控制系統 (Feedback control system) Final Exam

* Upload 5 Matlab scripts: Final_1.m, Final_2.m, Final_3.m, Final_4.m, and Final_5.m to iLMS by 12 pm (中午12點), June 23, 2020. Late submission would result in discounted scores. Your final scores will be evaluated on the basis of these 5 files, so no other files are required to be submitted.

* Throughout this exam, we assume a unity feedback system with plant $G(s)$ and controller $G_c(s) = K\frac{s+z}{s+p}$ in the feedforward loop, where $K, z,$ and p are parameters to be determined. To get the full grade of each problem, use the minimum number of parameters. For example, if a gain K is sufficient to meet the specs, we set $z = p = 0$.

* What you can do: Use Internet to search for relevant information, such as the functionality of Matlab commands.

* What you CANNOT do: Ask for answers in any form, e.g., posting the question online and waiting for the answer, using social media to ask for or send answers to your classmates. If you were to provide answers to your classmates, you would fail your final.

* Abbreviations and symbols: P. M.=phase margin; P. O.=percent overshoot; position constant K_p; velocity constant K_v; settling time T_s.

1(a)

$G_c = K\frac{p}{z}\frac{(s+z)}{(s+p)}$ for Bode

$K_v = \lim_{s\to 0} s\,G_c(s)G(s) = 3.6K = 18$

$\Rightarrow K = 5$

$\boxed{1(b)}$ $G_c(s) = K\frac{s+z}{s+p}$

$\Rightarrow K_v = K\frac{z}{p}\times 3.6 = 18$

1. (30%)

$$G(s) = \frac{3.6}{s(1+0.9s)(1+0.04s)}$$

(a) Use a phase-lag compensator $G_c(s)$ to meet the specs: $77^\circ \geq \text{P.M.} \geq 75^\circ$ and $K_v = 18$.

(b) Use a two-stage phase-lead compensator $G_{c1}(s)G_{c2}(s)$ to meet the specs in (a).

2. (20%)

$G_c(s) = K\frac{p}{z}\frac{(s+z)}{(s+p)}$

$$G(s) = \frac{s+5}{s^3+2s^2+20s+3}$$

Design a one-stage phase-lead or phase-lag compensator $G_c(s)$ to meet the specs: $K_p = 30$, $50^\circ \geq \text{P.M.} \geq 48^\circ$, $\text{P.O.} \leq 14.5\%$.

lag

用 root locus

$K_p = \lim_{s\to 0} G_c(s)G(s) = 30$

$= K\frac{z}{p}\times\frac{5}{3} = K\alpha\frac{5}{3}$

3. (20%)

$$G(s) = \frac{3.6}{(s+0.14)(0.1s+1)}$$

Specs: $\text{P.O.} \leq 11\%$, $T_s \leq 1.5$ seconds.

root locus

$T_s = \frac{4}{\zeta\omega_n}$

4. (20%)

$$G(s)=\frac{1}{(s-3)(s-2)}$$

Lead + rlocus

(a) Design a one-stage phase-lead or phase-lag compensator $G_c(s)$ to meet the specs: $\mathrm{T_s} \le$ 1 seconds.

$G_c(s)=K\frac{(s+z)}{(s+p)}$

(b) From (a), add a prefilter $G_p(s)$ to meet the specs: $\mathrm{T_s} \le 1$ seconds, $\mathrm{P.O.} \le 5\%$.

$G_p(s)=\frac{s+z_1}{z_1}$

5. (10%)

$$G(s)=\frac{1}{(s+0.1)(s+4)}$$

rlocus

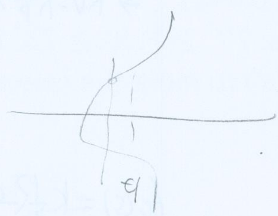
(a) meet the specs: steady-state error for a step input=0, $\mathrm{P.O.} \le 21\%$.

(b) From (a), add a prefilter $G_p(s)$ to meet the specs: steady-state error for a step input=0, $\mathrm{P.O.} \le 5\%$.

$K_s=\lim_{s\to 0} s\,G_c(s)G(s)=0$

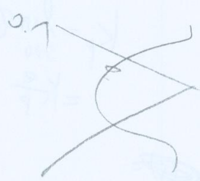
$G_c(s)=K\frac{s+z}{s+p}$ $\qquad G_p(s)=\frac{s+z}{z}$

$T_s=\frac{4}{\zeta\omega_n}$



$\frac{p}{s+p}$

$\frac{s+z}{z}$



s^2 acceleration acceleration

s step velocity

1 impulse position

Q1

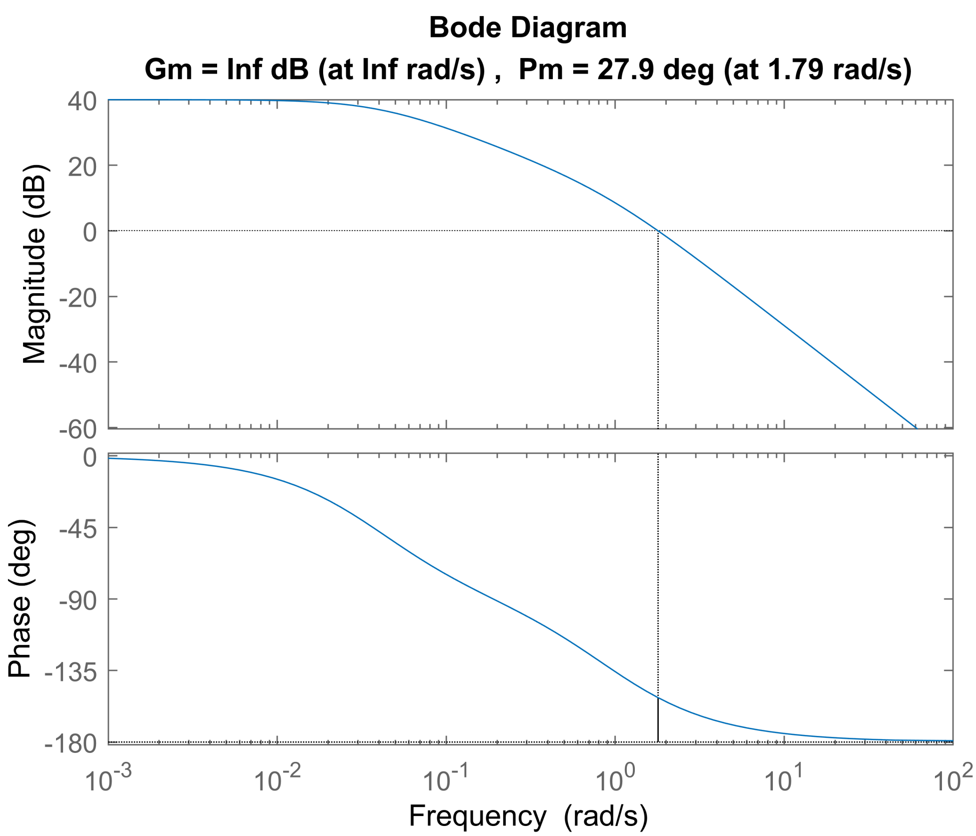
1(a)

$$\mathrm{Gc}(s) = K\frac{p}{z}\frac{s+z}{s+p}, K_v = \lim_{s\longrightarrow 0} \mathrm{sGc}(s)G(s) = 18$$

1(b)

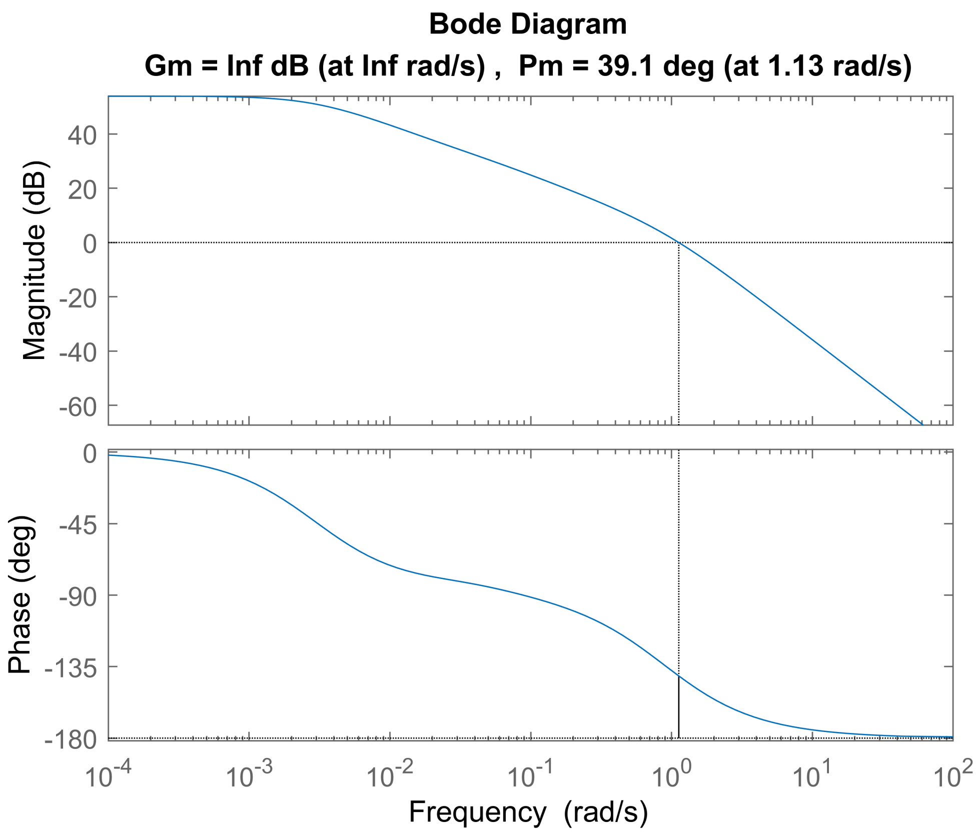
$$\mathrm{Gc}(s) = K\frac{s+z}{s+p}, K_v = \lim_{s\longrightarrow 0} \mathrm{sGc1}(s)\mathrm{Gc2}(s)G(s) = 18$$

```
clc;clear;close all;

clc;clear;close all;
a1 = [3.6];
b1 = conv(1, [1 0.9]); b1 = conv(b1, [1 0.04]);
G = tf(a1, b1);
% figure(1); rlocus(G);
figure(2); margin(G);
```

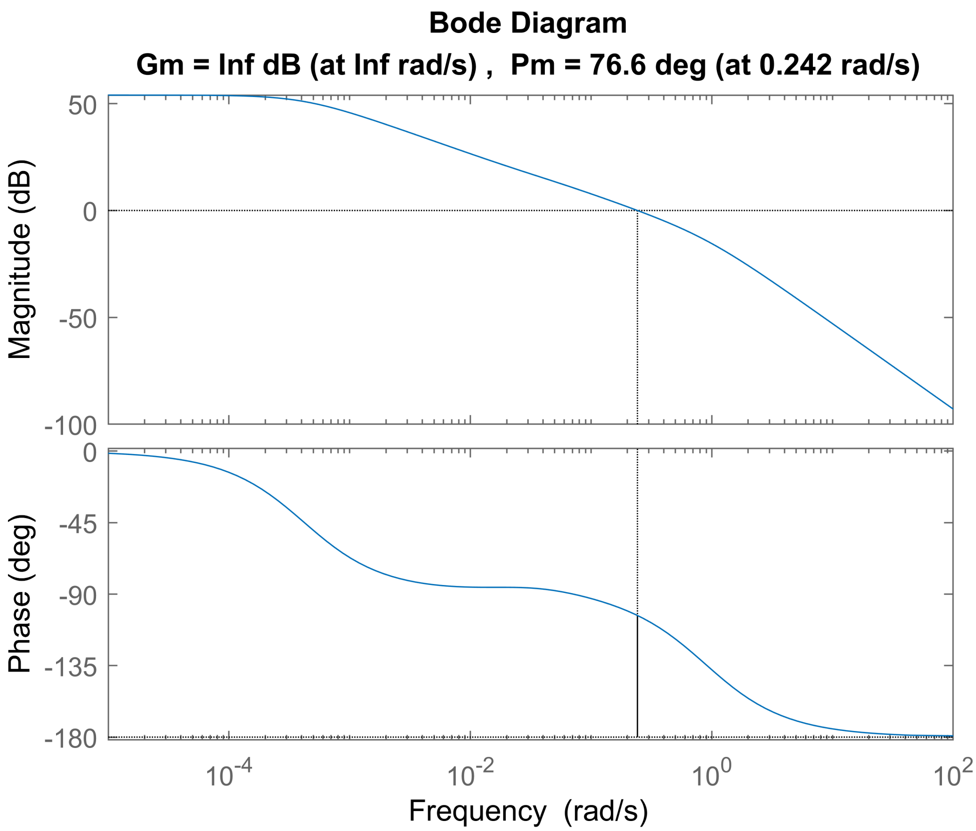



```
% 1a
K = 18/3.6; % make Kv = 18
omega_c = 0.334; % match PM
% omega_c: frequency when phase = PM-180+(small number)
attenuation_db = 20.9;
% attenuation_db: magnitude when frequency = omega_c
```

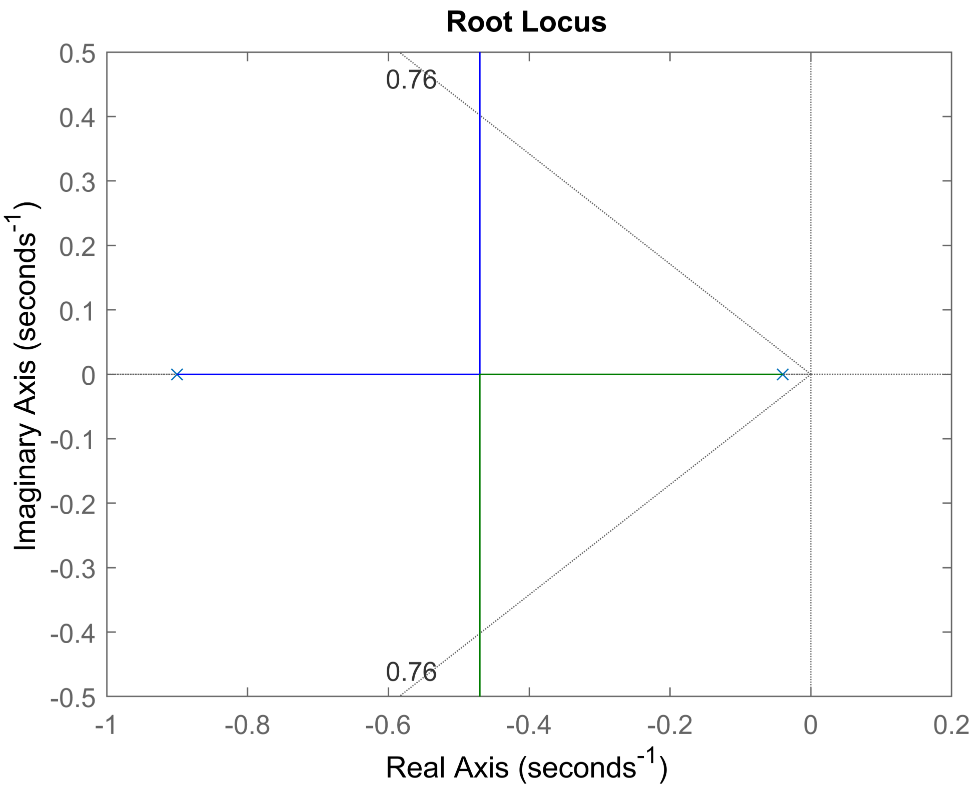
```
% fix omega_c then tune attenuation_db
Gca = Bode_lag(K,omega_c,attenuation_db);
La = Gca*G;
figure(3); margin(La);
```



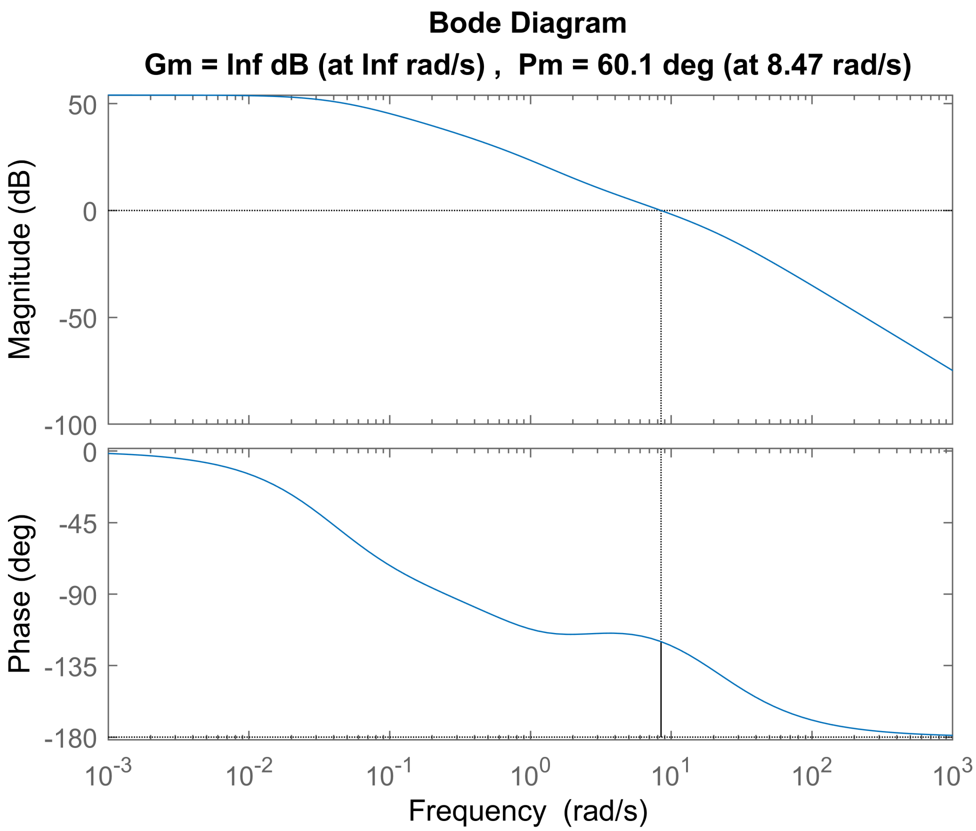

```
% 1a adjust
omega_c1 = 0.334;
attenuation_db1 = 38;
Gc1a = Bode_lag(K,omega_c1,attenuation_db1);
L1a=Gc1a*G;
figure(4); margin(L1a);
```



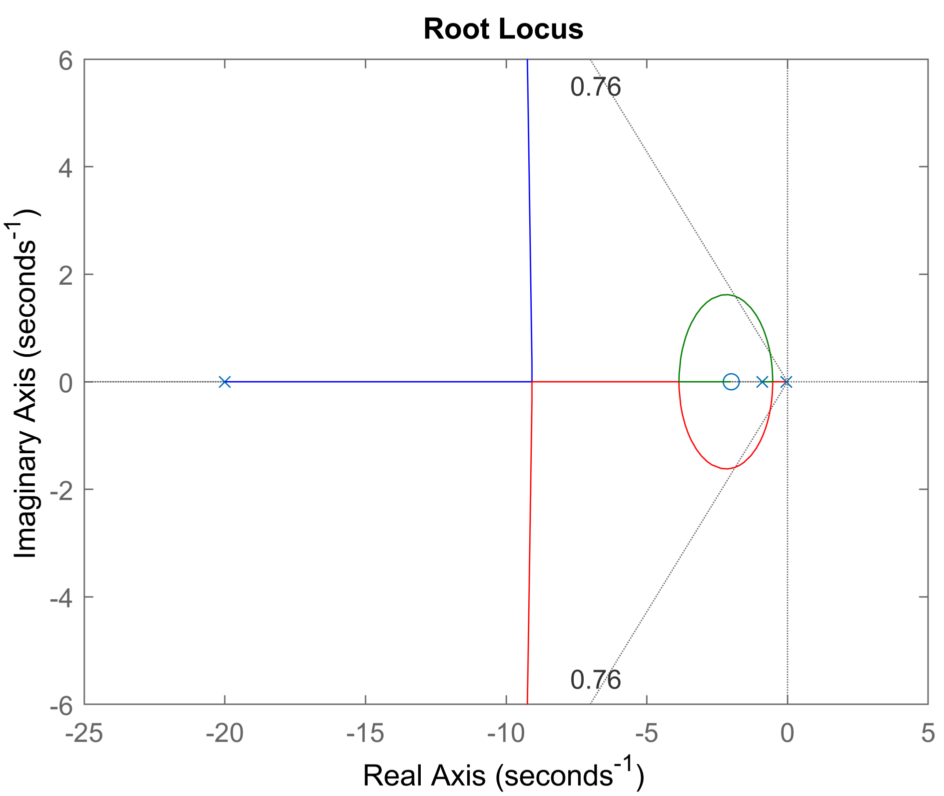

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% 1b
zeta = 0.01*76;
% zeta ~ 0.01*PM for second-order system
figure(5); rlocus(G); sgrid(zeta,10^4);
```




```
% K: gain s.t zeta <= 0.01*PM

% pull root locus left % Gc1
z1 = 2; p1 = 20; % make G as dominant poles
K1 = 18*(p1/z1)/3.6; % make Kv = 18
Gc1 = K1*tf([1 z1],[1 p1]);
L1b1 = G*Gc1;
figure(6); margin(L1b1);
```



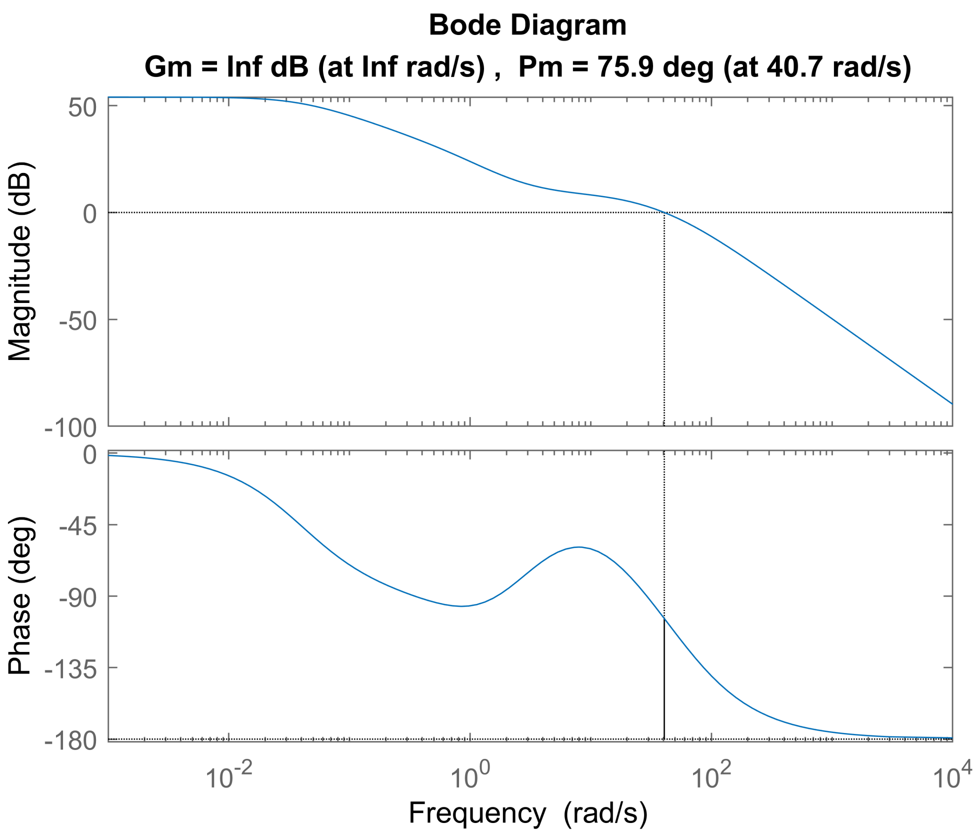

```
figure(7); rlocus(L1b1); sgrid(zeta,10^4);
```



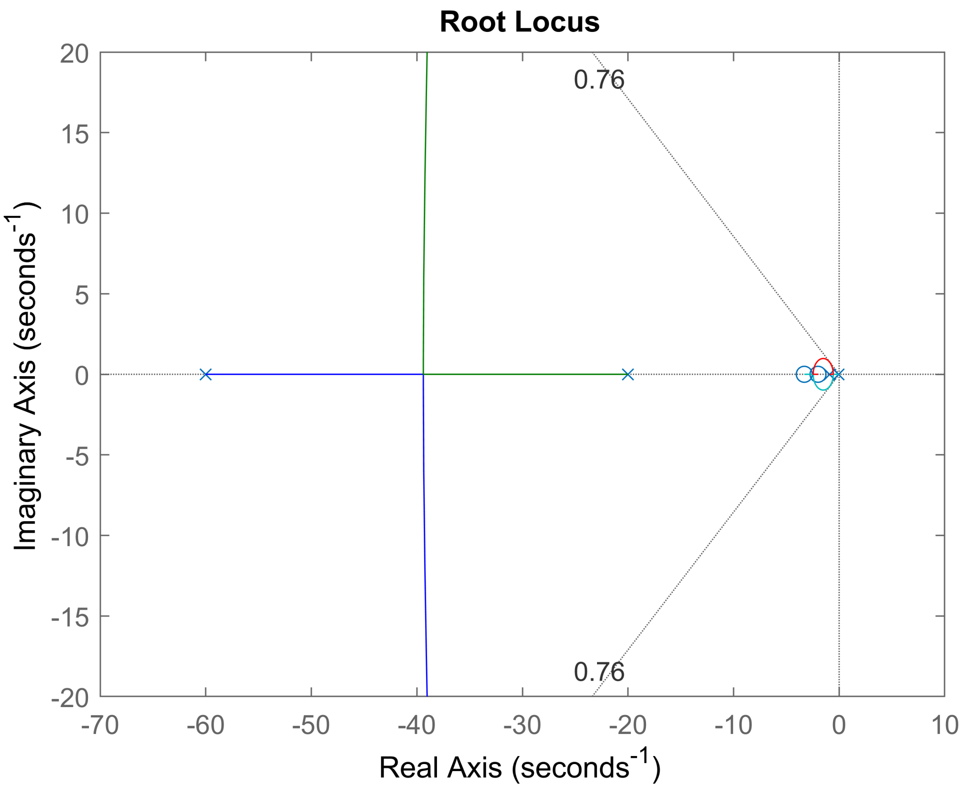
```
% match ess
[~,Pm1] = margin(L1b1);
disp(['phase margin = ' num2str(Pm1)]);
```

```
phase margin = 60.1009
```

```
% pull root locus left % Gc2
z2 = 3.3; p2 = 60; % make G as dominant poles
K2 = p2/z2; % make Kv = 18
Gc2 = K2*tf([1 z2],[1 p2]);
L1b = G*Gc1*Gc2;
figure(8); margin(L1b);
```




```
figure(9); rlocus(L1b); sgrid(zeta,10^4);
```




```
% match ess
[~,Pm1b] = margin(L1b);
disp(['phase margin = ' num2str(Pm1b)]);
```

```
phase margin = 75.908
```

Q2

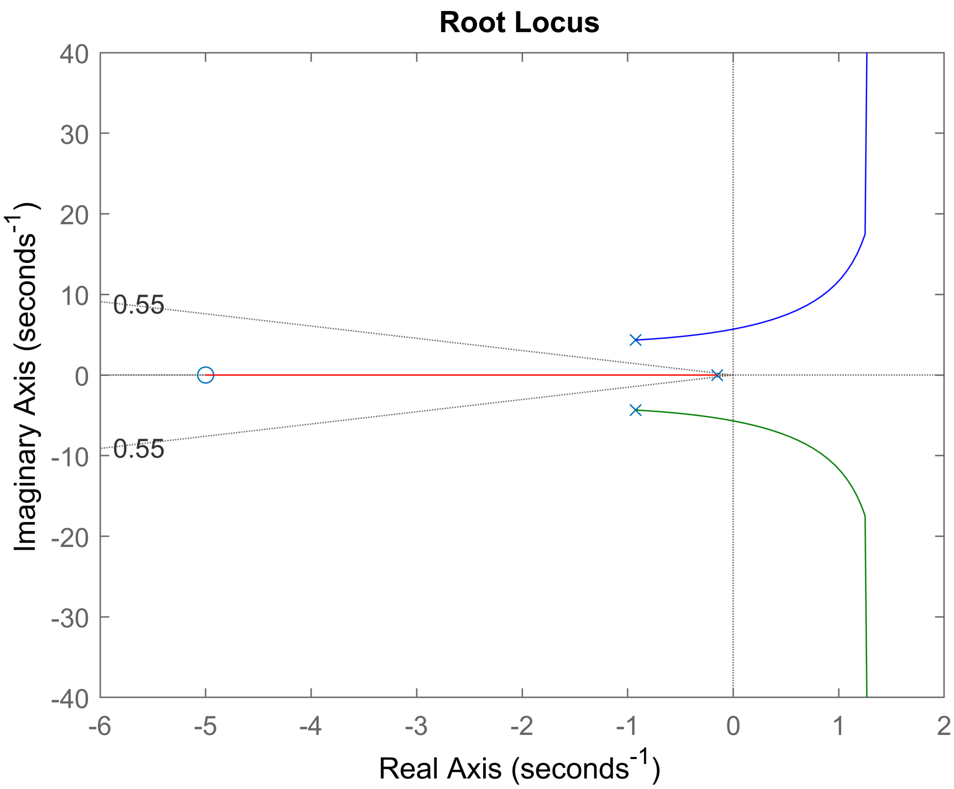
$$G(s) = \frac{s+5}{s^3 + 2s^2 + 20s + 3}$$

$$\mathrm{Gc}(s) = K\,\frac{p}{z}\frac{s+z}{s+p}, K_p = \lim_{s\longrightarrow 0} \mathrm{Gc}(s)G(s) = 30$$

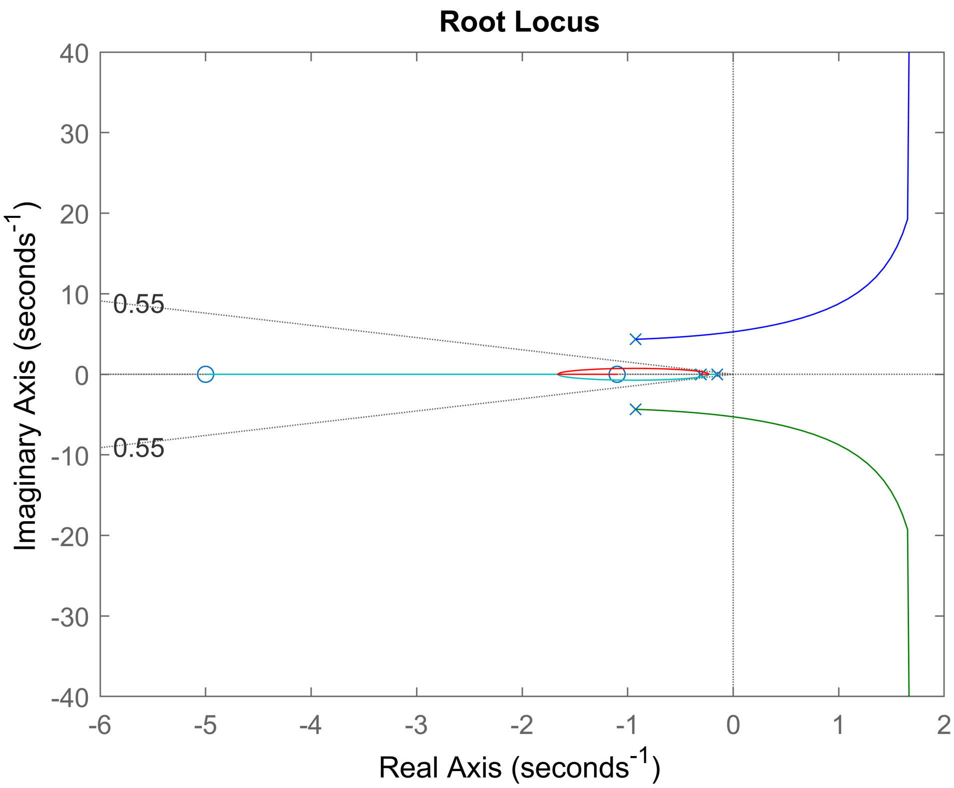
```
clc;clear;close all;

a = [1 5]; b = [1 2 20 3];
G = tf(a, b);

% Lag + rlocus
zeta = 0.55;
PO = 100*exp(-zeta*pi/(1-zeta^2)^0.5)
```

```
PO = 12.6324
```

```
figure(1); rlocus(G); sgrid(zeta,10^4);
```




```
K = 5;
my_alpha = 30*(3/5)*(1/K);  %  determined by the requirement on error constant
% K*(z/p)*(5/3) = 30, my_alpha = z/p
z = 1.1;
p = z/my_alpha;
Gc = tf([1 z],[1 p]);
L = G*Gc;
figure(2); rlocus(L); sgrid(zeta,10^4);
```




```
[~,Pm]=margin(K*L);
disp(['phase margin = ' num2str(Pm)]);
```

```
phase margin = 48.535
```

Q3

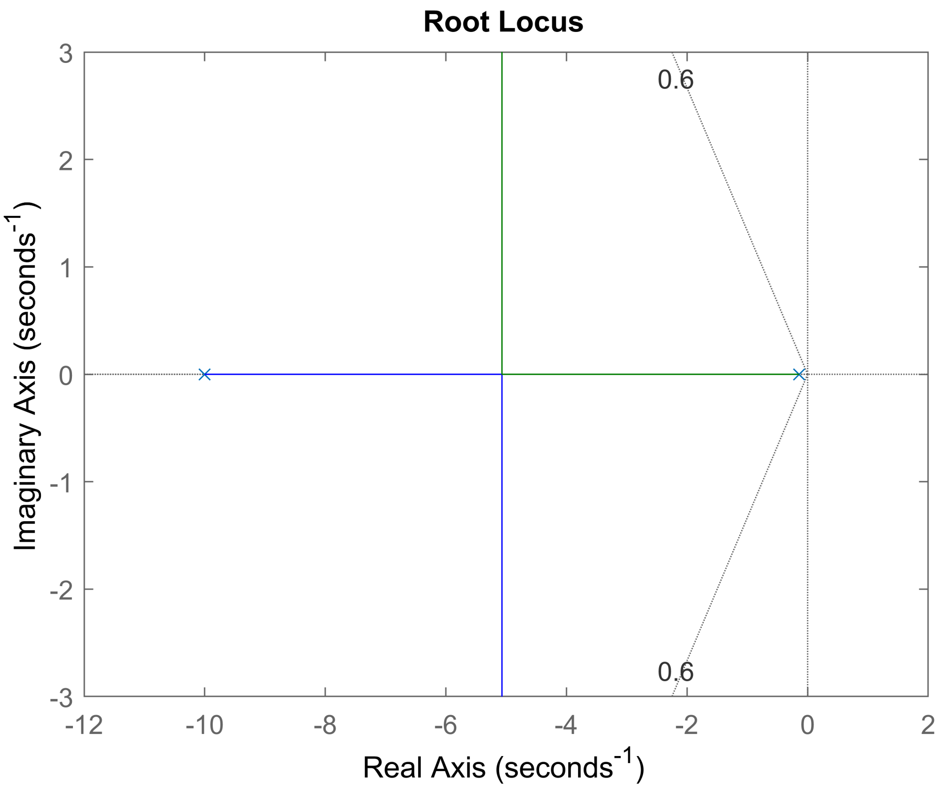
$$G(s) = \frac{3.6}{(s+0.14)(0.1s+1)}$$

$$\mathrm{Gc}(s) = K\frac{p}{z}\frac{s+z}{s+p}$$

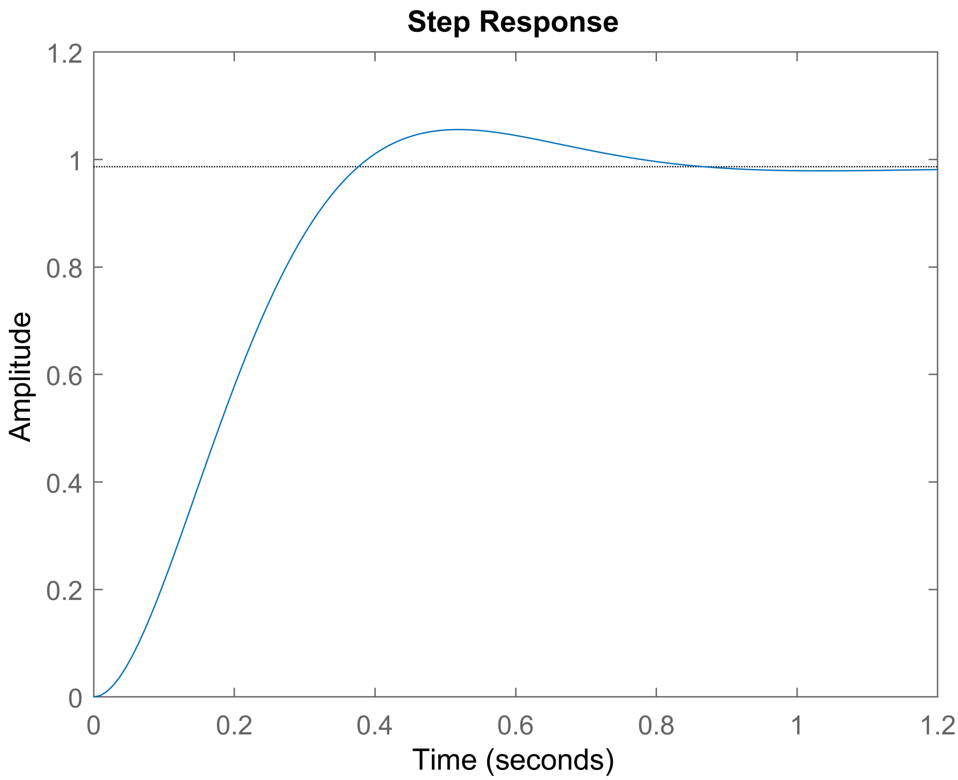
```
clc;clear;close all;

a = [3.6]; b = conv([1 0.14], [0.1 1]);
G = tf(a, b);

zeta = 0.6;
PO = 100*exp(-zeta*pi/(1-zeta^2)^0.5)
```

```
PO = 9.4780
```

```
figure(1); rlocus(G); sgrid(zeta,10^4);
```




```
K = 1.7;  % determined by the requirement on P.M.
my_alpha = K;  %  determined by the requirement on error constant
z = 0.1; p = z/my_alpha;

Gc = tf([1 z],[1 p]); Gc = K*Gc;
L = G*Gc;

T = feedback(L,1);
figure(2); step(T);
```




```
S = stepinfo(T)
```

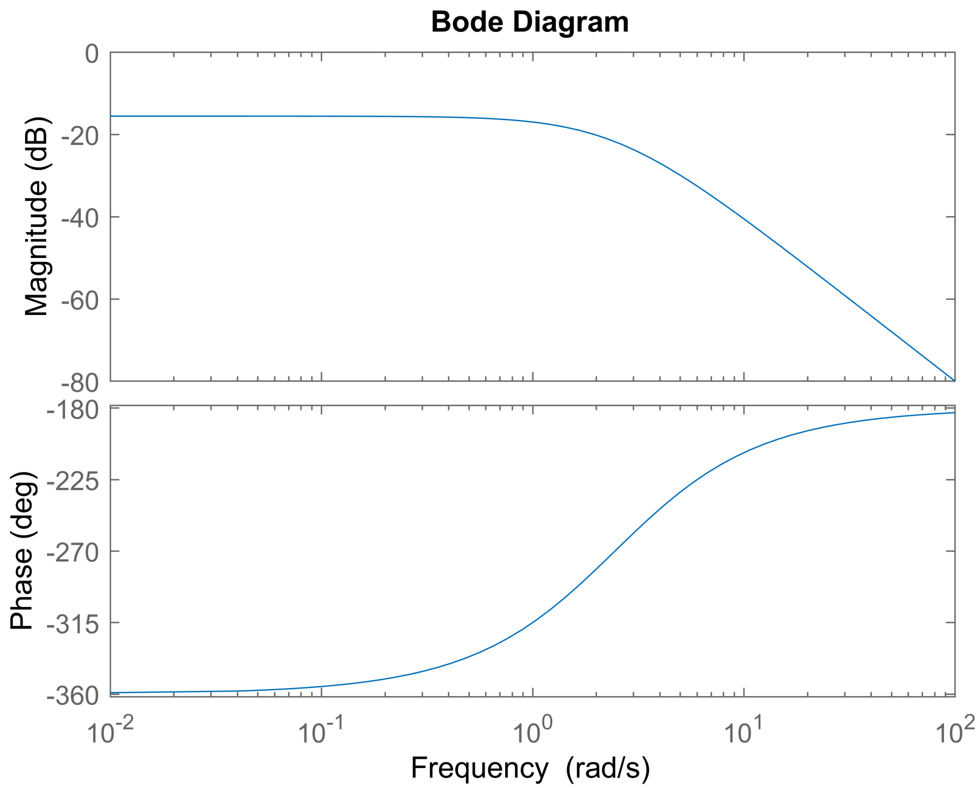
```
S = struct with fields:
        RiseTime: 0.2492
    SettlingTime: 0.7494
     SettlingMin: 0.9008
     SettlingMax: 1.0559
       Overshoot: 7.0110
      Undershoot: 0
            Peak: 1.0559
        PeakTime: 0.5198
```

Q4

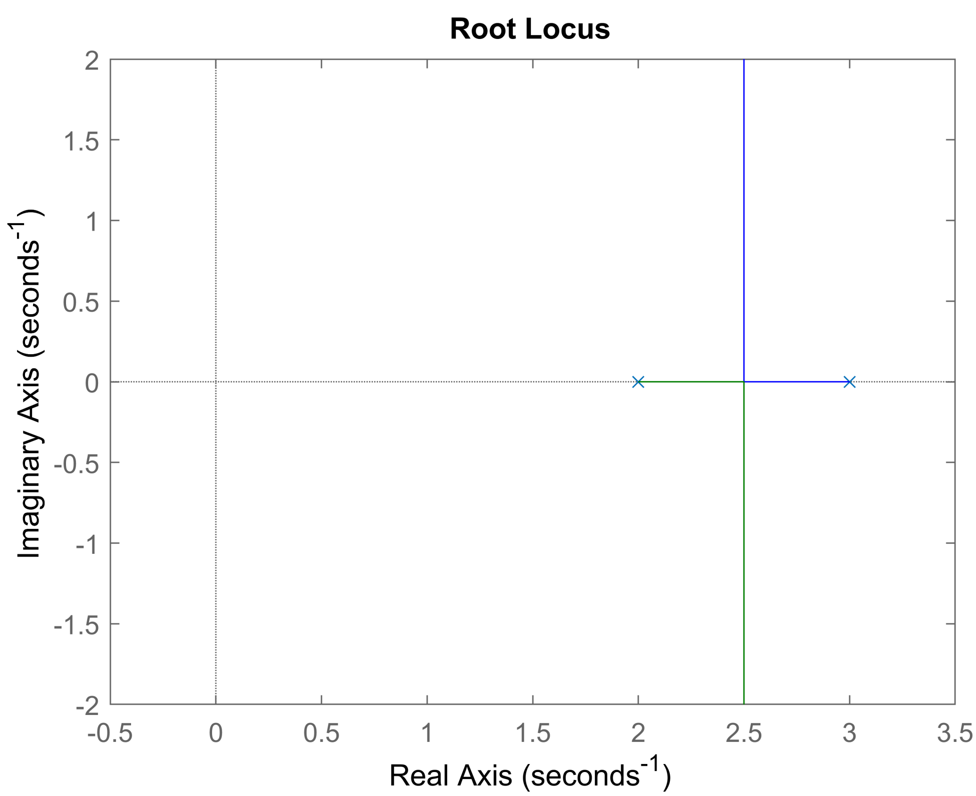
$$G(s) = \frac{1}{(s-3)(s-2)}$$

$$\mathrm{Gc}(s) = K\,\frac{s+z}{s+p}, \mathrm{Gp}(s) = \frac{s+z}{z}$$

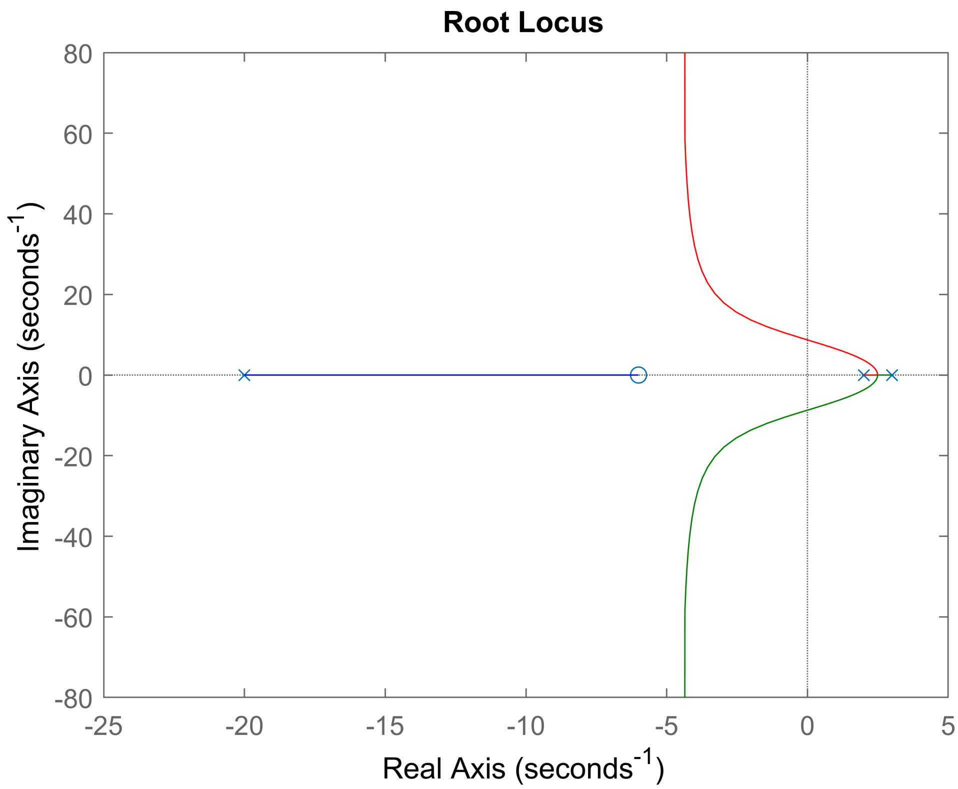
```
clc;clear;close all;

a = [1]; b = conv([1 -3], [1, -2]);
G = tf(a, b);
figure(1); bode(G);
```



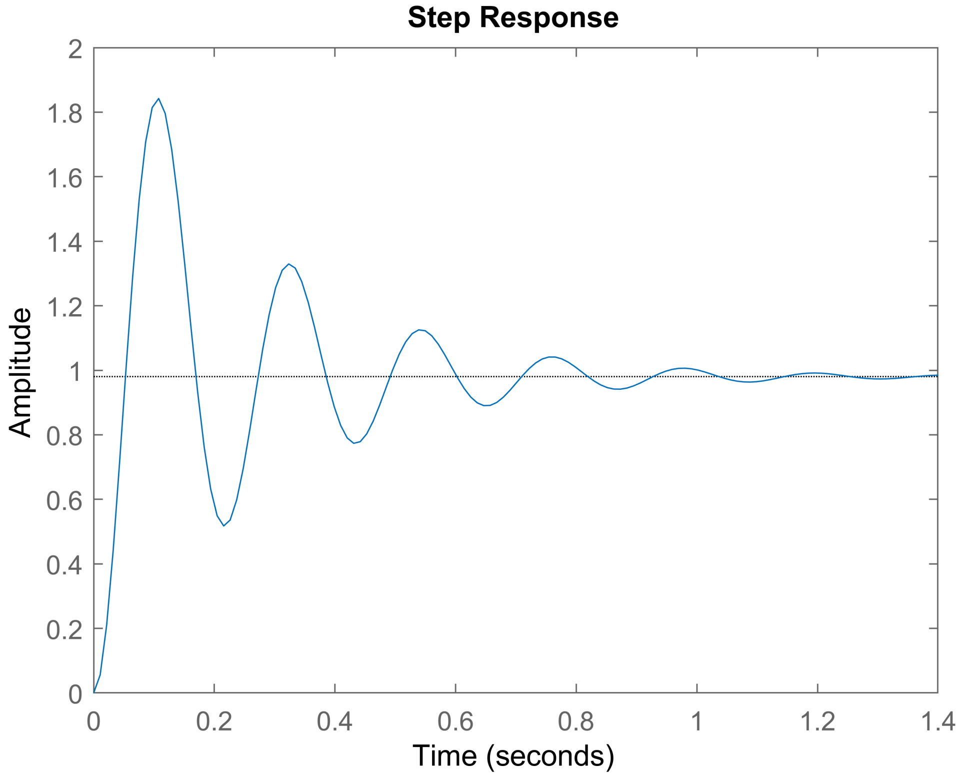

```
figure(2); rlocus(G);
```



```
z = 6; p = 20; K = 1000;
Gc = K*tf([1 z], [1 p]);

% 4a
L1 = G*Gc;
figure(3); rlocus(L1);
```



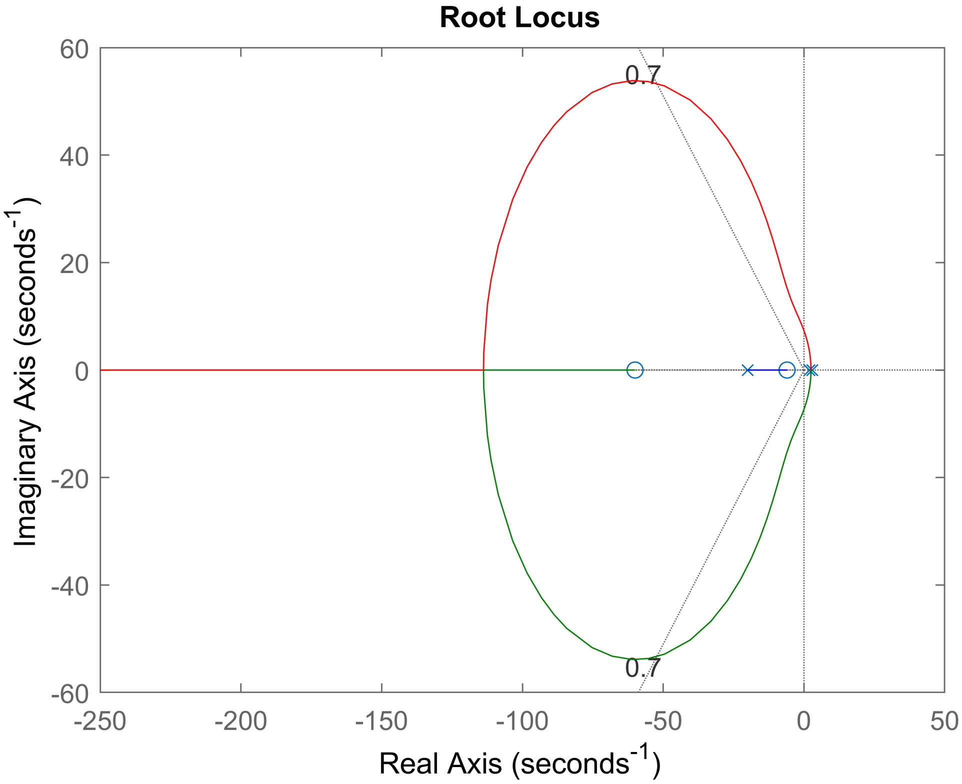

```
T1 = feedback(L1,1);
figure(4); step(T1);
```



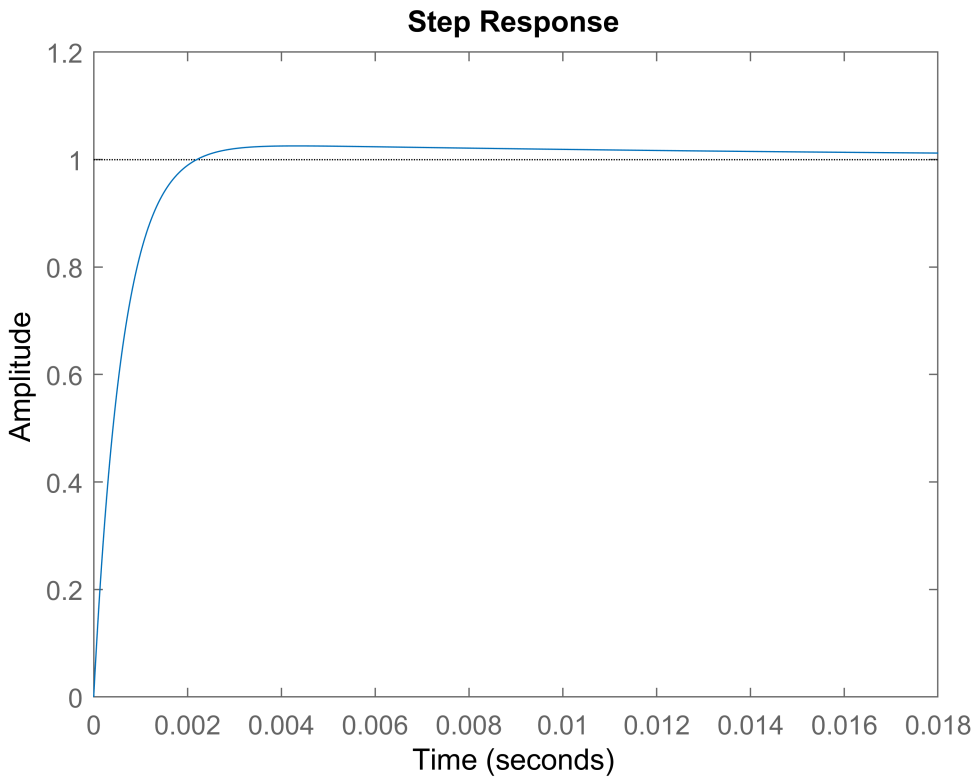

```
S1 = stepinfo(T1)
```

```
S1 = struct with fields:
        RiseTime: 0.0354
    SettlingTime: 1.0028
     SettlingMin: 0.5175
     SettlingMax: 1.8425
       Overshoot: 87.9332
      Undershoot: 0
            Peak: 1.8425
        PeakTime: 0.1078
```

```
% 4b
zeta = 0.7;
PO = 100*exp(-zeta*pi/(1-zeta^2)^0.5);
zp = 60; Kp = 100;
Gp = Kp*tf([1 zp], [zp]);
L = L1*Gp;
figure(5); rlocus(L); sgrid(zeta,10^4);
```




```
T = feedback(L,1);
figure(6); step(T);
```




```
S = stepinfo(T)
```

```
S = struct with fields:
        RiseTime: 0.0012
    SettlingTime: 0.0093
     SettlingMin: 0.9014
     SettlingMax: 1.0254
       Overshoot: 2.5613
      Undershoot: 0
            Peak: 1.0254
        PeakTime: 0.0043
```

Q5

$$G(s) = \frac{1}{(s+0.1)(s+4)}$$

$$\mathrm{Gc}(s) = K\ \frac{s+z}{s+p}, K_p = \lim_{s \longrightarrow 0} \mathrm{sGc}(s)G(s) = 30$$

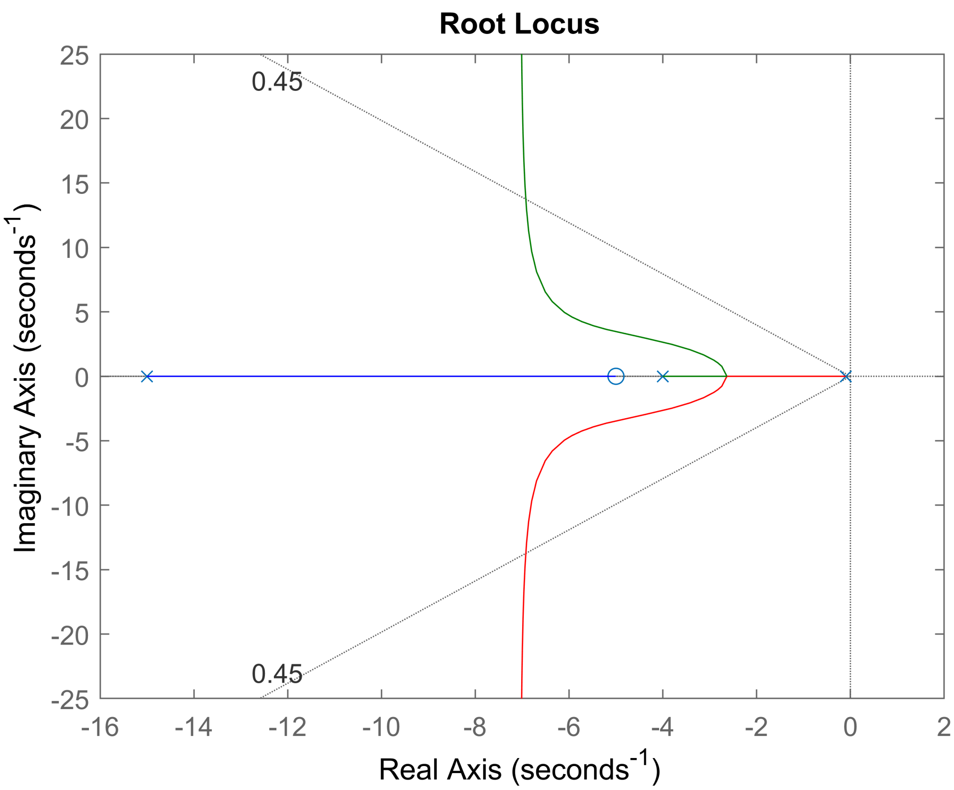
$$\mathrm{Gp}(s) = \frac{s+z}{z}, K_p = \lim_{s \longrightarrow 0} \mathrm{sGc}(s)\mathrm{Gp}(s)G(s) = 30$$

```
clc;clear;close all;

a = [1]; b = conv([1 0.1], [1 4]);
G = tf(a, b);
```

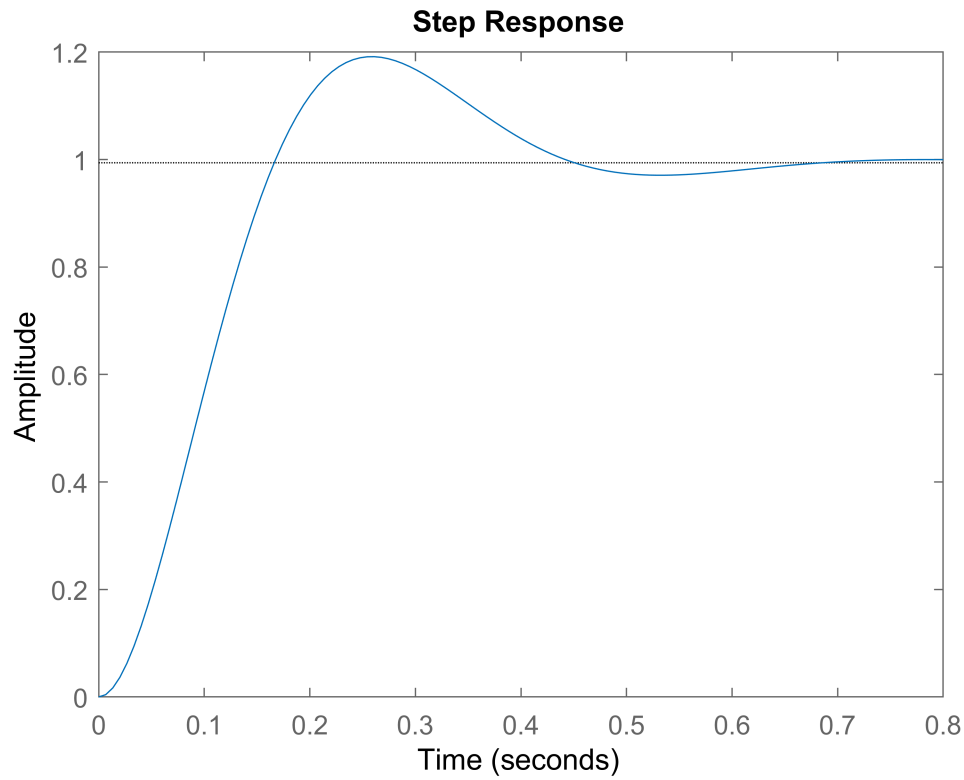
```
z = 5; p = 15; K = 200;
Gc = K*tf([1 z], [1 p]);

zeta1 = 0.45;
PO1 = 100*exp(-zeta1*pi/(1-zeta1^2)^0.5)
```

```
PO1 = 20.5346
```

```
L1 = Gc*G;
figure(1); rlocus(L1); sgrid(zeta1,10^4); % --> Get K = 200
```




```
T1 = feedback(L1,1);
figure(2); step(T1);
```



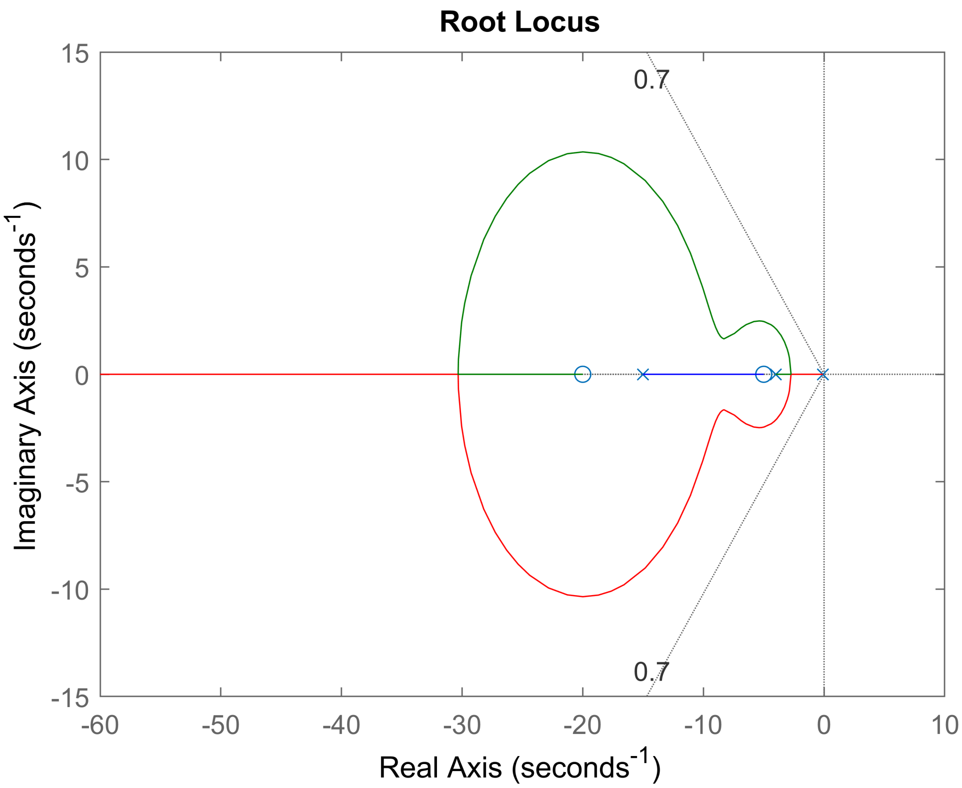

```
S1 = stepinfo(T1)
```

```
S1 = struct with fields:
        RiseTime: 0.1130
    SettlingTime: 0.5719
     SettlingMin: 0.9317
     SettlingMax: 1.1915
       Overshoot: 19.8636
      Undershoot: 0
            Peak: 1.1915
        PeakTime: 0.2610
```

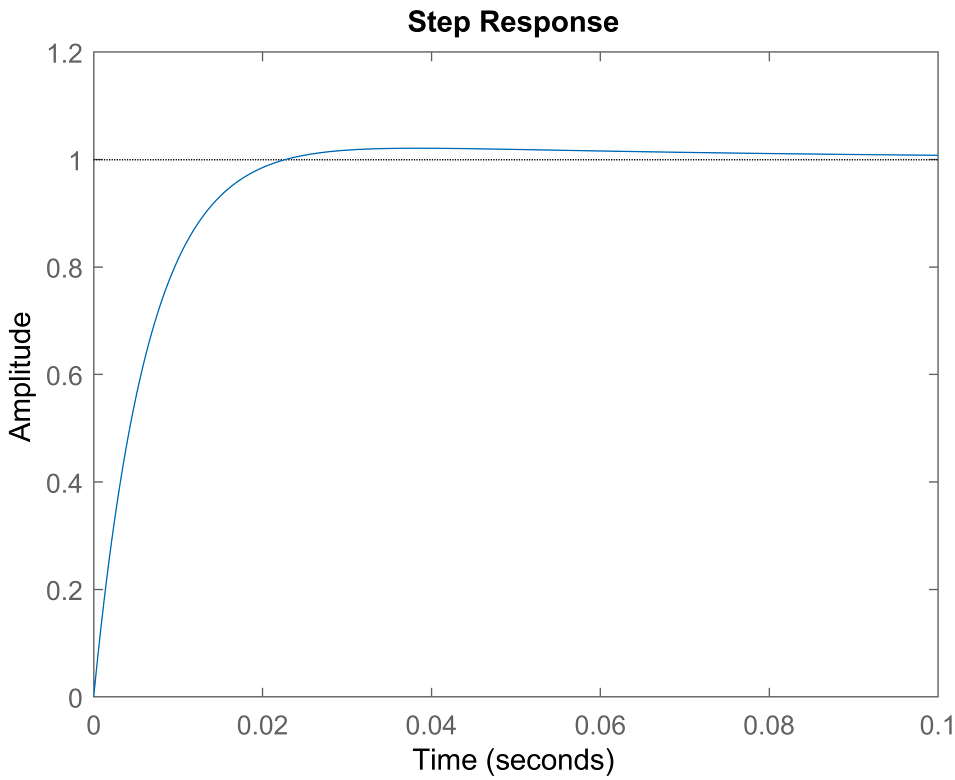
```
% add prefilter
zeta = 0.7;
PO = 100*exp(-zeta*pi/(1-zeta^2)^0.5)
```

```
PO = 4.5988
```

```
zp = 20; Kp = 16;
Gp = Kp*tf([1 zp], [zp]);

L = L1*Gp;
figure(3); rlocus(L); sgrid(zeta,10^4); % --> Get Kp = 16
```




```
T = feedback(L,1);
figure(4); step(T);
```




```
S = stepinfo(T)
```

```
S = struct with fields:
        RiseTime: 0.0125
    SettlingTime: 0.0472
     SettlingMin: 0.9004
     SettlingMax: 1.0211
       Overshoot: 2.1473
      Undershoot: 0
            Peak: 1.0211
        PeakTime: 0.0385
```

```
function Gc = Bode_lag(K,omega_c,attenuation_db)
    z = omega_c/10;
    alpha = 10^(attenuation_db/20);
    p = z/alpha;
    num = K*p*[1 z];
    den = z*[1 p];
    Gc = tf(num,den);
end

function [Gc] = Bode_lead(K, phi_m, wm)
    phi = phi_m*pi/180;
    a = (1+sin(phi))/(1-sin(phi));
    p = wm*(a^0.5); z = p/a;
    Gc = tf((K*(p/z)*[1 z]), [1 p]);
```

```
end
```