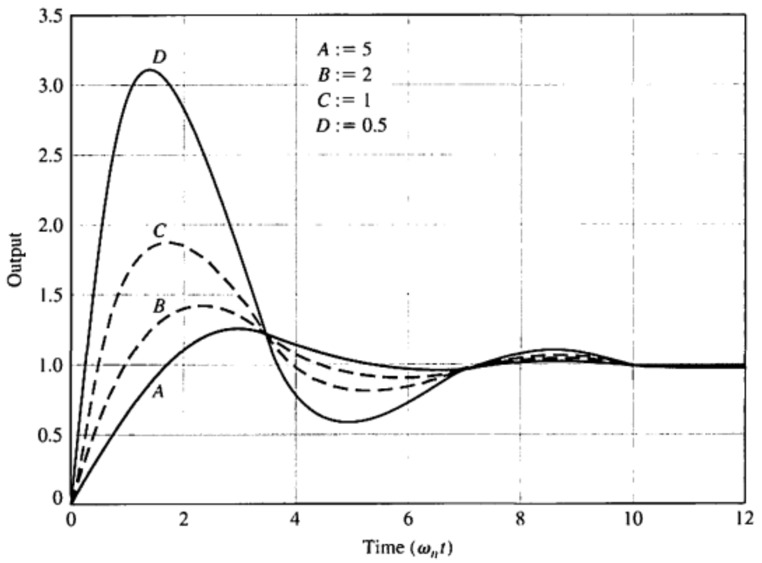
(E5.4) A feedback system with negative unity feedback has a loop transfer function

$$L(s) = G_c(s)G(s) = \frac{2(s+8)}{s(s+4)}$$

(a) Determine the closed-loop transfer function T(s) = Y(s)/R(s).
(b) Find the time response, y(t), for a step input r(t) = A for t > 0.
(c) Using Figure 5.13(a), determine the overshoot of the response.
(d) Using the final-value theorem, determine the steady-state value of y(t).



(b)

FIGURE 5.13 (a) Percent overshoot as a function of ζ and ω_n when a second-order transfer function contains a zero. Redrawn with permission from R. N. Clark, *Introduction to Automatic Control Systems* (New York: Wiley, 1962). (b) The response for the second-order transfer function with a zero for four values of the ratio $a/\zeta\omega_n$: $A = 5, B = 2, C = 1$, and $D = 0.5$ when $\zeta = 0.45$.

Ans.

(a) $T(s) = \frac{L(s)}{1+L(s)} = \frac{2(s+8)}{s^2+6s+16}$.

(b) $\frac{s-a}{(s-a)^2+b^2} \leftrightarrow e^{at}\cos(bt)\,,\ \frac{b}{(s-a)^2+b^2} \leftrightarrow e^{at}\sin(bt)$

Step input $R(s) = \frac{A}{s},\ Y(s) = \frac{2(s+8)}{s^2+6s+16}\frac{A}{s} = A\left(\frac{1}{s} - \frac{s+4}{s^2+6s+16}\right)$

$$\Rightarrow y(t) = A\left[1 - \left(e^{-3t}\cos\sqrt{7}t + \frac{1}{\sqrt{7}}e^{-3t}\sin\sqrt{7}t\right)\right]$$

$$= A\left[1 - 1.07e^{-3t}\sin\left(\sqrt{7}t + 1.21\right)\right]$$

(c) $T(s) = \frac{2(s+8)}{s^2+6s+16} = \frac{\cdots}{s^2+2\zeta\omega_n+\omega_n^2} \rightarrow \zeta = 0.75$ and $\omega_n = 4$.

$P.O. = 100e^{-\zeta\pi/\sqrt{1-\zeta^2}} = 2.83\%$

(d) This is a type 1 system, thus the steady-state error is zero and y(t) $\rightarrow$A as t $\rightarrow\infty$.

$$e_{ss} = \lim_{s\to 0} s[R(s) - Y(s)] = \lim_{s\to 0} A\left[1 - \frac{2(s+8)}{s^2+6s+16}\right] = 0$$

(E5.6) Consider the block diagram shown in Figure E5.6.

(a) Calculate the steady-state error for a ramp input.

(b) Select a value of K that will result in zero overshoot to a step input. Provide the most rapid response that is attainable.

Plot the poles and zeros of this system and discuss the dominance of the complex poles. What overshoot for a step input do you expect?

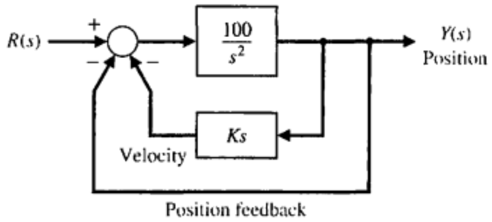


FIGURE E5.6 Block diagram with position and velocity feedback.

Ans.

(a) Ramp input $R(s) = \frac{A}{s^2}$, $T(s) = \frac{Y(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)} = \frac{100}{s^2+100Ks+100}$, where

$G(s) = \frac{100}{s^2}$ and $H(s) = 1 + sK$

- The steady-state error is

$$e_{ss} = \lim_{s\to 0} s[R(s) - Y(s)] = \lim_{s\to 0} s\left[\frac{A}{s^2} - T(s)\frac{A}{s^2}\right] = \lim_{s\to 0} \frac{A}{s}\left[\frac{s^2+100Ks}{s^2+100Ks+100}\right] = KA$$

(b) Natural frequency $\omega_n = 10$, damping ratio $\zeta = 5K$

ps. Overdamped $\zeta > 1$, Underdamped $\zeta < 1$, critically damping $\zeta = 1$

critically damping $\zeta = 1 = 5K \to K = 0.2$

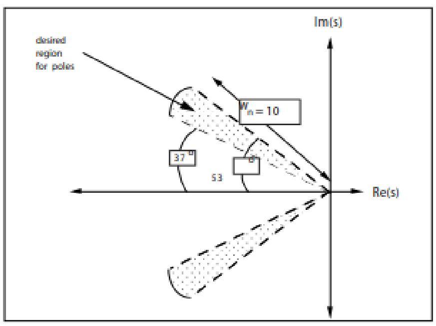
$\Rightarrow P.O. = 100e^{-5K\pi/\sqrt{1-(5K)^2}}$ for $0 < K < 0.2$

$P.O. = 0$ for $K > 0.2$

(E5.17)(a) A closed-loop control system transfer function T(s) has two dominant complex conjugate poles. Sketch the region in the left-hand 5-plane where the complex poles should be located to meet the given specifications.

$0.6 \le \zeta \le 0.8, \omega_n \le 10.$

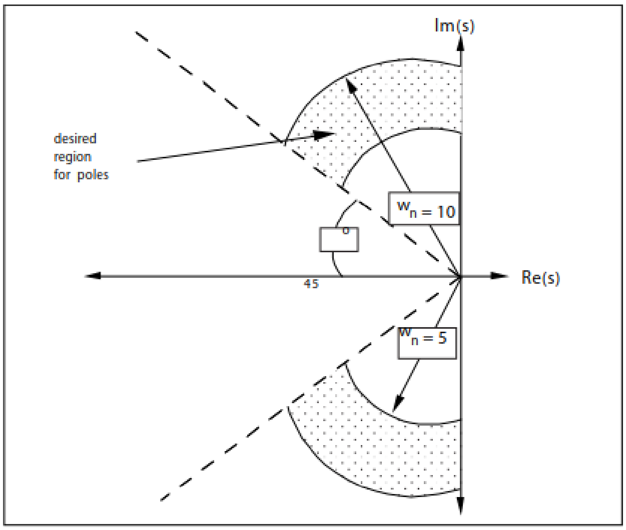
Ans. $s_{1,2} = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$



(a) 0.6 < z < 0.8 and w_n < 10

(E5.17)(d) A closed-loop control system transfer function T(s) has two dominant complex conjugate poles. Sketch the region in the left-hand 5-plane where the complex poles should be located to meet the given specifications.
$\zeta \leq 0.707, 5 \leq \omega_n \leq 10.$

Ans. $s_{1,2} = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$



(d) 0.707 > z and 5 < w_n <10

(E5.20) Consider the closed-loop system in Figure E5.19, where $G_C(s)G(s) = \frac{s+1}{s^2+0.3s}$, and $H(s) = K_a$.

(a) Determine the closed-loop transfer function T(s) = Y(s)/R(s).
(b) Determine the steady-state error of the closed-loop system response to a unit ramp input, $R(s) = 1/s^2$.
(c) Select a value for Ka so that the steady-state error of the system response to a unit step input, R(s) = 1/s, is zero.

Ans.

(a) $T(s) = \frac{Y(s)}{R(s)} = \frac{G_C(s)G(s)}{1+G_C(s)G(s)H(s)} = \frac{s+1}{s^2+0.3s+K_as+K_a}$

(b) $e_{ss} = \lim_{s\to 0} s[R(s)-Y(s)] = \lim_{s\to 0} \frac{1}{s}\left[1 - \frac{s+1}{s^2+0.3s+K_as+K_a}\right] = \infty$

(c) $e_{ss} = \lim_{s\to 0} s[R(s)-Y(s)] = \lim_{s\to 0} \left[1 - \frac{s+1}{s^2+0.3s+K_as+K_a}\right] = K_a - 1$

$\Rightarrow e_{ss} = 0 \text{ for } K_a = 1$

(P5.20) A system is shown in Figure P5.20.

(a) Determine the steady-state error for a unit step input in terms of K and K1, where E(s) = R(s) - Y(s).

(b) Select K1 so that the steady-state error is zero.

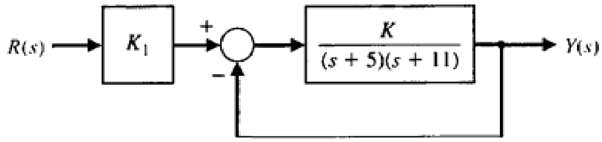


FIGURE P5.20 System with pregain, K_1.

Ans.

(a) $T(s) = \frac{Y(s)}{R(s)} = \frac{G(s)K_1}{G(s)+1}, G(s) = \frac{K}{(s+5)(s+11)}, R(s) = \frac{1}{s}$

$$e_{SS} = \lim_{s\to 0} s[R(s) - Y(s)] = \lim_{s\to 0}\left[1 - \frac{KK_1}{s^2+16s+55+K}\right] = \frac{55+K-KK_1}{K+55}$$

(b) To achieve zero steady-state error, $K_1 = K + \frac{55}{K}$

***** Important Equation in Chapter 5 *****

1. Second Order System
   - I. $P.O. = 100e^{-\zeta\pi/\sqrt{1-\zeta^2}}$
   - II. Overshoot $= M_{pt} - 1 = e^{-\zeta\pi/\sqrt{1-\zeta^2}}$
   - III. Peak time $T_P = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}$
   - IV. Settling time $T_S = 4\tau = \frac{4}{\zeta\omega_n}$
   - V. Poles $s_{1,2} = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$
2. Standard Test Input Signals
   - I. Step Signals: $\frac{A}{s}$
   - II. Ramp Signals: $\frac{A}{s^2}$
   - III. Parabolic Signals: $\frac{2A}{s^3}$