

1. Simulate spring-mass-damper system of unforced response with damping ratio equal to 0.3, 0.6, and 1 (see Fig. 2.46) (p.108)

Ans.

$$\ddot{y} + \frac{b}{M}\dot{y} + \frac{k}{M}y = -\frac{r(t)}{M_s}$$

$$\text{Condition : } \frac{b}{M} = 2\xi\omega_n, \frac{k}{M} = \omega_n^2 = 2\dot{y}(0) = 2, y(0) = -1$$

$$\Rightarrow (s^2Y(s) - sy(0) - \dot{y}(0)) + \frac{b}{M}(sY(s) - y(0)) + \frac{k}{M}Y(s) = -\frac{P}{s}$$

$$\Rightarrow Y(s) = \frac{-s^2 + \left(2 - \frac{b}{M}\right)s - P}{s\left(s^2 + \frac{b}{M}s + \frac{k}{M}\right)}$$

$$\Rightarrow y(t) = \frac{y(0)}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin(\omega_n \sqrt{1-\zeta^2} t + \theta), \theta = \cos^{-1}\zeta$$

```
close all; clear;
```

```
% Damping ratio is (b/M)/(2wn), wn = sqrt(k/M) = sqrt(2)
% the magnitude of the step response is P
```

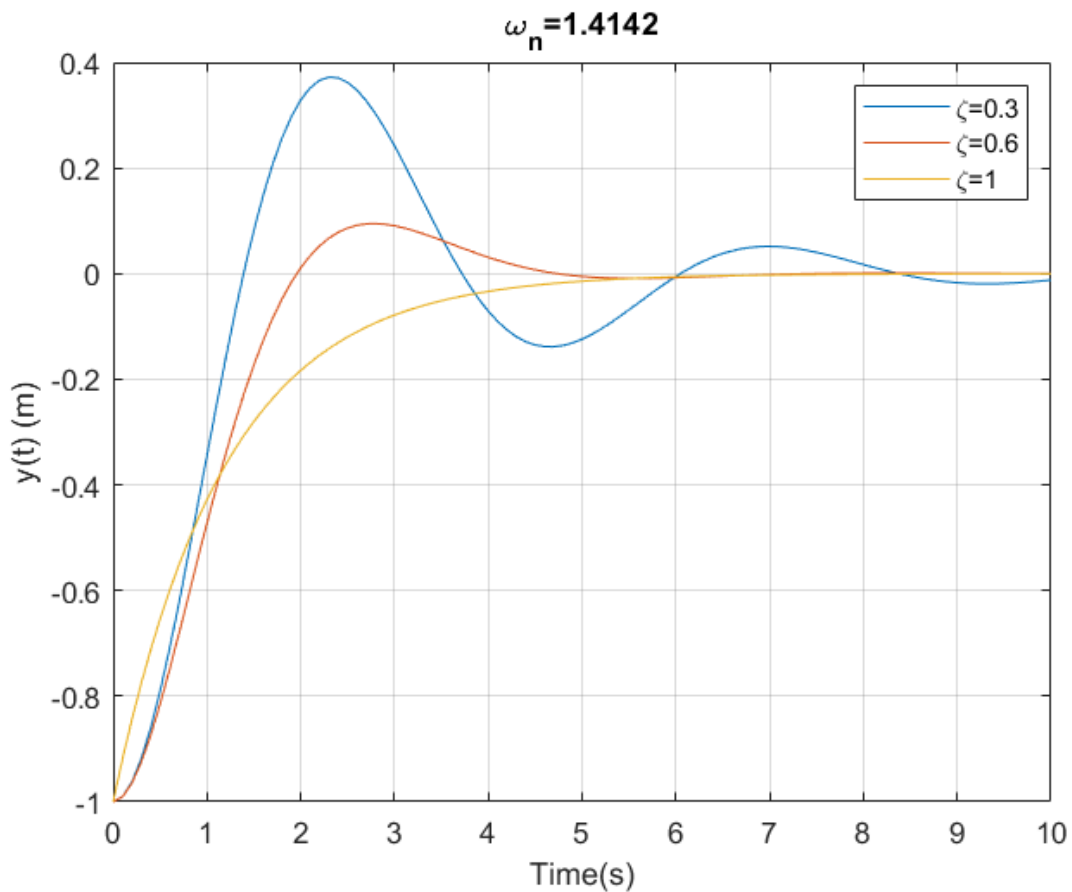
by equation of output signal in time domain

```
y0 = -1;
wn = sqrt(2);
zeta = [0.3 0.6 1];
t = [0:0.1:10];
y = zeros(3,length(t));

%Compute Unforced Response to an Initial Condition
for i=1:length(zeta)-1
    c = (y0/sqrt(1-zeta(i)^2));
    for j=1:length(t)
        y(i, j) = c*exp(-zeta(i)*wn*t(j)).*sin(wn*sqrt(1-zeta(i)^2)*t(j)+acos(zeta(i)));
    end
end

for j=1:length(t)
    y(length(zeta), j) = c*exp(-zeta(i)*wn*t(j)).*sin(wn*0*t(j)+acos(zeta(i)));
end

figure; plot(t,y); grid;
xlabel('Time(s)'), ylabel('y(t) (m)');
title(['\omega_n=', num2str(wn)]);
legend(['\zeta=', num2str(zeta(1))], ['\zeta=', num2str(zeta(2))], ['\zeta=', num2str(zeta(3))]);
```



2. Simulate Example 2.19 (Series connection) (p.120)

$$G(s) = \frac{1}{500s^2}, G_c(s) = \frac{s+1}{s+2}$$

$$\text{sys1} = G_c(s)G(s)$$

$$\text{sys2} = \frac{G_c(s)G(s)}{1 + G_c(s)G(s)}$$

```
close all; clear;
numg = [1]; deng = [500 0 0]; sysg = tf(numg,deng);
numgc = [1 1]; dengc = [1 2]; sysgc = tf(numgc,dengc);
sys1 = series(sysg,sysgc)
```

sys1 =

$$\frac{s + 1}{500 s^3 + 1000 s^2}$$

Continuous-time transfer function.

```
sys2_num = series(sysg,sysgc);
```

```
sys2 = feedback(sys2_num,[1])
```

sys2 =

$$\frac{s + 1}{500 s^3 + 1000 s^2 + s + 1}$$

Continuous-time transfer function.

3. Simulate Example 2.20 (The feedback function with unity feedback) (p.123)

$$G(s) = \frac{1}{500s^2}, G_c(s) = \frac{s+1}{s+2}$$

$$T_1(s) = \frac{G_c(s)G(s)}{1 + G_c(s)G(s)}, T_2(s) = \frac{G(s)}{1 \mp G_c(s)G(s)}$$

$$T_{2-1}(s) = \frac{G(s)}{1 + G_c(s)G(s)}, T_{2-2}(s) = \frac{G(s)}{1 - G_c(s)G(s)}$$

```
close all; clear;
numg = [1]; deng = [500 0 0]; sysg = tf(numg,deng);
numgc = [1 1]; dengc = [1 2]; sysgc = tf(numgc,dengc);
T1_num = series(sysg,sysgc);
T1 = feedback(T1_num,[1],-1)
```

T1 =

$$\frac{s + 1}{500 s^3 + 1000 s^2 + s + 1}$$

Continuous-time transfer function.

```
T2_1 = feedback(sysg,sysgc,-1)
```

T2_1 =

$$\frac{s + 2}{500 s^3 + 1000 s^2 + s + 1}$$

Continuous-time transfer function.

```
T2_2 = feedback(sysg,sysgc,+1)
```

T2_2 =

$$\frac{s + 2}{500 s^3 + 1000 s^2 - s - 1}$$

Continuous-time transfer function.