(E4.3) A robotic arm and camera could be used to pick fruit, as shown in Figure E4.3(a). The camera is used to close the feedback loop to a microcomputer, which controls the arm [8,9]. The transfer function for the process is

$$G(s) = \frac{K}{(s+5)^2}$$

(a) Calculate the expected steady-state error of the gripper for a step command A as a function of K.

(b) Name a possible disturbance signal for this system.

Ans.

(a) The tracking error, E(s) = R(s) – Y (s), is given by

$$\big(R(s) - Y(s)\big)G(s) = Y(s) \Rightarrow \frac{Y(s)}{R(s)} = \frac{G(s)}{1+G(s)}$$

$$\Rightarrow E(s) = R(s) - Y(s) = \frac{R(s)}{1+G(s)} = \frac{A/s}{1+K/(s+5)^2}$$

The steady-state error (computed using the final value theorem) is

$$e_{ss} = \lim_{s\to\infty} sE(s) = \frac{A}{1+K/25}$$

(b) A disturbance would be the wind shaking the robot arm.

(E4.9) Submersibles with clear plastic hulls have the potential to revolutionize underwater leisure. One small submersible vehicle has a depth-control system as illustrated in Figure E4.9.

(a) Determine the closed-loop transfer function T(s) = Y(s)/R(s).

(b) Determine the sensitivity $S^T_{K_1}$ and S^T_K.

(c) Determine the steady-state error due to a disturbance $T_d(s) = 1/s$.

(d) Calculate the response y(t) for a step input R(s) = 1/s when K = K2 = l and l < K1 < 10. Select K1 for the fastest response.

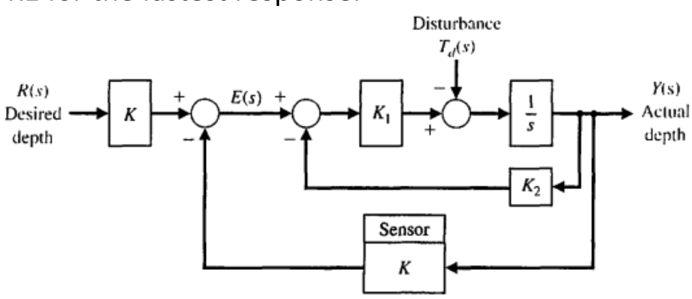

FIGURE E4.9 Depth control system.

Ans.

(a) $\frac{Y(s)}{E(s)} = \frac{(1/s)K_1}{1+(1/s)K_1K_2} = T_1(s), \frac{Y(s)}{R(s)} = T(s) = \frac{KT_1(s)}{1+KT_1(s)} = \frac{KK_1}{s+K_1(K+K_2)}$

(b) $S^T_{K_1} = \frac{\partial T/T}{\partial K_1/K_1} = \frac{\partial \ln T}{\partial \ln K_1} = \left(\frac{K}{s+K_1(K+K_2)} + \frac{-(KK_1)\times(K+K_2)}{(s+K_1(K+K_2))^2}\right)\left(\frac{K_1}{T}\right) =$

$$\left(\frac{sK}{(s+K_1(K+K_2))^2}\right)\left(\frac{s+K_1(K+K_2)}{K}\right)=\frac{s}{s+K_1(K+K_2)}$$

$$S_K^T=\frac{\frac{\partial T}{T}}{\frac{\partial K}{K}}=\left(\frac{K_1}{s+K_1(K+K_2)}+\frac{-(KK_1)\times(K_1)}{(s+K_1(K+K_2))^2}\right)\left(\frac{K}{T}\right)=\frac{sK_1+K_1^2K_2}{(s+K_1(K+K_2))^2}\times\frac{s+K_1(K+K_2)}{K_1}$$

$$=\frac{s+K_1K_2}{s+K_1(K+K_2)}$$

(c) $\left(K_1\left(E(s)-K_2Y(s)\right)-T_d(s)\right)\frac{1}{s}=Y(s), E(s)=-KY(s)\ (\text{when } R(s)=0)$

$$\Rightarrow\frac{1}{s}\left(K_1\left(-KY(s)-K_2Y(s)\right)-T_d(s)\right)=Y(s)$$

$$\Rightarrow\frac{Y(s)}{T_d(s)}=\frac{-1}{s+K_1(K+K_2)}, E(s)=-KY(s)=\frac{K}{s+K_1(K+K_2)}T_d(s), T_d(s)=\frac{1}{s}$$

$$\Rightarrow e_{ss}=\lim_{s\to0}sE(s)=\frac{K}{K_1(K+K_2)}$$

(d) $Y(s)=\frac{KK_1}{s+K_1(K+K_2)}R(s)=\frac{1}{s}\frac{K_1}{s+2K_1}=\frac{0.5}{s}+\frac{-0.5}{s+2K_1}$

$$\Rightarrow y(t)=\frac{1}{2}[1-e^{-2K_1t}]u(t)$$

where u(t) is the unit step function. Therefore, select K1 = 10 for the fastest response.

(P4.4) A control system has two forward paths, as shown in Figure P4.4.

(a) Determine the overall transfer function T(s) = Y(s)/R(s).

(b) Calculate the sensitivity, S_G^T, using Equation (4.16).

(c) Does the sensitivity depend on U(s) or M(s)?

$$T(s,\alpha)=\frac{N(s,\alpha)}{D(s,\alpha)}, S_\alpha^T=\frac{\partial\ln T}{\partial\ln\alpha}=\frac{\partial\ln N}{\partial\ln\alpha}\Big|_{\alpha_0}-\frac{\partial\ln D}{\partial\ln\alpha}\Big|_{\alpha_0}=S_\alpha^N-S_\alpha^D\ (4.16)$$

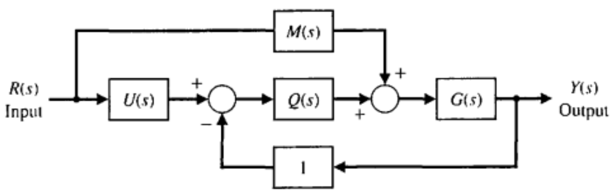

FIGURE P4.4
Two-path system.

Ans.

(a) $\left((RU-Y)Q+MR\right)G=Y\rightarrow T(s)=\frac{Y(s)}{R(s)}=\frac{G(s)U(s)Q(s)+G(s)M(s)}{1+G(s)Q(s)}$

(b) $S_G^T=S_G^N-S_G^D=\left(\frac{Q(s)G(s)}{1+Q(s)G(s)}\right)-(1)=\frac{1}{1+Q(s)G(s)}$

(c) The sensitivity does not depend upon U(s) or M(s).

(P4.6) An automatic speed control system will be necessary for passenger cars traveling on the automatic highways of the future. A model of a feedback speed control system for a standard vehicle is shown in Figure P4.6. The load disturbance due to a percent grade $\Delta T_d(s)$ is also shown. The engine gain Kt. varies within the range of 10 to 1000 for various models of automobiles. The engine time constant τ_e is 20 seconds

(a) Determine the sensitivity of the system to changes in the engine gain Ke.
(b) Determine the effect of the load torque on the speed.
(c) Determine the constant percent grade $\Delta T_d(s) = \Delta d/s$ for which the vehicle stalls (velocity V(s) = 0) in terms of the gain factors. Note that since the grade is constant, the steady-state solution is sufficient. Assume that R(s) = 30/s km/hr and that $K_e K_1$ » 1. When K_g/K_1 = 2, what percent grade Δd would cause the automobile to stall?

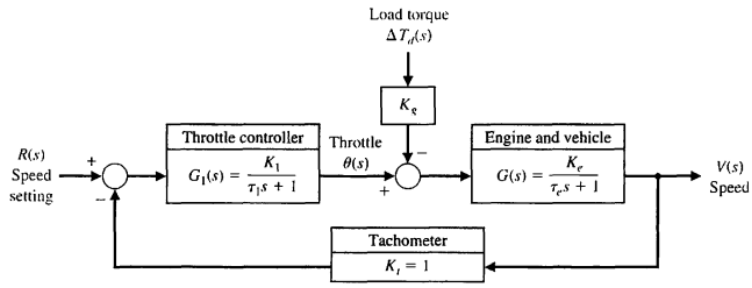

FIGURE P4.6 Automobile speed control.

Ans.

(a) $\frac{V(s)}{R(s)} = \frac{G_1(s)G(s)}{1+K_t(t)G_1(s)G(s)} = \frac{G_1(s)G(s)}{1+G_1(s)G(s)}, G_1(s) = \frac{K_1}{\tau_1 s+1}, G(s) = \frac{K_e}{\tau_e s+1}$

$$S_{K_e}^{T} = \frac{\ln T}{\ln K_e} = S_{K_e}^{N} - S_{K_e}^{D} = 1 - \frac{K_1 K_e}{(\tau_1 s+1)(\tau_e s+1)+K_1 K_e} = \frac{(\tau_1 s+1)(\tau_e s+1)}{(\tau_1 s+1)(\tau_e s+1)+K_1 K_e}$$

(b) $R(s) = 0, (-VG_1 - \Delta T_d K_g)G = V \rightarrow \frac{V(s)}{\Delta T_d(s)} = \frac{-K_g G(s)}{1+G_1(s)G(s)}$

(c) $R(s) = \frac{30}{s}, K_e K_1 \gg 1, \frac{K_g}{K_1} = 2, \Delta T_d(s) = \frac{\Delta d}{s}$. When the car stalls, $V(s) = 0$

$$V(s) = \left(\frac{-K_g G(s)}{1+G_1(s)G(s)}\right)\Delta T_d(s) + \left(\frac{G_1(s)G(s)}{1+G_1(s)G(s)}\right)R(s)$$

- Using value theorem

$$v_{ss} = \lim_{s\to 0} s\left(\left(\frac{K_1 K_e}{(\tau_1 s+1)(\tau_e s+1)+K_1 K_e}\frac{-(\tau_1 s+1)K_g}{K_1}\frac{\Delta d}{s}\right) + \left(\frac{K_1 K_e}{(\tau_1 s+1)(\tau_e s+1)+K_1 K_e}\frac{30}{s}\right)\right)$$

$$= \frac{-K_1 K_e K_g}{K_1 K_1 K_e}\Delta d + \frac{30 K_1 K_e}{K_1 K_e} = -\left(\frac{K_g}{K_1}\right)\Delta d + 30$$

- When $v_{ss} = 0, \Delta d = \frac{30 K_1}{K_g} = 15$

(P4.13) One form of a closed-loop transfer function is

$$T(s) = \frac{G_1(s) + kG_2(s)}{G_3(s) + kG_4(s)}$$

(a) Use Equation (4.16) to show that $S_k^T = \frac{k(G_2G_3 - G_1G_4)}{(G_3+kG_4)(G_1+kG_2)}$.

(b) Determine the sensitivity of the system shown in Figure P4.13, using the equation verified in part (a).

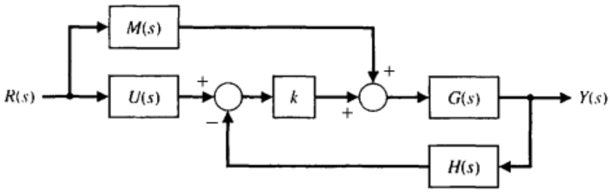

FIGURE P4.13 Closed-loop system.

Ans.

(a) $S_k^T = S_k^N - S_k^D = \left(\frac{kG_2(s)}{G_1(s)+kG_2(s)}\right) - \left(\frac{kG_4(s)}{G_3(s)+kG_4(s)}\right) = \frac{k(G_2(s)G_3(s) - G_1(s)G_4(s))}{(G_3(s)+kG_4(s))(G_1(s)+kG_2(s))}$

(b) $\left((RU - HY)k + RM\right)G = Y \rightarrow T(s) = \frac{Y(s)}{R(s)} = \frac{kG(s)U(s)+M(s)G(s)}{1+kH(s)G(s)} = \frac{G_1(s)+kG_2(s)}{G_3(s)+kG_4(s)}$

$S_k^T = \frac{k(G_2(s)G_3(s) - G_1(s)G_4(s))}{(G_3(s)+kG_4(s))(G_1(s)+kG_2(s))}, G_1(s) = M(s)G(s), G_2(s) = G(s)U(s),$

$G_3(s) = 1, G_4(s) = H(s)G(s)$