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(E3.5) A system is represented by a block diagram as shown in Figure E3.5. Write the state equations in the form of Equations (3.16) and (3.17).





Ans.

1

s(−fkX2(s) − dX2(s) + aX1(s) + fU(s)) = X2(s) ⇒ ẋ2 = −(fk + d)x2 + ax1 + fu

1

s(U(s) − kX2(s)) = X1(s) ⇒ ẋ1 = −kx2 + u

⚫ Therefore, ��̇ = ���� + ����, �� = ���� + ����

a −(fk + d)], �� = [1f], �� = [0 b], �� = [0], �� = [x1

�� = [0 −k

(E3.6) A system is represented by Equation (3.16), where �� = [0 1

0 0]

(a) Find the matrix ��(t)

(b) For the initial conditions x1(0) = x2(0) = l, find **x**(t). Ans.

x2], �� = [u],�� = [y]

(a) The state transition matrix is ��(t) = e��t = �� + ��t +12!��2t2 + ⋯ ��2 = [0 1

0 0][0 1

0 0] = 0,thus ��2 = ��3 = ��4 = ⋯ = 0

⇒ ��(t) = e��t = [1 0

0 1] + [0 1

0 0]t = [1 t

0 1]

(b) The state at any time t ≥ 0 is given by ��(t) = ��(t)��(0) Since x1(0) = x2(0) = 1

⇒ ��(t) = [1 + t

1], x1(t) = 1 + t, x2(t) = 1

1

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(E3.9) A multi-loop block diagram is shown in Figure E3.9. The state variables are denoted by x1 and x2.

(a) Determine a state variable representation of the closed-loop system where the output is denoted by y(t) and the input is r(t)

(b) Determine the characteristic equation.



Ans.

(a) X1(s) =1s(−X1(s) + R(s) +12X2(s)) ⇒ ẋ1 = −x1 +12x2 + r X2(s) =1s(− (−X1(s) + R(s) +12X2(s)) − X2(s)) ⇒ ẋ2 = x1 −32x2 − r Y(s) = −X2(s) − (−X1(s) + R(s) +12X2(s)) ⇒ y = x1 −32x2 − r ⚫ In state-variable form

1 −1.5] �� + [1−1] ��, �� = [1 −1.5]�� + [−1]��

��̇ = [−1 0.5

(b) The characteristic equation

det[−1 − s 0.5

1 −1.5 − s] = 0 → s2 +52s + 1 = (s +12)(s + 2) = 0

(E3.11) Determine a state variable representation for the system described by the transfer function

T(s) =Y(s)

R(s)=4(s + 3)

(s + 2)(s + 6)

Ans.

Y(s)

R(s)=4(s+3)

s2+8s+12=4s−1+12s−2 

(s+2)(s+6)=4s+12

1+8s−1+12s−2

X1(s) =1s(−12X1(s) + X2(s)) ⇒ ẋ1 = −12x1 + x2 X2(s) =1s(−8X2(s) + R(s)) ⇒ ẋ2 = −8x2 + r Y(s) = 12X1(s) + 4X2(s) ⇒ y = 12x1 + 4x2

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⚫ ��̇ = ���� + ����, �� = ����

⇒ �� = [−12 1

0 −8],�� = [01], �� = [12 4]

(E3.19) A single-input, single-output system has the matrix equations ��̇ = [0 1

−3 −4] �� + [01] ��, �� = [10 0]��

Determine the transfer function G(s) = Y(s)/U(s).

Ans.



U(s)=10s−2

G(s) =Y(s)

1+4s−1+3s−2 =10 s2+4s+3

(P3.12) A system is described by its transfer function T(s) =Y(s)

R(s)=8(s + 5)

s3 + 12s2 + 44s + 48

(a) Determine a state variable model.

(b) Determine ��(t), the state transition matrix. Ans.

(a) Y(s)

s3+12s2+44s+48=8s−2+40s−3

R(s)=8(s+5)

1+12−1+44s−2+48s−3

**3 1 -** 

**--**

��̇ = [

0 1 0 0 0 1 −48 −44 −12

] �� + [

0 0 1

] ��, �� = [40 8 0]��

(b) ��(t) = e��t = [Φ1(t) Φ2(t) Φ3(t)] 0 1 0

�� = [

0 0 1 −48 −44 −12

]��(s) = [s�� − ��]−1 ⇒ ��(t)

3