & 0 \\ -2 & 0 & 4 \\ 5 & 4 & -7 \end{bmatrix}, B = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}, C = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}$

```
a2 = [1 1 0; -2 0 4; 5 4 -7]; b2 = [-1; 0; 1];
c2 = [0 1 0]; d2 = [0];
sys2_ss = ss(a2, b2, c2, d2);
sys2_tf = tf(sys2_ss)
```

```
sys2_tf =

         6 s - 10
  -----------------------
  s^3 + 6 s^2 - 21 s + 10

Continuous-time transfer function.
```

(c) $A = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C = \begin{bmatrix} -2 & 1 \end{bmatrix}$

```
a3 = [0 1; -1 -2]; b3 = [0; 1];
c3 = [-2 1]; d3 = [0];
sys3_ss = ss(a3, b3, c3, d3);
sys3_tf = tf(sys3_ss)
```

```
sys3_tf =

      s - 2
  -------------
  s^2 + 2 s + 1

Continuous-time transfer function.
```

CP3.4 Consider the system

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -3 & -2 & -5 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u, y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} x$$

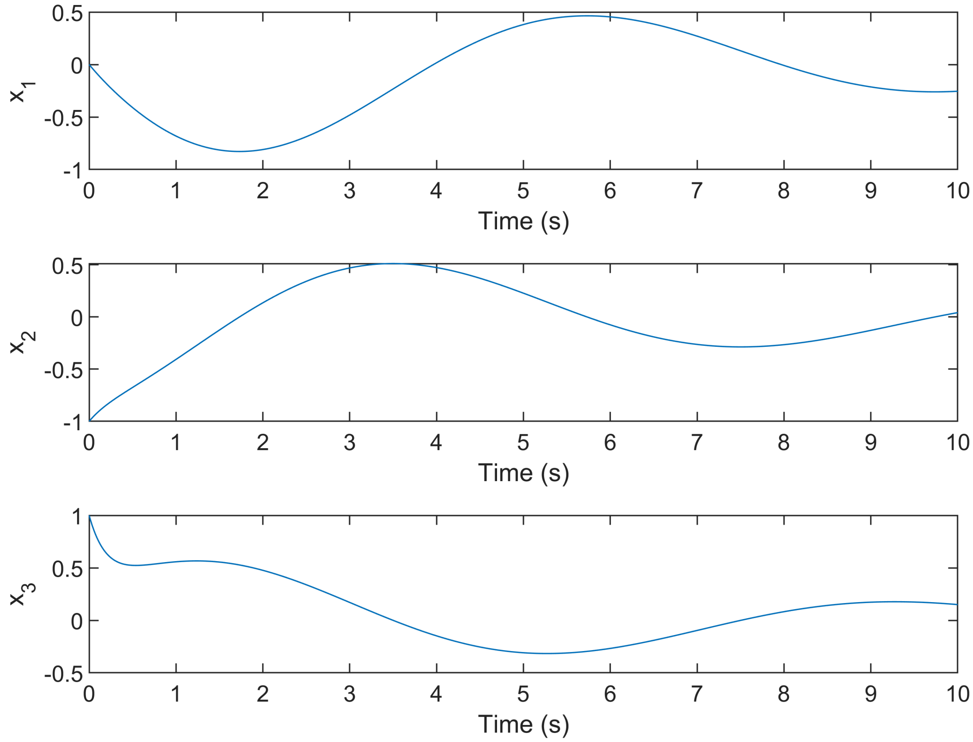
(a) Using the tf function, determine the transfer function Y(s)/U(s).

```
close all; clear;

a = [0 1 0; 0 0 1; -3 -2 -5]; b = [0; 0; 1];
c = [1 0 0]; d = [0];
sys_ss = ss(a, b, c, d);
sys_tf = tf(sys_ss)
```

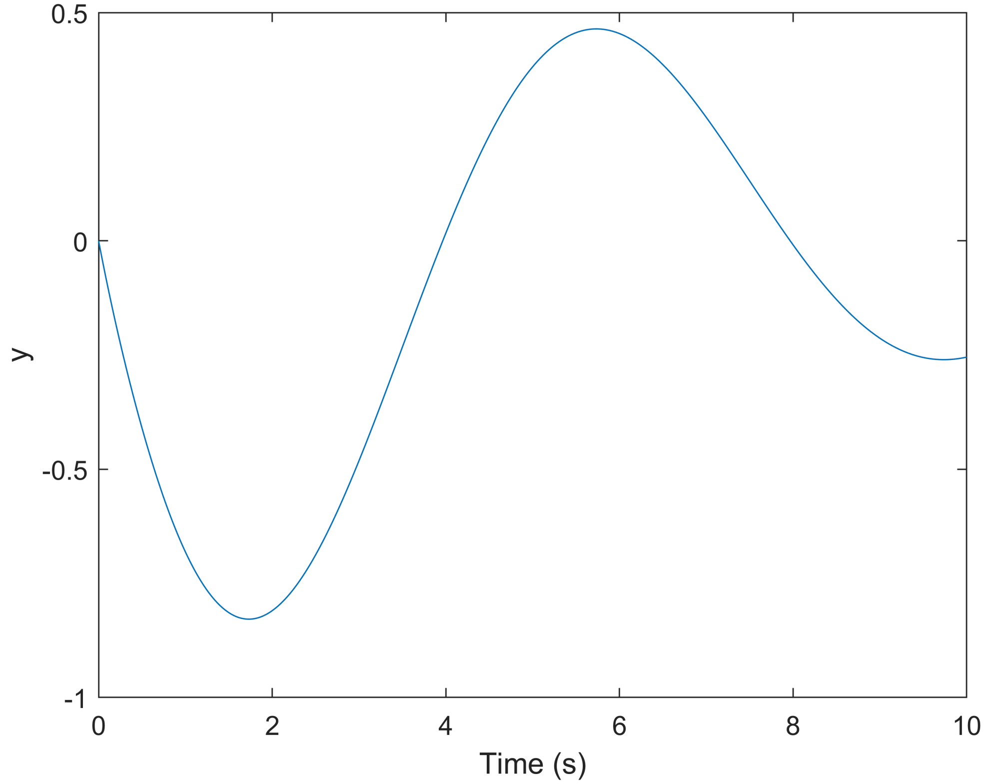
```
sys_tf =

            1
  ---------------------
  s^3 + 5 s^2 + 2 s + 3

Continuous-time transfer function.
```

(b) Plot the response of the system to the initial condition $x(0) = [0 \quad -1 \quad 1]^T$ for $0 \le t \le 10$.

```
x0 = [0; -1; 1]; % initial condition
t = [0:0.01:10];
u = 0*t; % zero input

[y,T,x]=lsim(sys_ss,u,t,x0);
figure;
subplot(3,1,1); plot(T,x(:,1));
xlabel('Time (s)'); ylabel('x_1');
subplot(3,1,2); plot(T,x(:,2));
xlabel('Time (s)'); ylabel('x_2');
subplot(3,1,3); plot(T,x(:,3));
xlabel('Time (s)'); ylabel('x_3');
```




```
figure; plot(T,y);
xlabel('Time (s)'); ylabel('y');
```



(c) Compute the state transition matrix using the expm function,and determine x(t) at t = 10 for the initial condition given in part (b). Compare the result with the system response obtained in part (b).

```
dt = 10;
phi = expm(a*dt);
fprintf("the result from part(c):");
```

```
the result from part(c):
```

```
x3 = phi*x0
```

```
x3 = 3×1
   -0.2545
    0.0418
    0.1500
```

```
fprintf("the result from part(b):");
```

```
the result from part(b):
```

```
ans_b = x(1001,:)
```

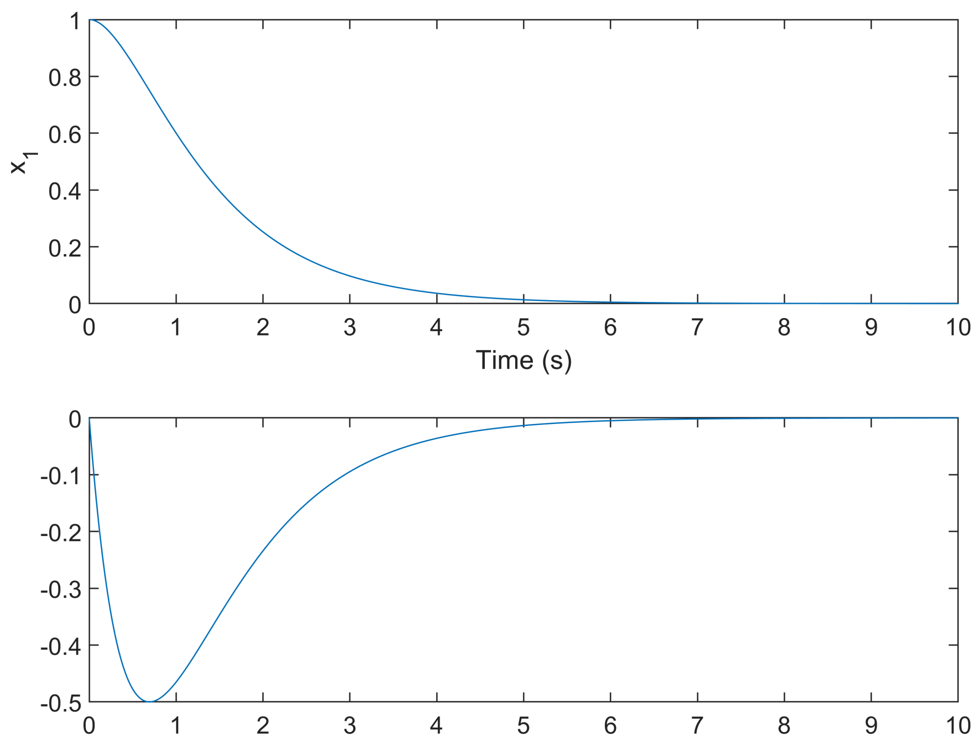
```
ans_b = 1×3
   -0.2545    0.0418    0.1500
```

We can find that the results from part (b) and part (c) are the same.

CP3.7 Consider the following system: $\dot{x} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u,\ y = [1 \quad 0] x,\ x(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$.

Using the lsim function obtain and plot the system response (for x1(t) and x2(t)) when u(t) = 0.

```
close all; clear;

a = [0 1; -2 -3]; b = [0; 1];
c = [1 0]; d = [0];
sys_ss = ss(a, b, c, d);

x0 = [1; 0]; % initial condition
t = [0:0.01:10];
u = 0*t; % zero input

[y,T,x]=lsim(sys_ss,u,t,x0);
figure;
subplot(2,1,1); plot(T,x(:,1));
xlabel('Time (s)'); ylabel('x_1');
subplot(2,1,2); plot(T,x(:,2));
```

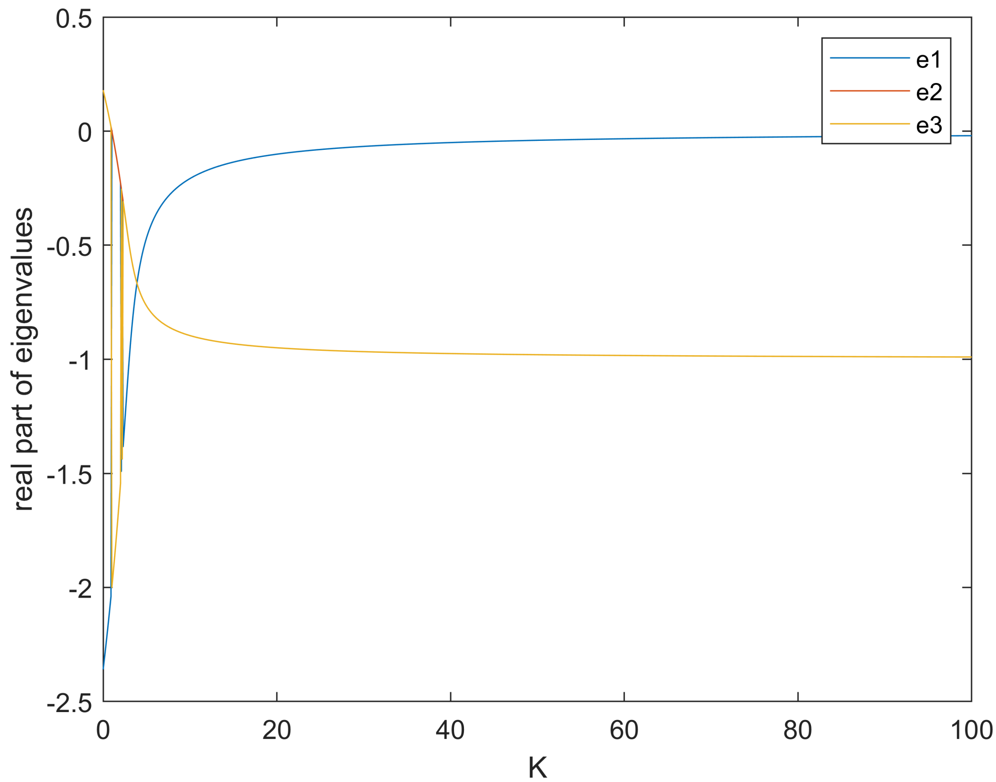


CP3.8 Consider the state variable model with parameter K given by

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & -K & -2 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u, \, y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} x$$

Plot the characteristic values of the system as a function of K in the range $0 \le K \le 100$. Determine that range of K for which all the characteristic values lie in the left half-plane.

```
close all; clear;

dk = 0.1; K_fv = 100;
K = [0: dk: K_fv];
eigenA_real = zeros(3, (dk^-1)*K_fv+1);

i = 1;
for j = 0: dk: K_fv
    a = [0 1 0; 0 0 1; -2 -j -2];
    eigenA = eig(a);
    eigenA_real(:,i) = real(eigenA);
    i = i + 1;
end

figure;
plot(K, eigenA_real(1,:)); hold on;
plot(K, eigenA_real(2,:));
plot(K, eigenA_real(3,:)); hold off;
xlabel('K'); ylabel('real part of eigenvalues');
legend(["e1"], ["e2"], ["e3"]);
```



We can find that all the characteristic values lie in the left half-plane in the range $1 \le K \le 100$.